Ninth Edition

Automatic Control Systems

Farid Golnaraghi • Benjamin C. Kuo

Laplace Transform Table

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	$t^n(n = \text{positive integer})$
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{\left(s+\alpha\right)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ ($n = \text{positive integer}$)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})(\alpha\neq\beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta - \alpha} (\beta e^{-\beta t} - \alpha e^{-\alpha t}) (\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha t e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t - 1 + e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2} \left[t - \frac{2}{\alpha} + \left(t + \frac{2}{\alpha} \right) e^{-\alpha t} \right]$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{\omega_n}{s^2 + \omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2 + \omega_n^2}$	$\cos \omega_n t$

Laplace Transform Table (cont.)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1 - \cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n \sqrt{\alpha^2 + \omega_n^2} \sin(\omega_n t + \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s+\alpha)\left(s^2+\omega_n^2\right)}$	$\frac{\omega_n}{\alpha^2 + \omega_n^2} e^{-\alpha t} + \frac{1}{\sqrt{\alpha^2 + \omega_n^2}} \sin(\omega_n t - \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_ns+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t (\zeta<1)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1 - \zeta^2} t + \theta\right)$ where $\theta = \cos^{-1} \zeta$ ($\zeta < 1$)
$\frac{s\omega_n^2}{s^2+2\xi\omega_ns+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}t-\theta\right)$ where $\theta = \cos^{-1}\zeta$ ($\zeta < 1$)
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_ns+\omega_n^2}$	$\omega_n \sqrt{\frac{\alpha^2 - 2\alpha\zeta\omega_n + \omega_n^2}{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1 - \zeta^2} t + \theta\right)$ where $\theta = \tan^{-1} \frac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta\omega_n} (\zeta < 1)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n \sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1 - \zeta^2} t + \theta\right)$ where $\theta = \cos^{-1}(2\zeta^2 - 1)$ ($\zeta < 1$)

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ISBN-13 978-0470-04896-2 Printed in the United States of America 10 9 8 7 6 5 4 3 2 1 To my wife, Mitra, and to Sophia and Carmen, the joys of my life. —M. Farid Golnaraghi This is the ninth edition of the text but the first with Farid Golnaraghi as the lead author. For this edition, we increased the number of examples, added MATLAB^(R-1) toolboxes, and enhanced the MATLAB GUI software, **ACSYS**. We added more computer-aided tools for students and teachers. The prepublication manuscript was reviewed by many professors, and most of the relevant suggestions have been adopted. In this edition, Chapters 1 through 4 are organized to contain all background material, while Chapters 5 through 10 contain material directly related to the subject of control.

In this edition, the following materials have been moved into appendices on this book's Web site at www.wiley.com/college/golnaraghi.

Appendix A: Elementary Matrix Theory and Algebra

Appendix B: Difference Equations

Appendix C: Laplace Transform Table

Appendix D: z-Transform Table

Appendix E: Properties and Construction of the Root Loci

Appendix F: General Nyquist Criterion

Appendix G: ACSYS 2008: Description of the Software

Appendix H: Discrete-Data Control Systems

In addition, the Web site contains the MATLAB files for ACSYS, which are software tools for solving control-system problems, and PowerPoint files for the illustrations in the text.

The following paragraphs are aimed at three groups: professors who have adopted the book or who we hope will select it as their text; practicing engineers looking for answers to solve their day-to-day design problems; and, finally, students who are going to live with the book because it has been assigned for the control-systems course they are taking.

To the Professor: The material assembled in this book is an outgrowth of senior-level control-system courses taught by the authors at their universities throughout their teaching careers. The first eight editions have been adopted by hundreds of universities in the United States and around the world and have been translated into at least six languages. Practically all the design topics presented in the eighth edition have been retained.

This text contains not only conventional MATLAB toolboxes, where students can learn MATLAB and utilize their programming skills, but also a graphical MATLAB-based software, ACSYS. The ACSYS software added to this edition is very different from the software accompanying any other control book. Here, through extensive use of MATLAB GUI programming, we have created software that is easy to use. As a result, students will need to focus only on learning control problems, not programming! We also have added two new applications, SIMLab and Virtual Lab, through which students work on realistic problems and conduct speed and position control labs in a software environment. In SIMLab, students have access to the system parameters and can alter them (as in any simulation). In Virtual Lab, we have introduced a black-box approach in which the students

¹ MATLAB^R is a registered trademark of The MathWorks, Inc.

have no access to the plant parameters and have to use some sort of system identification technique to find them. Through Virtual Lab we have essentially provided students with a realistic online lab with all the problems they would encounter in a real speed- or position-control lab—for example, amplifier saturation, noise, and nonlinearity. We welcome your ideas for the future editions of this book.

Finally, a sample section-by-section for a one-semester course is given in the *Instructor's Manual*, which is available from the publisher to qualified instructors. The *Manual* also contains detailed solutions to all the problems in the book.

To Practicing Engineers: This book was written with the readers in mind and is very suitable for self-study. Our objective was to treat subjects clearly and thoroughly. The book does not use the theorem-proof-Q.E.D. style and is without heavy mathematics. The authors have consulted extensively for wide sectors of the industry for many years and have participated in solving numerous control-systems problems, from aerospace systems to industrial controls, automotive controls, and control of computer peripherals. Although it is difficult to adopt all the details and realism of practical problems in a textbook at this level, some examples and problems reflect simplified versions of real-life systems.

To Students: You have had it now that you have signed up for this course and your professor has assigned this book! You had no say about the choice, though you can form and express your opinion on the book after reading it. Worse yet, one of the reasons that your professor made the selection is because he or she intends to make you work hard. But please don't misunderstand us: what we really mean is that, though this is an easy book to study (in our opinion), it is a no-nonsense book. It doesn't have cartoons or nice-looking photographs to amuse you. From here on, it is all business and hard work. You should have had the prerequisites on subjects found in a typical linear-systems course, such as how to solve linear ordinary differential equations, Laplace transform and applications, and timeresponse and frequency-domain analysis of linear systems. In this book you will not find too much new mathematics to which you have not been exposed before. What is interesting and challenging is that you are going to learn how to apply some of the mathematics that you have acquired during the past two or three years of study in college. In case you need to review some of the mathematical foundations, you can find them in the appendices on this book's Web site. The Web site also contains lots of other goodies, including the ACSYS software, which is GUI software that uses MATLAB-based programs for solving linear control systems problems. You will also find the Simulink^(R-2)-based SIMLab and Virtual Lab, which will help you to gain understanding of real-world control systems.

This book has numerous illustrative examples. Some of these are deliberately simple for the purpose of illustrating new ideas and subject matter. Some examples are more elaborate, in order to bring the practical world closer to you. Furthermore, the objective of this book is to present a complex subject in a clear and thorough way. One of the important learning strategies for you as a student is not to rely strictly on the textbook assigned. When studying a certain subject, go to the library and check out a few similar texts to see how other authors treat the same subject. You may gain new perspectives on the subject and discover that one author may treat the material with more care and thoroughness than the others. Do not be distracted by written-down coverage with oversimplified examples. The minute you step into the real world, you will face the design of control systems with nonlinearities and/or time-varying elements as well as orders that can boggle your mind. It

²Simulink^³ is a registered trademark of The MathWorks, Inc.

may be discouraging to tell you now that strictly linear and first-order systems do not exist in the real world.

Some advanced engineering students in college do not believe that the material they learn in the classroom is ever going to be applied directly in industry. Some of our students come back from field and interview trips totally surprised to find that the material they learned in courses on control systems is actually being used in industry today. They are surprised to find that this book is also a popular reference for practicing engineers. Unfortunately, these fact-finding, eye-opening, and self-motivating trips usually occur near the end of their college days, which is often too late for students to get motivated.

There are many learning aids available to you: the MATLAB-based ACSYS software will assist you in solving all kinds of control-systems problems. The SIMLab and Virtual Lab software can be used for simulation of virtual experimental systems. These are all found on the Web site. In addition, the Review Questions and Summaries at the end of each chapter should be useful to you. Also on the Web site, you will find the errata and other supplemental material.

We hope that you will enjoy this book. It will represent another major textbook acquisition (investment) in your college career. Our advice to you is not to sell it back to the bookstore at the end of the semester. If you do so but find out later in your professional career that you need to refer to a control systems book, you will have to buy it again at a higher price.

Special Acknowledgments: The authors wish to thank the reviewers for their invaluable comments and suggestions. The prepublication reviews have had a great impact on the revision project. Dr. Earl Foster, Dr. Vahe Caliskan,

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2009

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Introduction

▶ 1-1 INTRODUCTION

The main objectives of this chapter are:

- 1. To define a control system.
- 2. To explain why control systems are important.
- 3. To introduce the basic components of a control system.
- 4. To give some examples of control-system applications.
- 5. To explain why feedback is incorporated into most control systems.
- 6. To introduce types of control systems.

One of the most commonly asked questions by a novice on a control system is: What is a control system? To answer the question, we can say that in our daily lives there are numerous "objectives" that need to be accomplished. For instance, in the domestic domain, we need to regulate the temperature and humidity of homes and buildings for comfortable living. For transportation, we need to control the automobile and airplane to go from one point to another accurately and safely. Industrially, manufacturing processes contain numerous objectives for products that will satisfy the precision and costeffectiveness requirements. A human being is capable of performing a wide range of tasks, including decision making. Some of these tasks, such as picking up objects and walking from one point to another, are commonly carried out in a routine fashion. Under certain conditions, some of these tasks are to be performed in the best possible way. For instance, an athlete running a 100-yard dash has the objective of running that distance in the shortest possible time. A marathon runner, on the other hand, not only must run the distance as quickly as possible, but, in doing so, he or she must control the consumption of energy and devise the best strategy for the race. The means of achieving these "objectives" usually involve the use of control systems that implement certain control strategies.

In recent years, control systems have assumed an increasingly important role in the development and advancement of modern civilization and technology. Practically every aspect of our day-to-day activities is affected by some type of control system. Control systems are found in abundance in all sectors of industry, such as quality control of manufactured products, automatic assembly lines, machine-tool control, space technology and weapon systems, computer control, transportation systems, power systems, robotics, Micro-Electro-Mechanical Systems (MEMS), nanotechnology, and many others. Even the control of inventory and social and economic systems may be approached from the theory of automatic control.

 Control systems are in abundance in modern civilization.



1-1-1 Basic Components of a Control System

The basic ingredients of a control system can be described by:

- 1. Objectives of control.
- 2. Control-system components.
- 3. Results or outputs.

The basic relationship among these three components is illustrated in Fig. 1-1. In more technical terms, the **objectives** can be identified with **inputs**, or **actuating signals**, u, and the results are also called **outputs**, or **controlled variables**, y. In general, the objective of the control system is to control the outputs in some prescribed manner by the inputs through the elements of the control system.

1-1-2 Examples of Control-System Applications

Intelligent Systems

Applications of control systems have significantly increased through the development of new materials, which provide unique opportunities for highly efficient actuation and sensing, thereby reducing energy losses and environmental impacts. State-of-the-art actuators and sensors may be implemented in virtually any system, including biological propulsion; locomotion; robotics; material handling; biomedical, surgical, and endoscopic; aeronautics; marine; and the defense and space industries. Potential applications of control of these systems may benefit the following areas:

- Machine tools. Improve precision and increase productivity by controlling chatter.
- Flexible robotics. Enable faster motion with greater accuracy.
- **Photolithography.** Enable the manufacture of smaller microelectronic circuits by controlling vibration in the photolithography circuit-printing process.
- Biomechanical and biomedical. Artificial muscles, drug delivery systems, and other assistive technologies.
- Process control. For example, on/off shape control of solar reflectors or aerodynamic surfaces.

Control in Virtual Prototyping and Hardware in the Loop

The concept of virtual prototyping has become a widely used phenomenon in the automotive, aerospace, defense, and space industries. In all these areas, pressure to cut costs has forced manufacturers to design and test an entire system in a computer environment before a physical prototype is made. Design tools such as **MATLAB** and **Simulink** enable companies to design and test controllers for different components (e.g., suspension, ABS, steering, engines, flight control mechanisms, landing gear, and specialized devices) within the system and examine the behavior of the control system on the virtual prototype in real time. This allows the designers to change or adjust controller parameters online before the actual hardware is developed. Hardware in the loop terminology is a new approach of testing individual components by attaching them to the virtual and controller prototypes. Here the physical controller hardware is interfaced with the computer and replaces its mathematical model within the computer!

Smart Transportation Systems

The automobile and its evolution in the last two centuries is arguably the most transformative invention of man. Over years innovations have made cars faster, stronger, and aesthetically appealing. We have grown to desire cars that are "intelligent" and provide maximum levels of comfort, safety, and fuel efficiency. Examples of intelligent systems in cars include climate control, cruise control, anti-lock brake systems (ABSs), active suspensions that reduce vehicle vibration over rough terrain or improve stability, air springs that self-level the vehicle in high-G turns (in addition to providing a better ride), integrated vehicle dynamics that provide yaw control when the vehicle is either over- or understeering (by selectively activating the brakes to regain vehicle control), traction control systems to prevent spinning of wheels during acceleration, and active sway bars to provide "controlled" rolling of the vehicle. The following are a few examples.

Drive-by-wire and Driver Assist Systems The new generations of intelligent vehicles will be able to understand the driving environment, know their whereabouts, monitor their health, understand the road signs, and monitor driver performance, even overriding drivers to avoid catastrophic accidents. These tasks require significant overhaul of current designs. Drive-by-wire technology replaces the traditional mechanical and hydraulic systems with electronics and control systems, using electromechanical actuators and human-machine interfaces such as pedal and steering feel emulators-otherwise known as haptic systems. Hence, the traditional components-such as the steering column, intermediate shafts, pumps, hoses, fluids, belts, coolers, brake boosters, and master cylinders-are eliminated from the vehicle. Haptic interfaces that can offer adequate transparency to the driver while maintaining safety and stability of the system. Removing the bulky mechanical steering wheel column and the rest of the steering system has clear advantages in terms of mass reduction and safety in modern vehicles, along with improved ergonomics as a result of creating more driver space. Replacing the steering wheel with a haptic device that the driver controls through the sense of touch would be useful in this regard. The haptic device would produce the same sense to the driver as the mechanical steering wheel but with improvements in cost, safety, and fuel consumption as a result of eliminating the bulky mechanical system.

Driver assist systems help drivers to avoid or mitigate an accident by sensing the nature and significance of the danger. Depending on the significance and timing of the threat, these on-board safety systems will initially alert the driver as early as possible to an impending danger. Then, they will actively assist or, ultimately, intervene in order to avert the accident or mitigate its consequences. Provisions for automatic over-ride features when the driver loses control due to fatigue or lack of attention will be an important part of the system. In such systems, the so-called advanced vehicle control system monitors the longitudinal and lateral control, and by interacting with a central management unit, it will be ready to take control of the vehicle whenever the need arises. The system can be readily integrated with sensor networks that monitor every aspect of the conditions on the road and are prepared to take appropriate action in a safe manner.

Integration and Utilization of Advanced Hybrid Powertrains Hybrid technologies offer improved fuel consumption while enhancing driving experience. Utilizing new energy storage and conversion technologies and integrating them with powertrains would be prime objectives of this research activity. Such technologies must be compatible with current platforms and must enhance, rather than compromise, vehicle function. Sample applications would include developing plug-in hybrid technology, which would enhance the vehicle cruising distance based on using battery power alone, and utilizing sustainable energy resources, such as solar and wind power, to charge the batteries. The smart plug-in vehicle can be a part of an integrated smart home and grid energy system of the future, which would utilize smart energy metering devices for optimal use of grid energy by avoiding peak energy consumption hours.

High Performance Real-time Control, Health Monitoring, and Diagnosis Modern vehicles utilize an increasing number of sensors, actuators, and networked embedded computers. The need for high performance computing would increase with the introduction of such revolutionary features as drive-by-wire systems into modern vehicles. The tremendous computational burden of processing sensory data into appropriate control and monitoring signals and diagnostic information creates challenges in the design of embedded computing technology. Towards this end, a related challenge is to incorporate sophisticated computational techniques that control, monitor, and diagnose complex automotive systems while meeting requirements such as low power consumption and cost effectiveness.

The following represent more traditional applications of control that have become part of our daily lives.

Steering Control of an Automobile

As a simple example of the control system, as shown in Fig. 1-1, consider the steering control of an automobile. The direction of the two front wheels can be regarded as the controlled variable, or the output, y; the direction of the steering wheel is the actuating signal, or the input, u. The control system, or process in this case, is composed of the steering mechanism and the dynamics of the entire automobile. However, if the objective is to control the speed of the automobile, then the amount of pressure exerted on the accelerator is the actuating signal, and the vehicle speed is the controlled variable. As a whole, we can regard the simplified automobile control system as one with two inputs (steering and accelerator) and two outputs (heading and speed). In this case, the two controls are coupled. Systems with more than one input and one output are called **multivariable systems**.

Idle-Speed Control of an Automobile

As another example of a control system, we consider the idle-speed control of an automobile engine. The objective of such a control system is to maintain the engine idle speed at a relatively low value (for fuel economy) regardless of the applied engine loads (e.g., transmission, power steering, air conditioning). Without the idle-speed control, any sudden engine-load application would cause a drop in engine speed that might cause the engine to stall. Thus the main objectives of the idle-speed control system are (1) to eliminate or minimize the speed droop when engine loading is applied and (2) to maintain the engine idle speed at a desired value. Fig. 1-2 shows the block diagram of the idle-speed control system from the standpoint of inputs-system-outputs. In this case, the throttle angle α and the load torque T_L (due to the application of air conditioning, power steering, transmission, or power brakes, etc.) are the inputs, and the engine speed ω is the output. The engine is the controlled process of the system.

Sun-Tracking Control of Solar Collectors

To achieve the goal of developing economically feasible non-fossil-fuel electrical power, the U.S. government has sponsored many organizations in research and development of solar power conversion methods, including the solar-cell conversion techniques. In most of





Figure 1-3 Solar collector field.

these systems, the need for high efficiencies dictates the use of devices for sun tracking. Fig. 1-3 shows a solar collector field. Fig. 1-4 shows a conceptual method of efficient water extraction using solar power. During the hours of daylight, the solar collector would produce electricity to pump water from the underground water table to a reservoir (perhaps on a nearby mountain or hill), and in the early morning hours, the water would be released into the irrigation system.

One of the most important features of the solar collector is that the collector dish must track the sun accurately. Therefore, the movement of the collector dish must be controlled by sophisticated control systems. The block diagram of Fig. 1-5 describes the general philosophy of the sun-tracking system together with some of the most important components. The controller ensures that the tracking collector is pointed toward the sun in the morning and sends a "start track" command. The controller constantly calculates the sun's rate for the two axes (azimuth and elevation) of control during the day. The controller uses the sun rate and sun sensor information as inputs to generate proper motor commands to slew the collector.

1-1-3 Open-Loop Control Systems (Nonfeedback Systems)

• Open-loop systems are economical but usually inaccurate.

The idle-speed control system illustrated in Fig. 1-2, shown previously, is rather unsophisticated and is called an **open-loop control system**. It is not difficult to see that the system as shown would not satisfactorily fulfill critical performance requirements. For instance, if the throttle angle α is set at a certain initial value that corresponds to a certain



Figure 1-4 Conceptual method of efficient water extraction using solar power.



Figure 1-5 Important components of the sun-tracking control system.

engine speed, then when a load torque T_L is applied, there is no way to prevent a drop in the engine speed. The only way to make the system work is to have a means of adjusting α in response to a change in the load torque in order to maintain ω at the desired level. The conventional electric washing machine is another example of an open-loop control system because, typically, the amount of machine wash time is entirely determined by the judgment and estimation of the human operator.

The elements of an open-loop control system can usually be divided into two parts: the **controller** and the **controlled process**, as shown by the block diagram of Fig. 1-6. An input signal, or command, r, is applied to the controller, whose output acts as the actuating signal u; the actuating signal then controls the controlled process so that the controlled variable y will perform according to some prescribed standards. In simple cases, the controller can be



Figure 1-6 Elements of an open-loop control system.

an amplifier, a mechanical linkage, a filter, or other control elements, depending on the nature of the system. In more sophisticated cases, the controller can be a computer such as a microprocessor. Because of the simplicity and economy of open-loop control systems, we find this type of system in many noncritical applications.

1-1-4 Closed-Loop Control Systems (Feedback Control Systems)

What is missing in the open-loop control system for more accurate and more adaptive control is a link or feedback from the output to the input of the system. To obtain more accurate control, the controlled signal y should be fed back and compared with the reference input, and an actuating signal proportional to the difference of the input and the output must be sent through the system to correct the error. A system with one or more feedback paths such as that just described is called a **closed-loop system**.

• Closed-loop systems have many advantages over open-loop systems.

A closed-loop idle-speed control system is shown in Fig. 1-7. The reference input ω_r sets the desired idling speed. The engine speed at idle should agree with the reference value ω_r , and any difference such as the load torque T_L is sensed by the speed transducer and the error detector. The controller will operate on the difference and provide a signal to adjust the throttle angle α to correct the error. Fig. 1-8 compares the typical performances of open-loop and closed-loop idle-speed control systems. In Fig. 1-8(a), the idle speed of the open-loop system will drop and settle at a lower value after a load torque is applied. In Fig. 1-8 (b), the idle speed of the closed-loop system is shown to recover quickly to the preset value after the application of T_L .

The objective of the idle-speed control system illustrated, also known as a **regulator** system, is to maintain the system output at a prescribed level.



Figure 1-7 Block diagram of a closed-loop idle-speed control system.





1-2 WHAT IS FEEDBACK, AND WHAT ARE ITS EFFECTS?

The motivation for using feedback, as illustrated by the examples in Section 1-1, is somewhat oversimplified. In these examples, feedback is used to reduce the error between the reference input and the system output. However, the significance of the effects of feedback in control systems is more complex than is demonstrated by these simple examples. The reduction of system error is merely one of the many important effects that feedback may have upon a system. We show in the following sections that feedback also has effects on such system performance characteristics as stability, bandwidth, overall gain, impedance, and sensitivity.

· Feedback exists whenever there is a closed sequence of cause-andeffect relationships.

To understand the effects of feedback on a control system, it is essential to examine this phenomenon in a broad sense. When feedback is deliberately introduced for the purpose of control, its existence is easily identified. However, there are numerous situations where a physical system that we recognize as an inherently nonfeedback system turns out to have feedback when it is observed in a certain manner. In general, we can state that whenever a closed sequence of cause-and-effect relationships exists among the variables of a system, feedback is said to exist. This viewpoint will inevitably admit feedback in a large number of systems that ordinarily would be identified as nonfeedback systems. However, control-system theory allows numerous systems, with or without physical feedback, to be studied in a systematic way once the existence of feedback in the sense mentioned previously is established.

We shall now investigate the effects of feedback on the various aspects of system performance. Without the necessary mathematical foundation of linear-system theory, at this point we can rely only on simple static-system notation for our discussion. Let us consider the simple feedback system configuration shown in Fig. 1-9, where r is the input signal; y, the output signal; e, the error; and b, the feedback signal. The parameters G and H may be considered as constant gains. By simple algebraic manipulations, it is simple to show that the input-output relation of the system is

$$M = \frac{y}{r} = \frac{G}{1 + GH} \tag{1-1}$$

Using this basic relationship of the feedback system structure, we can uncover some of the significant effects of feedback.

1-2-1 Effect of Feedback on Overall Gain

· Feedback may increase frequency range but decrease it in another.

As seen from Eq. (1-1), feedback affects the gain G of a nonfeedback system by a factor of the gain of a system in one 1 + GH. The system of Fig. 1-9 is said to have **negative feedback**, because a minus sign is assigned to the feedback signal. The quantity GH may itself include a minus sign, so the general effect of feedback is that it may increase or decrease the gain G. In a practical control system, G and H are functions of frequency, so the magnitude of 1 + GH may be



Figure 1-9 Feedback system.

greater than 1 in one frequency range but less than 1 in another. Therefore, *feedback could increase the gain of system in one frequency range but decrease it in another*.

1-2-2 Effect of Feedback on Stability

• A system is unstable if its output is out of control.

Stability is a notion that describes whether the system will be able to follow the input command, that is, be useful in general. In a nonrigorous manner, a system is said to be unstable if its output is out of control. To investigate the effect of feedback on stability, we can again refer to the expression in Eq. (1-1). If GH = -1, the output of the system is infinite for any finite input, and the system is said to be unstable. Therefore, we may state that feedback can cause a system that is originally stable to become unstable. Certainly, feedback is a double-edged sword; when it is improperly used, it can be harmful. It should be pointed out, however, that we are only dealing with the static case here, and, in general, GH = -1 is not the only condition for instability. The subject of system stability will be treated formally in Chapters 2, 5, 7, and 8.

It can be demonstrated that one of the advantages of incorporating feedback is that it can stabilize an unstable system. Let us assume that the feedback system in Fig. 1-9 is unstable because GH = -1. If we introduce another feedback loop through a negative feedback gain of *F*, as shown in Fig. 1-10, the input–output relation of the overall system is

$$\frac{y}{r} = \frac{G}{1 + GH + GF} \tag{1-2}$$

It is apparent that although the properties of *G* and *H* are such that the inner-loop feedback system is unstable, because GH = -1, the overall system can be stable by properly selecting the outer-loop feedback gain *F*. In practice, *GH* is a function of frequency, and the stability condition of the closed-loop system depends on the **magnitude** and **phase** of *GH*. The bottom line is that *feedback can improve stability or be harmful to stability if it is not properly applied*.

Sensitivity considerations often are important in the design of control systems. Because all physical elements have properties that change with environment and age, we cannot always consider the parameters of a control system to be completely stationary over the entire operating life of the system. For instance, the winding resistance of an electric motor changes as the temperature of the motor rises during operation. Control systems with electric components may not operate normally when first turned on because



Figure 1-10 Feedback system with two feedback loops.

• Feedback can improve stability or be harmful to stability.

of the still-changing system parameters during warmup. This phenomenon is sometimes called "morning sickness." Most duplicating machines have a warmup period during which time operation is blocked out when first turned on.

In general, a good control system should be very insensitive to parameter variations but sensitive to the input commands. We shall investigate what effect feedback has on sensitivity to parameter variations. Referring to the system in Fig. 1-9, we consider G to be a gain parameter that may vary. The sensitivity of the gain of the overall system M to the variation in G is defined as

$$S_G^M = \frac{\partial M/M}{\partial G/G} = \frac{\text{percentage change in } M}{\text{percentage change in } G}$$
(1-3)

• Note: Feedback can increase or decrease the sensitivity of a system.

where ∂M denotes the incremental change in M due to the incremental change in G, or ∂G . By using Eq. (1-1), the sensitivity function is written

$$S_G^M = \frac{\partial M}{\partial G} \quad \frac{G}{M} = \frac{1}{1+GH} \tag{1-4}$$

This relation shows that if *GH* is a positive constant, the magnitude of the sensitivity function can be made arbitrarily small by increasing *GH*, provided that the system remains stable. It is apparent that, in an open-loop system, the gain of the system will respond in a one-to-one fashion to the variation in *G* (i.e., $S_G^M = 1$). Again, in practice, *GH* is a function of frequency; the magnitude of 1 + GH may be less than unity over some frequency ranges, so feedback could be harmful to the sensitivity to parameter variations in certain cases. In general, the sensitivity of the system gain of a feedback system to parameter variations depends on where the parameter is located. The reader can derive the sensitivity of the system in Fig. 1-9 due to the variation of *H*.

1-2-3 Effect of Feedback on External Disturbance or Noise

All physical systems are subject to some types of extraneous signals or noise during operation. Examples of these signals are thermal-noise voltage in electronic circuits and brush or commutator noise in electric motors. External disturbances, such as wind gusts acting on an antenna, are also quite common in control systems. Therefore, control systems should be designed so that they are insensitive to noise and disturbances and sensitive to input commands.

The effect of feedback on noise and disturbance depends greatly on where these extraneous signals occur in the system. No general conclusions can be reached, but in many situations, *feedback can reduce the effect of noise and disturbance on system performance*. Let us refer to the system shown in Fig. 1-11, in which *r* denotes the command



Figure 1-11 Feedback system with a noise signal.

• Feedback can reduce the effect of noise.

signal and n is the noise signal. In the absence of feedback, that is, H=0, the output y due to n acting alone is

$$y = G_2 n \tag{1-5}$$

With the presence of feedback, the system output due to n acting alone is

$$y = \frac{G_2}{1 + G_1 G_2 H} \quad n \tag{1-6}$$

t Comparing Eq. (1-6) with Eq. (1-5) shows that the noise component in the output of Eq. (1-6) is reduced by the factor $1 + G_1G_2H$ if the latter is greater than unity and the system is kept stable.

In Chapter 9, the feedforward and forward controller configurations are used along with feedback to reduce the effects of disturbance and noise inputs. In general, feedback also has effects on such performance characteristics as bandwidth, impedance, transient response, and frequency response. These effects will be explained as we continue.

▶ 1-3 TYPES OF FEEDBACK CONTROL SYSTEMS

Feedback control systems may be classified in a number of ways, depending upon the purpose of the classification. For instance, according to the method of analysis and design, control systems are classified as **linear** or **nonlinear**, and **time-varying** or **time-invariant**. According to the types of signal found in the system, reference is often made to **continuous-data** or **discrete-data** systems, and **modulated** or **unmodulated** systems. Control systems are often classified according to the main purpose of the system. For instance, a **position-control system** and a **velocity-control system** control the output variables just as the names imply. In Chapter 9, the **type** of control system is defined according to the form of the open-loop transfer function. In general, there are many other ways of identifying control systems according to some special features of the system. It is important to know some of the more common ways of classifying control systems before embarking on the analysis and design of these systems.

1-3-1 Linear versus Nonlinear Control Systems

• Most real-life control systems have nonlinear characteristics to some extent.

This classification is made according to the methods of analysis and design. Strictly speaking, linear systems do not exist in practice, because all physical systems are nonlinear to some extent. Linear feedback control systems are idealized models fabricated by the analyst purely for the simplicity of analysis and design. When the magnitudes of signals in a control system are limited to ranges in which system components exhibit linear characteristics (i.e., the principle of superposition applies), the system is essentially linear. But when the magnitudes of signals are extended beyond the range of the linear operation, depending on the severity of the nonlinearity, the system should no longer be considered linear. For instance, amplifiers used in control systems often exhibit a saturation effect when their input signals become large; the magnetic field of a motor usually has saturation properties. Other common nonlinear effects found in control systems are the backlash or dead play between coupled gear members, nonlinear spring characteristics, nonlinear friction force or torque between moving members, and so on. Quite often, nonlinear characteristics are intentionally introduced in a control system to improve its performance

or provide more effective control. For instance, to achieve minimum-time control, an onoff (bang-bang or relay) type controller is used in many missile or spacecraft control systems. Typically in these systems, jets are mounted on the sides of the vehicle to provide reaction torque for attitude control. These jets are often controlled in a full-on or full-off fashion, so a fixed amount of air is applied from a given jet for a certain time period to control the attitude of the space vehicle.

• There are no general methods for solving a wide class of nonlinear systems.

For linear systems, a wealth of analytical and graphical techniques is available for design and analysis purposes. A majority of the material in this text is devoted to the analysis and design of linear systems. Nonlinear systems, on the other hand, are usually difficult to treat mathematically, and there are no general methods available for solving a wide class of nonlinear systems. It is practical to first design the controller based on the linear-system model by neglecting the nonlinearities of the system. The designed controller is then applied to the nonlinear system model for evaluation or redesign by computer simulation. The Virtual Lab introduced in Chapter 6 is mainly used to model the characteristics of practical systems with realistic physical components.

1-3-2 Time-Invariant versus Time-Varying Systems

When the parameters of a control system are stationary with respect to time during the operation of the system, the system is called a time-invariant system. In practice, most physical systems contain elements that drift or vary with time. For example, the winding resistance of an electric motor will vary when the motor is first being excited and its temperature is rising. Another example of a time-varying system is a guided-missile control system in which the mass of the missile decreases as the fuel on board is being consumed during flight. Although a time-varying system without nonlinearity is still a linear system, the analysis and design of this class of systems are usually much more complex than that of the linear time-invariant systems.

Continuous-Data Control Systems

A continuous-data system is one in which the signals at various parts of the system are all functions of the continuous time variable t. The signals in continuous-data systems may be further classified as ac or dc. Unlike the general definitions of ac and dc signals used in electrical engineering, ac and dc control systems carry special significance in control systems terminology. When one refers to an **ac control system**, it usually means that the signals in the system are *modulated* by some form of modulation scheme. A **dc control system**, on the other hand, simply implies that the signals are *unmodulated*, but they are still ac signals according to the conventional definition. The schematic diagram of a closed-loop dc control system is shown in Fig. 1-12. Typical waveforms of the signals in response to a step-function input are shown in the figure. Typical components of a dc control system are potentiometers, dc amplifiers, dc motors, dc tachometers, and so on.

Figure 1-13 shows the schematic diagram of a typical ac control system that performs essentially the same task as the dc system in Fig. 1-12. In this case, the signals in the system are modulated; that is, the information is transmitted by an ac carrier signal. Notice that the output controlled variable still behaves similarly to that of the dc system. In this case, the modulated signals are demodulated by the low-pass characteristics of the ac motor. Ac control systems are used extensively in aircraft and missile control systems in which noise and disturbance often create problems. By using modulated ac control systems with carrier frequencies of 400 Hz or higher, the system will be less susceptible to low-frequency noise. Typical components of an ac control system are synchros, ac amplifiers, ac motors, gyroscopes, accelerometers, and so on.



Figure 1-12 Schematic diagram of a typical dc closed-loop system.



Figure 1-13 Schematic diagram of a typical ac closed-loop control system.

In practice, not all control systems are strictly of the ac or dc type. A system may incorporate a mixture of ac and dc components, using modulators and demodulators to match the signals at various points in the system.

Discrete-Data Control Systems

Discrete-data control systems differ from the continuous-data systems in that the signals at one or more points of the system are in the form of either a pulse train or a digital code. Usually, discrete-data control systems are subdivided into **sampled-data** and **digital control systems**. Sampled-data control systems refer to a more general class of



Figure 1-14 Block diagram of a sampled-data control system.



Figure 1-15 Digital autopilot system for a guided missile.

discrete-data systems in which the signals are in the form of pulse data. A digital control system refers to the use of a digital computer or controller in the system so that the signals are digitally coded, such as in binary code.

In general, a sampled-data system receives data or information only intermittently at specific instants of time. For example, the error signal in a control system can be supplied only in the form of pulses, in which case the control system receives no information about the error signal during the periods between two consecutive pulses. Strictly, a sampled-data system can also be classified as an ac system, because the signal of the system is pulse modulated.

Figure 1-14 illustrates how a typical sampled-data system operates. A continuous-data input signal r(t) is applied to the system. The error signal e(t) is sampled by a sampling device, the **sampler**, and the output of the sampler is a sequence of pulses. The sampling rate of the sampler may or may not be uniform. There are many advantages to incorporating sampling into a control system. One important advantage is that expensive equipment used in the system may be time-shared among several control channels. Another advantage is that pulse data are usually less susceptible to noise.

Because digital computers provide many advantages in size and flexibility, computer control has become increasingly popular in recent years. Many airborne systems contain digital controllers that can pack thousands of discrete elements into a space no larger than the size of this book. Figure 1-15 shows the basic elements of a digital autopilot for guided-missile control.

• Digital control systems are usually less susceptible to noise.

▶ 1-4 SUMMARY

In this chapter, we introduced some of the basic concepts of what a control system is and what it is supposed to accomplish. The basic components of a control system were described. By demonstrating the effects of feedback in a rudimentary way, the question of why most control systems are closed-loop systems was also clarified. Most important, it was pointed out that feedback is a double-edged sword—it can benefit as well as harm the system to be controlled. This is part of the challenging task of designing a control system, which involves consideration of such performance criteria as stability,

sensitivity, bandwidth, and accuracy. Finally, various types of control systems were categorized according to the system signals, linearity, and control objectives. Several typical control-system examples were given to illustrate the analysis and design of control systems. Most systems encountered in real life are nonlinear and time-varying to some extent. The concentration on the studies of linear systems is due primarily to the availability of unified and simple-to-understand analytical methods in the analysis and design of linear systems.

REVIEW QUESTIONS

- 1. List the advantages and disadvantages of an open-loop system.
- 2. List the advantages and disadvantages of a closed-loop system.
- 3. Give the definitions of ac and dc control systems.
- 4. Give the advantages of a digital control system over a continuous-data control system.
- 5. A closed-loop control system is usually more accurate than an open-loop system. (T) (F)
- 6. Feedback is sometimes used to improve the sensitivity of a control system. (T) (F)
- If an open-loop system is unstable, then applying feedback will always improve its stability.
 Feedback can increase the gain of a system in one frequency range but decrease it in another.
 Nonlinear elements are sometimes intentionally introduced to a control system to improve its performance.
 (T) (F)
- Discrete-data control systems are more susceptible to noise due to the nature of their signals.
 (T) (F)

Answers to these review questions can be found on this book's companion Web site: www.wiley.com/college/golnaraghi.

Mathematical Foundation

The studies of control systems rely to a great extent on applied mathematics. One of the major purposes of control-system studies is to develop a set of analytical tools so that the designer can arrive with reasonably predictable and reliable designs without depending solely on the drudgery of experimentation or extensive computer simulation.

In this chapter, it is assumed that the reader has some level of familiarity with these concepts through earlier courses. Elementary matrix algebra is covered in Appendix A. Because of space limitations, as well as the fact that most subjects are considered as review material for the reader, the treatment of these mathematical subjects is not exhaustive. The reader who wishes to conduct an in-depth study of any of these subjects should refer to books that are devoted to them.

The main objectives of this chapter are:

- 1. To introduce the fundamentals of complex variables.
- 2. To introduce frequency domain analysis and frequency plots.
- 3. To introduce differential equations and state space systems.
- 4. To introduce the fundamentals of Laplace transforms.
- To demonstrate the applications of Laplace transforms to solve linear ordinary differential equations.
- To introduce the concept of transfer functions and how to apply them to the modeling of linear time-invariant systems.
- 7. To discuss stability of linear time-invariant systems and the Routh-Hurwitz criterion.
- 8. To demonstrate the MATLAB tools using case studies.

2-1 COMPLEX-VARIABLE CONCEPT

To understand complex variables, it is wise to start with the concept of complex numbers and their mathematical properties.

2-1-1 Complex Numbers

CHAPTER

A complex number is represented in rectangular form as

$$z = x + jy \tag{2-1}$$

where, $j = \sqrt{-1}$ and (x, y) are real and imaginary coefficients of z respectively. We can treat (x, y) as a point in the **Cartesian** coordinate frame shown in Fig. 2-1. A point in a



Figure 2-1 Complex number *z* representation in rectangular and polar forms.

rectangular coordinate frame may also be defined by a vector R and an angle θ . It is then easy to see that

$$\begin{aligned} x &= R\cos\theta\\ y &= R\sin\theta \end{aligned} \tag{2-2}$$

where,

R =magnitude of z

 θ = phase of z and is measured from the x axis. Right-hand rule convention: positive phase is in counter clockwise direction.

Hence,

$$R = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$
(2-3)

Introducing Eq. (2-2) into Eq. (2-1), we get

$$z = R\cos\theta + jR\sin\theta \tag{2-4}$$

Upon comparison of Taylor series of the terms involved, it is easy to confirm

$$e^{j\theta} = \cos\theta + j\sin\theta \tag{2-5}$$

Eq. (2-5) is also known as the **Euler formula**. As a result, Eq. (2-1) may also be represented in **polar form** as

$$z = R e^{j\theta} = R \angle \theta \tag{2-6}$$

We define the conjugate of the complex number z in Eq. (2-1) as

$$z^* = x - jy \tag{2-7}$$

Or, alternatively,

$$z^* = R\cos\theta - jR\sin\theta = Re^{-j\theta}$$
(2-8)

Note:

$$zz^* = R^2 = x^2 + y^2 \tag{2-9}$$

Table 2-1 shows basic mathematical properties of complex numbers.

and a second sec		
Addition	$\begin{cases} z_1 = x_1 + jy_1 \\ z_2 = x_2 + jy_2 \\ \rightarrow z = (x_1 + x_2) + j(y_1 + y_2) \end{cases}$	$\begin{cases} z_1 = R_1 e^{j\theta_1} \\ z_2 = R_2 e^{j\theta_2} \\ \rightarrow z = R_1 e^{j\theta_1} + R_2 e^{j\theta_2} \end{cases}$
Subtraction	$\begin{cases} z_1 = x_1 + jy_1 \\ z_2 = x_2 + jy_2 \\ \rightarrow z = (x_1 - x_2) + j(y_1 - y_2) \end{cases}$	$\begin{cases} z_1 = R_1 e^{j\theta_1} \\ z_2 = R_2 e^{j\theta_2} \end{cases}$ $\implies z = R_1 e^{j\theta_1} + R_2 - i\theta_2$
Multiplication	$\begin{cases} z_1 = x_1 + jy_1 \\ z_2 = x_2 + jy_2 \\ \rightarrow z = (x_1x_2 - y_1y_2) + j(x_1y_2 + x_2y_1) \\ j^2 = -1 \end{cases}$	$ \begin{array}{l} \begin{array}{l} \begin{array}{c} \begin{array}{c} z = R_1 e^{j \theta_1} \\ z_1 = R_1 e^{j \theta_1} \\ z_2 = R_2 e^{j \theta_2} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} z = (R_1 R_2) e^{j (\theta_1 + \theta_2)} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} $
Division	$\begin{cases} z_1 = x_1 + jy_1 \\ z_2 = x_2 + jy_2 \\ z_1^* = x_1 - jy_1 \\ z_2^* = x_2 - jy_2 \end{cases}$ Complex Conjugate $\rightarrow z = \frac{z_1}{z_2}$	$\begin{cases} z_1 = R_1 e^{j\theta_1} \\ z_2 = R_2 e^{j\theta_2} \\ \begin{cases} z_1^* = R_1 e^{-j\theta_1} \\ z_2^* = R_2 e^{-j\theta_2} \end{cases} \\ \rightarrow z = \left(\frac{R_1}{R_2}\right) e^{j(\theta_1 - \theta_2)} \end{cases}$
	$\rightarrow z = \frac{z_1}{z_2} \frac{z_2^*}{z_2^*} = \frac{(x_1 x_2 + y_1 y_2) + j(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2}$	$\rightarrow z = \left(\frac{R_1}{R_2}\right) / \left(\theta_1 - \theta_2\right)$

 TABLE 2-1
 Basic Properties of Complex Numbers

EXAMPLE 2-1-1 Find j^3 and j^4 .

$$j = \sqrt{-1} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = e^{j\frac{\pi}{2}}$$
$$j^{3} = \sqrt{-1}\sqrt{-1}\sqrt{-1} = -\sqrt{-1} = -j$$
$$j^{3} = e^{j\frac{3\pi}{2}} = e^{-j\frac{\pi}{2}}$$
$$j^{4} = j^{3}j = -j^{2} = 1$$

EXAMPLE 2-1-2 Find z^n using Eq. (2-6).

$$z^{n} = \left(R e^{j\theta}\right)^{n} = R^{n} e^{jn\theta} = R^{n} \angle n\theta$$
(2-10)

:

2-1-2 Complex Variables

A complex variable s has two components: a real component σ and an imaginary component ω . Graphically, the real component of s is represented by a σ axis in the horizontal direction, and the imaginary component is measured along the vertical j ω axis, in the complex s-plane. Fig. 2-2 illustrates the complex s-plane, in which any arbitrary point $s = s_1$ is defined by the coordinates $\sigma = \sigma_1$, and $\omega = \omega_1$, or simply $s_1 = \sigma_1 + j\omega_1$.



2-1-3 Functions of a Complex Variable

The function G(s) is said to be a function of the complex variable s if, for every value of s, there is one or more corresponding values of G(s). Because s is defined to have real and imaginary parts, the function G(s) is also represented by its real and imaginary parts; that is,

$$G(s) = \operatorname{Re}[G(s)] + j\operatorname{Im}[G(s)]$$
(2-11)

where $\operatorname{Re}[G(s)]$ denotes the real part of G(s), and $\operatorname{Im}[G(s)]$ represents the imaginary part of G(s). The function G(s) is also represented by the complex G(s)-plane, with $\operatorname{Re}[G(s)]$ as the real axis and $\operatorname{Im}[G(s)]$ as the imaginary axis. If for every value of s there is only one corresponding value of G(s) in the G(s)-plane, G(s) is said to be a **single-valued function**, and the mapping from points in the s-plane onto points in the G(s)-plane is described as **single-valued** (Fig. 2-3). If the mapping from the G(s)-plane to the s-plane is also single-valued, the mapping is called **one-to-one**. However, there are many functions for which the mapping from the function plane to the complex-variable plane is not single-valued. For instance, given the function

$$G(s) = \frac{1}{s(s+1)}$$
 (2-12)



Figure 2-3 Single-valued mapping from the s-plane to the G(s)-plane.

it is apparent that, for each value of s, there is only one unique corresponding value for G(s). However, the inverse mapping is not true; for instance, the point $G(s) = \infty$ is mapped onto two points, s = 0 and s = -1, in the s-plane.

2-1-4 Analytic Function

A function G(s) of the complex variable s is called an analytic function in a region of the splane if the function and all its derivatives exist in the region. For instance, the function given in Eq. (2-12) is analytic at every point in the s-plane except at the points s = 0 and s = -1. At these two points, the value of the function is infinite. As another example, the function G(s) = s + 2 is analytic at every point in the finite s-plane.

2-1-5 Singularities and Poles of a Function

The singularities of a function are the points in the *s*-plane at which the function or its derivatives do not exist. A pole is the most common type of singularity and plays a very important role in the studies of classical control theory.

The definition of a **pole** can be stated as: If a function G(s) is analytic and singlevalued in the neighborhood of point p_i , it is said to have a pole of order r at $s = p_i$ if the limit $\lim_{s \to p_i} [(s - p_i)^r G(s)]$ has a finite, nonzero value. In other words, the denominator of G(s) must include the factor $(s - p_i)^r$, so when $s = p_i$, the function becomes infinite. If r = 1, the pole at $s = p_i$ is called a **simple pole**. As an example, the function

$$G(s) = \frac{10(s+2)}{s(s+1)(s+3)^2}$$
(2-13)

has a pole of order 2 at s = -3 and simple poles at s = 0 and s = -1. It can also be said that the function G(s) is analytic in the s-plane except at these poles. See Fig. 2-4 for the graphical representation of the finite poles of the system.

2-1-6 Zeros of a Function

The definition of a zero of a function can be stated as: If the function G(s) is analytic at $s = z_i$, it is said to have a zero of order r at $s = z_i$ if the limit $\lim_{s \to z_i} [(s - z_i)^{-r}G(s)]$ has a finite, nonzero value. Or, simply, G(s) has a zero of order r at $s = z_i$ if 1/G(s) has an rth-order pole at $s = z_i$. For example, the function in Eq. (2-13) has a simple zero at s = -2.

If the function under consideration is a rational function of s, that is, a quotient of two polynomials of s, the total number of poles equals the total number of zeros, counting the multiple-order poles and zeros and taking into account the poles and zeros at infinity. The function in Eq. (2-13) has four finite poles at s = 0, -1, -3, and -3; there is one finite zero at s = -2, but there are three zeros at infinity, because

$$\lim_{s \to \infty} G(s) = \lim_{s \to \infty} \frac{10}{s^3} = 0$$
 (2-14)

Therefore, the function has a total of four poles and four zeros in the entire s-plane, including infinity. See Fig. 2-4 for the graphical representation of the finite zeros of the system.



Figure 2-4 Graphical representation of $G(s) = \frac{10(s+2)}{s(s+1)(s+3)^2}$ in the s-plane: × poles and O zeros.

Toolbox 2-1-1

For Eq. (2-13), use "zpk" to create zero-pole-gain models by the following sequence of MATLAB functions

>> G = zpk([-2], [0 - 1 - 3 - 3], 10)

Zero/pole/gain: 10 (s + 2) s (s + 1) (s + 3)²

Convert the transfer function to polynomial form

>> Gp = tf(G)

Use "pole" and "zero" to obtain the poles and zeros of the transfer function

>> pole(Gp) ans = 0 -1 -3 -3

>> zero(Gp)

ans = -2 Alternatively use:

>> clear all
>> s = tf('s');
>> Gp = 10*(s + 2)/(s*(s + 1)*(s + 3)²)

Transfer function:

10 s + 20s^4 + 7 s^3 + 15 s^2 + 9 s

Convert the transfer function Gp to zero-pole-gain form

>> Gzpk = zpk(Gp)

```
Zero/pole/gain:
10 (s + 2)
s (s + 3)<sup>2</sup> (s + 1)
```

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2-1-7 Polar Representation

To find the **polar representation** of G(s) in Eq. (2-12) at s = 2j, we look at individual components. That is

$$G(s) = \frac{1}{s(s+1)}$$
 (2-15)

$$s = 2j = R e^{j\theta} = 2e^{j\frac{\pi}{2}}$$

$$\begin{cases} s + 1 - 2j + 1 = R e^{j\theta} \\ R = \sqrt{2^2 + 1} = \sqrt{5} \\ \theta = \tan^{-1}\frac{2}{1} = 1.11 rad(= 63.43^\circ) \end{cases}$$

$$G(2j) = \frac{1}{2j(2j+1)} = \frac{1}{2}e^{-j\frac{\pi}{2}}\frac{1}{\sqrt{5}}e^{-j\tan^{-1}\frac{1}{2}} = \frac{1}{2\sqrt{5}}e^{-j(\frac{\pi}{2} + \tan^{-1}\frac{1}{2})}$$
(2-17)

See Fig. 2-5 for a graphical representation of $s_1 = 2j + 1$ in the s-plane.

EXAMPLE 2-1-3 Find the polar representation of G(s) given below for $s = j\omega$, where ω is a constant varying from zero to infinity.

$$G(s) = \frac{16}{s^2 + 10s + 16} = \frac{16}{(s+2)(s+8)}$$
(2-18)

To evaluate Eq. (2-18) at $s = j\omega$, we look at individual components. Thus,

$$j\omega + 2 = \sqrt{2^2 + \omega^2} e^{j\phi_1}$$
(2-19)

$$\omega = R_1 \sin \phi_1 \tag{2-20}$$

$$2 = R_1 \cos \phi_1 \tag{2-21}$$

$$R_1 = \sqrt{2^2 + \omega^2}$$
 (2-22)

$$\phi_1 = \tan^{-1} \frac{\omega/R_1}{2/R_1} \tag{2-23}$$



Figure 2-5 Graphical representation of $s_1 = 2j + 1$ in the *s*-plane.

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Figure 2-6 Graphical representation of components of $\frac{16}{(\omega j+2)(\omega j+8)}$.

$$j\omega + 2 = R_1(j\sin\phi_1 + \cos\phi_1)$$
 (2-24)

$$j\omega + 2 = R_1 e^{j\phi_1} \tag{2-25}$$

$$j\omega + 8 = \sqrt{8^2 + \omega^2} e^{j\phi_2}$$
(2-26)

$$\phi_2 = \tan^{-1} \frac{\omega/R_2}{8/R_2} \tag{2-27}$$

$$16 = 16 e^0 (2-28)$$

See Fig. 2-6 for a graphical representation of components of $\frac{16}{(\omega j + 2)(\omega j + 8)}$. Hence,

nce,

$$\frac{1}{j\omega+2} = \frac{1}{\sqrt{2^2 + \omega^2} e^{j\phi_1}}$$

$$\frac{1}{j\omega+8} = \frac{1}{\sqrt{8^2 + \omega^2} e^{j\phi_2}}$$
(2-29)

As a result, $G(s = j\omega)$ becomes:

$$G(j\omega) = \frac{16}{\sqrt{2^2 + \omega^2}\sqrt{8^2 + \omega^2}} e^{-j(\phi_1 + \phi_2)} = |G(j\omega)|e^{j\phi}$$
(2-30)

where

$$R = G(\omega) = |G(j\omega)| = \frac{16}{\sqrt{(\omega^2 + 4)(\omega^2 + 64)}}$$
(2-31)

Similarly, we can define

$$\phi = \tan^{-1} \frac{\operatorname{Im} G(j\omega)}{\operatorname{R} e G(j\omega)} = \angle G(s = j\omega) = -\phi_1 - \phi_2$$
(2-32)

Table 2-2 describes different R and ϕ values as ω changes. As shown, the magnitude decreases as the frequency increases. The phase goes from 0° to -180° .

ω rad/s	R	φ
0.1	0.999	-3.58
1	0.888	-33.69
10	0.123	-130.03
100	0.0016	-174.28

 TABLE 2-2
 Numerical Values of Sample Magnitude and Phase of the

 System in Example 2-1-3

Alternative Approach: If we multiply both numerator and denominator of Eq. (2-18) by the

complex conjugate of the denominator, i.e. $\frac{(-j\omega+2)(-j\omega+8)}{(-j\omega+2)(-j\omega+8)} = 1$, we get

$$G(j\omega) = \frac{16(-j\omega+2)(-j\omega+8)}{(\omega^2+2^2)(\omega^2+8^2)}$$

$$= \frac{16}{(\omega^2+4)(\omega^2+64)} [(16-\omega^2) - j10\omega]$$

= Real + Imaginary

$$= \frac{16\sqrt{(16-\omega^2)^2 + (10\omega)^2}}{(\omega^2+4)(\omega^2+64)} e^{j\phi}$$

$$= \frac{16}{\sqrt{(\omega^2+4)(\omega^2+64)}} e^{j\phi}$$

$$= Re^{j\phi}$$

where $\phi = \tan^{-1} \frac{-10\omega/R}{(16-\omega^2)/R} = \frac{\mathrm{Im}(G(j\omega))}{\mathrm{Re}(G(j\omega))}$
16

See Fig. 2-7 for a graphical representation of $\frac{10}{(\omega j+2)(\omega j+8)}$ for a fixed value of ω .

So as you have noticed, the frequency response can be determined graphically. Consider the following second order system:

$$G(s) = \frac{K}{(s+p_1)(s+p_2)}$$
(2-34)



Figure 2-7 Graphical representation of $\frac{16}{(\omega j+2)(\omega j+8)}$ for a fixed value of ω .

```
Toolbox 2-1-2
Here are MATLAB commands to treat complex variables:
Z = complex(a,b)
creates a complex output, Z, from the two real inputs Z = a + bi
ZC = conj(Z)
returns the complex conjugate of the elements of Z
X = real(Z)
returns the real part of the elements of the complex array Z
Y = imag(Z)
returns the imaginary part of the elements of array Z
R = abs(Z)
returns the complex modulus (magnitude), which is the same as
R = sqrt(real(Z).^2 + imag(Z).^2)
theta = angle(Z)
returns the phase angles, in radians, for each element of complex array Z
The angles lie between the "real axis" in the s-plane and the magnitude R
Z = R.*exp(i*theta)
converts back to the original complex Z
>> Z = complex(3,2)
Z =
 3,0000 + 2,0000i
>> ZC = conj(Z)
ZC =
 3.0000 - 2.0000i
>> R = abs(Z)
R =
   3.6056
>> theta = angle(Z)
theta =
  0.5880
>> ZRT = R.*exp(i*theta)
ZRT =
  3.0000+2.0000i
```

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where $(-p_1)$ and $(-p_2)$ are poles of the function G(s). By definition, if $s = j\omega$, $G(j\omega)$ is the **frequency** response function of G(s), because ω has a unit of frequency (rad/s):

$$G(s) = \frac{K}{(j\omega + p_1)(j\omega + p_2)}$$
 (2-35)

The magnitude of $G(j\omega)$ is

$$R = |G(j\omega)| = \frac{K}{|j\omega + p_1||j\omega + p_2|}$$
(2-36)

and the phase angle of $G(j\omega)$ is

$$\phi = /G(j\omega) = /K - /j\omega + p_1 - /j\omega + p_2$$

= $-\phi_1 - \phi_2$ (2-37)

For the general case, where

$$G(s) = K \frac{\sum_{k=1}^{m} (s + z_k)}{\sum_{i=-1}^{n} (s + p_i)}$$
(2-38)

The magnitude and phase of G(s) are as follows

$$R = |G(j\omega)| = K \frac{|j\omega + z_1| \cdots |j\omega + z_m|}{|j\omega + p_1| \cdots |j\omega + p_n|}$$

$$\phi = /G(j\omega) = (\psi_1 + \cdots + \psi_m) - (\phi_1 + \cdots + \phi_n)$$
(2-39)

2-2 FREQUENCY-DOMAIN PLOTS

Let G(s) be the forward-path transfer function¹ of a linear control system with unity feedback. The frequency-domain analysis of the closed-loop system can be conducted from the frequency-domain plots of G(s) with s replaced by $j\omega$.

The function $G(j\omega)$ is generally a complex function of the frequency ω and can be written as

$$G(j\omega) = |G(j\omega)| \angle G(j\omega)$$
(2-40)

where $|G(j\omega)|$ denotes the magnitude of $G(j\omega)$, and $\angle G(j\omega)$ is the phase of $G(j\omega)$.

The following frequency-domain plots of $G(j\omega)$ versus ω are often used in the analysis and design of linear control systems in the frequency domain.

- 1. Polar plot. A plot of the magnitude versus phase in the polar coordinates as ω is varied from zero to infinity
- 2. Bode plot. A plot of the magnitude in decibels versus ω (or $\log_{10}\omega$) in semilog (or rectangular) coordinates
- 3. Magnitude-phase plot. A plot of the magnitude (in decibels) versus the phase on rectangular coordinates, with ω as a variable parameter on the curve

2-2-1 Computer-Aided Construction of the Frequency-Domain Plots

The data for the plotting of the frequency-domain plots are usually quite time consuming to generate if the computation is carried out manually, especially if the function is of high order. In this textbook, we use MATLAB and the **ACSYS** software for this purpose.

¹ For the formal definition of a "transfer function," refer to Section 2-7-2.



Figure 2-8 Polar plot shown as a mapping of the positive half of the $j\omega$ -axis in the s-plane onto the $G(j\omega)$ -plane.

From an analytical standpoint, the analyst and designer should be familiar with the properties of the frequency-domain plots so that proper interpretations can be made on these computer-generated plots.

2-2-2 Polar Plots

The polar plot of a function of the complex variable s, G(s), is a plot of the magnitude of $G(j\omega)$ versus the phase of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity. From a mathematical viewpoint, the process can be regarded as the mapping of the positive half of the imaginary axis of the s-plane onto the $G(j\omega)$ -plane. A simple example of this mapping is shown in Fig. 2-8. For any frequency $\omega = \omega_1$, the magnitude and phase of $G(j\omega_1)$ are represented by a vector in the $G(j\omega)$ -plane. In measuring the phase, counterclockwise is referred to as positive, and clockwise is negative.

EXAMPLE 2-2-1 To illustrate the construction of the polar plot of a function G(s), consider the function

$$G(s) = \frac{1}{1+Ts} \tag{2-41}$$

where T is a positive constant. Setting $s = j\omega$, we have

$$G(j\omega) = \frac{1}{1+j\omega T}$$
(2-42)

In terms of magnitude and phase, Eq. (2-42) is rewritten as

$$G(j\omega) = \frac{1}{\sqrt{1+\omega^2 T^2}} \angle -\tan^{-1} \omega T$$
 (2-43)

When ω is zero, the magnitude of $G(j\omega)$ is unity, and the phase of $G(j\omega)$ is at 0°. Thus, at $\omega = 0$, $G(j\omega)$ is represented by a vector of unit length directed in the 0° direction. As ω increases, the magnitude of $G(j\omega)$ decreases, and the phase becomes more negative. As ω increases, the length of the vector in the polar coordinates decreases and the vector rotates in the clockwise (negative) direction. When ω approaches infinity, the magnitude of $G(j\omega)$ becomes zero, and the phase reaches -90° . This is presented by a vector with an infinitesimally small length directed along the -90° -axis in the $G(j\omega)$ -plane. By substituting other finite values of ω into Eq. (2-43), the exact plot of $G(j\omega)$ turns out to be a semicircle, as shown in Fig. 2-9.



Figure 2-9 Polar plot of $G(j\omega) = \frac{1}{(1+j\omega T)}$.

EXAMPLE 2-2-2 As a second illustrative example, consider the function

$$G(j\omega) = \frac{1+j\omega T_2}{1+j\omega T_1}$$
(2-44)

where T_1 and T_2 are positive real constants. Eq. (2-44) is re-written as

$$G(j\omega) = \sqrt{\frac{1+\omega^2 T_2^2}{1+\omega^2 T_1^2}} \, \ell \left(\tan^{-1} \omega T_2 - \tan^{-1} \omega T_1 \right) \tag{2-45}$$

The polar plot of $G(j\omega)$, in this case, depends on the relative magnitudes of T_1 and T_2 . If T_2 is greater than T_1 , the magnitude of $G(j\omega)$ is always greater than unity as ω is varied from zero to infinity, and the phase of $G(j\omega)$ is always positive. If T_2 is less than T_1 , the magnitude of $G(j\omega)$ is always less than unity, and the phase is always negative. The polar plots of $G(j\omega)$ of Eq. (2-45) that correspond to these two conditions are shown in Fig. 2-10.

The general shape of the polar plot of a function $G(j\omega)$ can be determined from the following information.

- 1. The behavior of the magnitude and phase of $G(j\omega)$ at $\omega = 0$ and $\omega = \infty$.
- 2. The intersections of the polar plot with the real and imaginary axes, and the values of ω at these intersections



Figure 2-10 Polar plots of $G(j\omega) = \frac{(1+j\omega T_2)}{(1+j\omega T_1)}$.



Comparing the results in Toolbox 2-2-1 and Fig. 2-10, it is clear that the polar plot reflects only a portion of the Nyquist diagram. In many control-system applications, such as the Nyquist stability criterion (see Chapter 8), an exact plot of the frequency response is not essential. Often, a rough sketch of the polar plot of the transfer function is adequate for stability analysis in the frequency domain.

EXAMPLE 2-2-3 In frequency-domain analyses of control systems, often we have to determine the basic properties of a polar plot. Consider the following transfer function:

$$G(s) = \frac{10}{s(s+1)}$$
(2-46)

By substituting $s = j\omega$ in Eq. (2-46), the magnitude and phase of $G(j\omega)$ at $\omega = 0$ and $\omega = \infty$ are computed as follows:

$$\lim_{\omega \to 0} |G(j\omega)| = \lim_{\omega \to 0} \frac{10}{\omega} = \infty$$
(2-47)

$$\lim_{\omega \to 0} \angle G(j\omega) = \lim_{\omega \to 0} \angle 10/j\omega = -90^{\circ}$$
(2-48)

$$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{10}{\omega^2} = 0$$
 (2-49)

$$\lim_{\omega \to \infty} \angle G(j\omega) = \lim_{\omega \to \infty} \angle 10/(j\omega)^2 = -180^{\circ}$$
 (2-50)

Thus, the properties of the polar plot of $G(j\omega)$ at $\omega = 0$ and $\omega = \infty$ are ascertained. Next, we determine the intersections, if any, of the polar plot with the two axes of the $G(j\omega)$ -plane. If the polar plot of $G(j\omega)$ intersects the real axis, at the point of intersection, the imaginary part of $G(j\omega)$ is zero; that is,

$$\operatorname{Im}[G(j\omega)] = 0 \tag{2-51}$$

To express $G(j\omega)$ as the sum of its real and imaginary parts, we must rationalize $G(j\omega)$ by multiplying its numerator and denominator by the complex conjugate of its denominator. Therefore, $G(j\omega)$ is written

$$G(j\omega) = \frac{10(-j\omega)(-j\omega+1)}{j\omega(j\omega+1)(-j\omega)(-j\omega+1)} = \frac{-10\omega^2}{\omega^4 + \omega^2} - j\frac{10\omega}{\omega^4 + \omega^2}$$

= Re[G(j\omega)] + j Im[G(j\omega)] (2-52)

When we set $\text{Im}[G(j\omega)]$ to zero, we get $\omega = \infty$, meaning that the $G(j\omega)$ plot intersects only with the real axis of the $G(j\omega)$ -plane at the origin.

Similarly, the intersection of $G(j\omega)$ with the imaginary axis is found by setting $\operatorname{Re}[G(j\omega)]$ of Eq. (2-52) to zero. The only real solution for ω is also $\omega = \infty$, which corresponds to the origin of the $G(j\omega)$ -plane. The conclusion is that the polar plot of $G(j\omega)$ does not intersect any one of the axes at any finite nonzero frequency. Under certain conditions, we are interested in the properties of the $G(j\omega)$ at infinity, which corresponds to $\omega = 0$ in this case. From Eq. (2-52), we see that $\operatorname{Im}[G(j\omega)] = \infty$ and $\operatorname{Re}[G(j\omega)] = -10$ at $\omega = 0$. Based on this information as well as knowledge of the angles of $G(j\omega)$ at $\omega = 0$ and $\omega = \infty$, the polar plot of $G(j\omega)$ is easily sketched without actual plotting, as shown in Fig. 2-11.



Figure 2-11 Polar plot of $G(s) = \frac{10}{s(s+1)}$.

EXAMPLE 2-2-4 Given the transfer function

$$G(s) = \frac{10}{s(s+1)(s+2)}$$
(2-53)

we want to make a rough sketch of the polar plot of $G(j\omega)$. The following calculations are made for the properties of the magnitude and phase of $G(j\omega)$ at $\omega = 0$ and $\omega = \infty$:

$$\lim_{\omega \to 0} |G(j\omega)| = \lim_{\omega \to 0} \frac{5}{\omega} = \infty$$
(2-54)

$$\lim_{\omega \to 0} \langle G(j\omega) = \lim_{\omega \to 0} \langle 5/j\omega = -90^{\circ}$$
(2-55)

$$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{10}{\omega^3} = 0$$
 (2-56)

To find the intersections of the $G(j\omega)$ plot on the real and imaginary axes of the $G(j\omega)$ -plane, we rationalize $G(j\omega)$ to give

$$G(j\omega) = \frac{10(-j\omega)(-j\omega+1)(-j\omega+2)}{j\omega(j\omega+1)(j\omega+2)(-j\omega)(-j\omega+1)(-j\omega+2)}$$
(2-57)

After simplification, the last equation is written

$$G(j\omega) = \operatorname{Re}[G(j\omega)] + j\operatorname{Im}[G(j\omega)] = \frac{-30}{9\omega^2 + (2-\omega^2)^2} - \frac{j10(2-\omega^2)}{9\omega^3 + \omega(2-\omega^2)^2}$$
(2-58)

Setting Re[$G(j\omega)$] to zero, we have $\omega = \infty$, and $G(j\infty) = 0$, which means that the $G(j\omega)$ plot intersects the imaginary axis only at the origin. Setting $Im[G(j\omega)]$ to zero, we have $\omega = \pm \sqrt{2}$ rad/sec. This gives the point of intersection on the real axis at

$$G\left(\pm j\sqrt{2}\right) = -5/3 \tag{2-59}$$

The result, $\omega = -\sqrt{2}$ rad/sec, has no physical meaning, because the frequency is negative; it simply represents a mapping point on the negative $j\omega$ -axis of the s-plane. In general, if G(s) is a rational function of s (a quotient of two polynomials of s), the polar plot of $G(j\omega)$ for negative values of ω is the mirror image of that for positive ω , with the mirror placed on the real axis of the $G(j\omega)$ -plane. From Eq. (2-58), we also see that $\operatorname{Re}[G(j0)] = \infty$ and $\operatorname{Im}[G(j0)] = \infty$. With this information, it is now possible to make a sketch of the polar plot for the transfer function in Eq. (2-53), as shown in Fig. 2-12.

Although the method of obtaining the rough sketch of the polar plot of a transfer function as described is quite straightforward, in general, for complicated transfer functions that may have multiple crossings on the real and imaginary axes of the transfer-function plane, the algebraic manipulation may again be quite involved. Furthermore, the polar plot is basically a tool for analysis; it is somewhat awkward for design purposes. We shall show in the next section that approximate information on the polar plot can always be obtained from the Bode plot, which can be sketched



Figure 2-12 Polar plot of $G(s) = \frac{10}{s(s+1)(s+2)}$.

without any calculations. Thus, for more complicated transfer functions, sketches of the polar plots can be obtained with the help of the Bode plots, unless MATLAB is used.

2-2-3 Bode Plot (Corner Plot or Asymptotic Plot)

The Bode plot of the function $G(j\omega)$ is composed of two plots, one with the amplitude of $G(j\omega)$ in decibels (dB) versus $\log_{10}\omega$ or ω and the other with the phase of $G(j\omega)$ in degrees as a function of $\log_{10}\omega$ or ω . A Bode plot is also known as a **corner plot** or an **asymptotic plot** of $G(j\omega)$. These names stem from the fact that the Bode plot can be constructed by using straight-line approximations that are asymptotic to the actual plot. In simple terms, the Bode plot has the following features:

- 1. Because the magnitude of $G(j\omega)$ in the Bode plot is expressed in dB, product and division factors in $G(j\omega)$ became additions and subtractions, respectively. The phase relations are also added and subtracted from each other algebraically.
- 2. The magnitude plot of the Bode plot of $G(j\omega)$ can be approximated by straight-line segments, which allow the simple sketching of the plot without detailed computation.

Because the straight-line approximation of the Bode plot is relatively easy to construct, the data necessary for the other frequency-domain plots, such as the polar plot and the magnitude-versus-phase plot, can be easily generated from the Bode plot.

Consider the function

$$G(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{s^j(s+p_1)(s+p_2)\cdots(s+p_n)}e^{-T_d s}$$
(2-60)

where K and T_d are real constants, and the z's and the p's may be real or complex (in conjugate pairs) numbers. In Chapter 7, Eq. (2-60) is the preferred form for root-locus construction, because the poles and zeros of G(s) are easily identified. For constructing the Bode plot manually, G(s) is preferably written in the following form:

$$G(s) = \frac{K_1(1+T_1s)(1+T_2s)\cdots(1+T_ms)}{s^j(1+T_qs)(1+T_bs)\cdots(1+T_ns)}e^{-T_ds}$$
(2-61)

where K_1 is a real constant, the T's may be real or complex (in conjugate pairs) numbers, and T_d is the real time delay. If the Bode plot is to be constructed with a computer program, then either form of Eq. (2-60) or Eq. (2-61) can be used.

Because practically all the terms in Eq. (2-61) are of the same form, then without loss of generality, we can use the following transfer function to illustrate the construction of the Bode diagram.

$$G(s) = \frac{K(1+T_1s)(1+T_2s)}{s(1+T_as)(1+2\zeta s/\omega_n + s^2/\omega_n^2)}e^{-T_ds}$$
(2-62)

where K, T_d , T_1 , T_2 , T_a , ζ , and ω_n are real constants. It is assumed that the second-order polynomial in the denominator has complex-conjugate zeros.

The magnitude of $G(j\omega)$ in dB is obtained by multiplying the logarithm (base 10) of $|G(j\omega)|$ by 20; we have

$$|G(j\omega)|_{dB} = 20 \log_{10} |G(j\omega)|$$

= 20 log₁₀|K| + 20 log₁₀|1 + j\omegaT_1| + 20 log₁₀|1 + j\omegaT_2|
- 20 log₁₀|j\omega| - 20 log₁₀|1 + j\omegaT_a| - 20 log₁₀|1 + j2\zeta\omega - \omega^2/\omega_n^2| (2-63)

The phase of $G(j\omega)$ is

$$\mathcal{L}G(j\omega) = \mathcal{L}K + \mathcal{L}(1 + j\omega T_1) + \mathcal{L}(1 + j\omega T_2) - \mathcal{L}j\omega - \mathcal{L}(1 + j\omega T_a) - \mathcal{L}(1 + 2\zeta\omega/\omega_n - \omega^2/\omega_n^2) - \omega T_d \text{ rad}$$
(2-64)

In general, the function $G(j\omega)$ may be of higher order than that of Eq. (2-62) and have many more factored terms. However, Eqs. (2-63) and (2-64) indicate that additional terms in $G(j\omega)$ would simply produce more similar terms in the magnitude and phase expressions, so the basic method of construction of the Bode plot would be

Toolbox 2-2-2

The Bode plot for Example 2-1-3, using the MATLAB "bode" function, is obtained by the following sequence of MATLAB functions.

Approach 1 num = [16]; S = tf('s'); den = [11016]; G=16/(s^2 + 10*s + 16); bode(G); bode(G);

The "bode" function computes the magnitude and phase of the frequency response of linear time invariant models. The magnitude is plotted in decibels (dB) and the phase in degrees. Compare the results to the values in Table 2-2.

