# Introduction to Electric Circuits
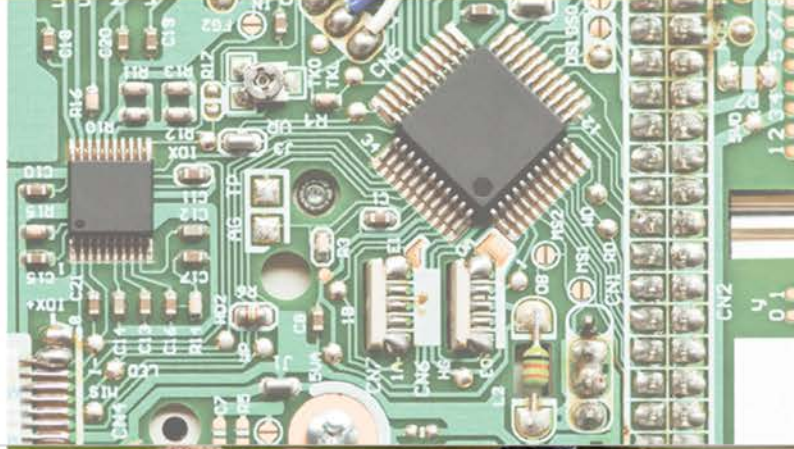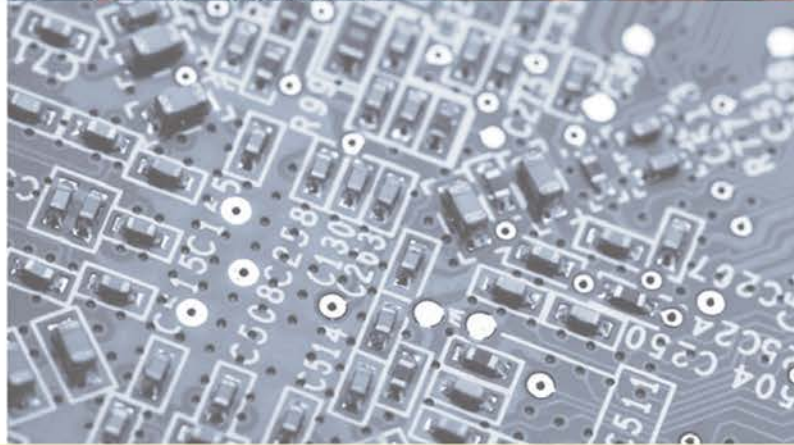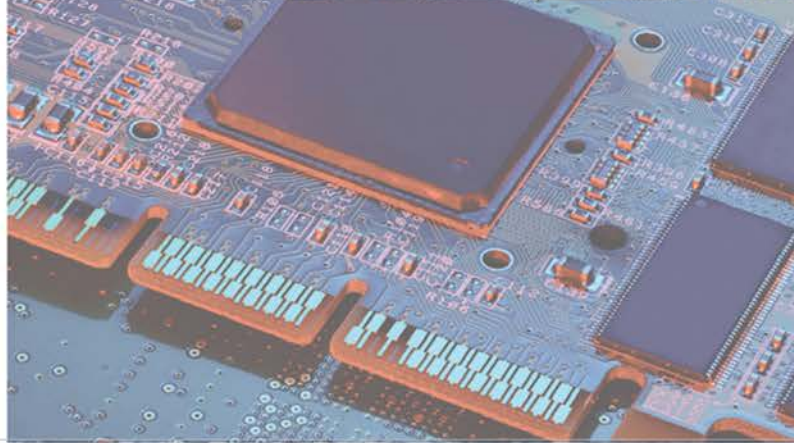
JAMES A. SVOBODA
RICHARD C. DORF

9th edition

9TH EDITION  ● *Introduction to Electric Circuits*

James A. Svoboda
*Clarkson University*

Richard C. Dorf
*University of California*

WILEY

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The scientific nature of the ordinary man
Is to go on out and do the best he can.
—John Prine

But, Captain, I cannot change the laws of physics.
—Lt. Cmdr. Montogomery Scott (Scotty), USS *Enterprise*

*Dedicated to our grandchildren:*

Ian Christopher Boilard, Kyle Everett Schafer, and Graham Henry Schafer
and
Heather Lynn Svoboda, James Hugh Svoboda, Jacob Arthur Leis,
Maxwell Andrew Leis, and Jack Mandlin Svoboda

# About the Authors

**James A. Svoboda** is an associate professor of electrical and computer engineering at Clarkson University, where he teaches courses on topics such as circuits, electronics, and computer programming. He earned a PhD in electrical engineering from the University of Wisconsin at Madison, an MS from the University of Colorado, and a BS from General Motors Institute.

Sophomore Circuits is one of Professor Svoboda's favorite courses. He has taught this course to 6,500 undergraduates at Clarkson University over the past 35 years. In 1986, he received Clarkson University's Distinguished Teaching Award.

Professor Svoboda has written several research papers describing the advantages of using nullors to model electric circuits for computer analysis. He is interested in the way technology affects engineering education and has developed several software packages for use in Sophomore Circuits.
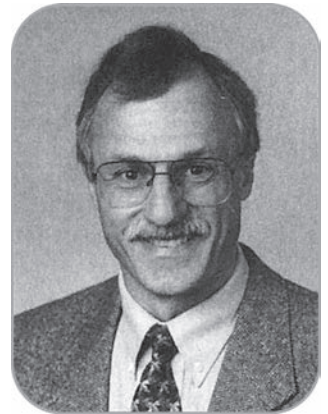
**Richard C. Dorf**, professor of electrical and computer engineering at the University of California, Davis, teaches graduate and undergraduate courses in electrical engineering in the fields of circuits and control systems. He earned a PhD in electrical engineering from the U.S. Naval Postgraduate School, an MS from the University of Colorado, and a BS from Clarkson University. Highly concerned with the discipline of electrical engineering and its wide value to social and economic needs, he has written and lectured internationally on the contributions and advances in electrical engineering.

Professor Dorf has extensive experience with education and industry and is professionally active in the fields of robotics, automation, electric circuits, and communications. He has served as a visiting professor at the University of Edinburgh, Scotland, the Massachusetts Institute of Technology, Stanford University, and the University of California at Berkeley.

A Fellow of the Institute of Electrical and Electronic Engineers and the American Society for Engineering Education, Dr. Dorf is widely known to the profession for his *Modern Control Systems*, twelfth edition (Pearson, 2011) and *The International Encyclopedia of Robotics* (Wiley, 1988). Dr. Dorf is also the coauthor of *Circuits, Devices and Systems* (with Ralph Smith), fifth edition (Wiley, 1992). Dr. Dorf edited the widely used *Electrical Engineering Handbook*, third edition (CRC Press and IEEE press), published in 2011. His latest work is Technology Ventures, fourth edition (McGraw-Hill 2013).

# Preface

The central theme of *Introduction to Electric Circuits* is the concept that electric circuits are part of the basic fabric of modern technology. Given this theme, we endeavor to show how the analysis and design of electric circuits are inseparably intertwined with the ability of the engineer to design complex electronic, communication, computer, and control systems as well as consumer products.

## Approach and Organization

This book is designed for a one- to three-term course in electric circuits or linear circuit analysis and is structured for maximum *flexibility*. The flowchart in Figure 1 demonstrates alternative chapter organizations that can accommodate different course outlines without disrupting continuity.

The presentation is geared to readers who are being exposed to the basic concepts of electric circuits for the first time, and the scope of the work is broad. Students should come to the course with the basic knowledge of differential and integral calculus.

This book endeavors to prepare the reader to solve realistic problems involving electric circuits. Thus, circuits are shown to be the results of real inventions and the answers to real needs in industry, the office, and the home. Although the tools of electric circuit analysis may be partially abstract, electric circuits are the building blocks of modern society. The analysis and design of electric circuits are critical skills for all engineers.

## What's New in the 9th Edition

### Revisions to Improve Clarity

Chapter 10, covering AC circuits, has been largely rewritten to improve clarity of exposition. In addition, revisions have been made through the text to improve clarity. Sometimes these revisions are small, involving sentences or paragraphs. Other larger revisions involved pages or even entire sections. Often these revisions involve examples. Consequently, the 9th edition contains 36 new examples.

### More Problems

The 9th edition contains 180 new problems, bringing the total number of problems to more than 1,400. This edition uses a variety of problem types and they range in difficulty from simple to challenging, including:

- Straightforward analysis problems.
- Analysis of complicated circuits.
- Simple design problems. (For example, given a circuit and the specified response, determine the required *RLC* values.)
- Compare and contrast, multipart problems that draw attention to similarities or differences between two situations.
- MATLAB and PSpice problems.
- Design problems. (Given some specifications, devise a circuit that satisfies those specifications.)
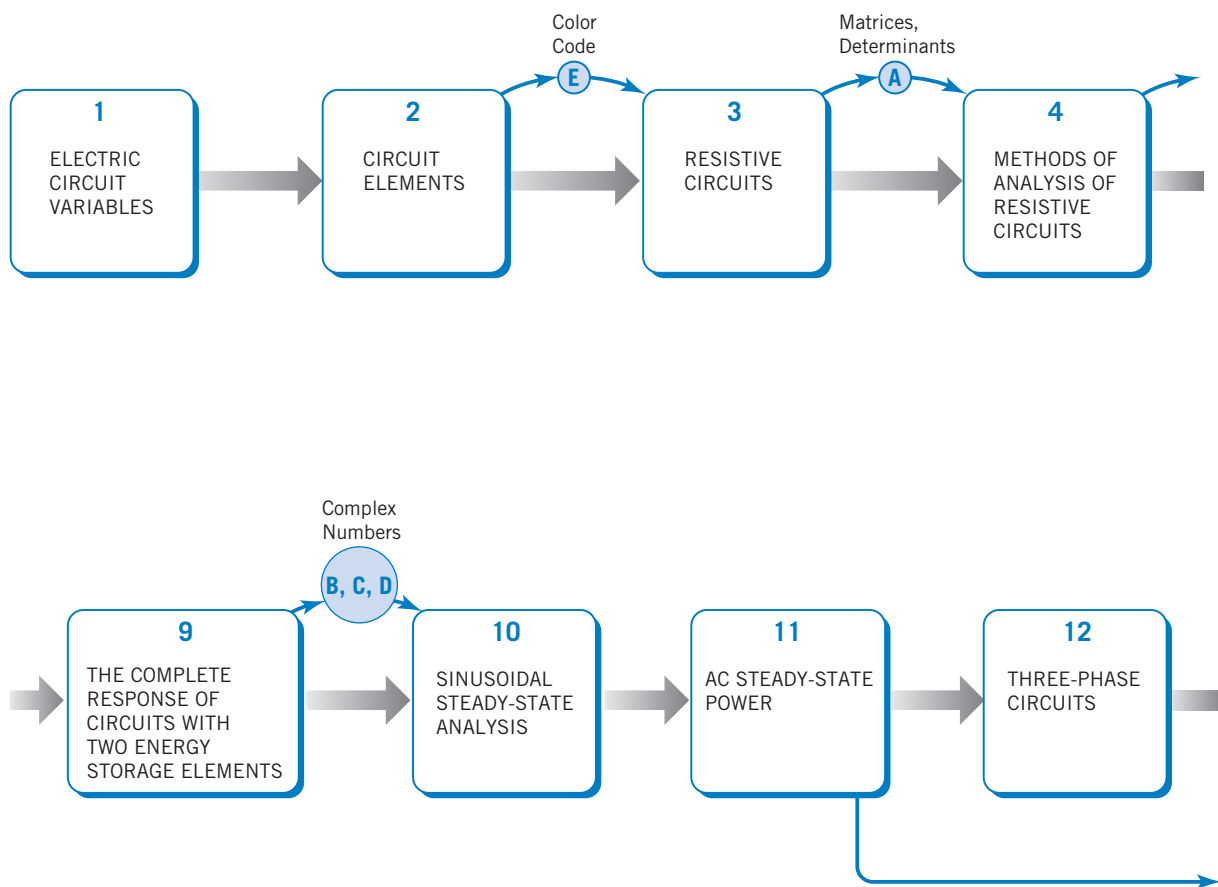- How Can We Check . . . ? (Verify that a solution is indeed correct.)

**FIGURE 1**   Flow chart showing alternative paths through the topics in this textbook.

## *Features Retained from Previous Editions*

### Introduction

Each chapter begins with an introduction that motivates consideration of the material of that chapter.

### Examples

Because this book is oriented toward providing expertise in problem solving, we have included more than 260 illustrative examples. Also, each example has a title that directs the student to exactly what is being illustrated in that particular example.

Various methods of solving problems are incorporated into select examples. These cases show students that multiple methods can be used to derive similar solutions or, in some cases, that multiple solutions can be correct. This helps students build the critical thinking skills necessary to discern the best choice between multiple outcomes.

Much attention has been given to using PSpice and MATLAB to solve circuits problems. Two appendices, one introducing PSpice and the other introducing MATLAB, briefly describe the capabilities of the programs and illustrate the steps needed to get started using them. Next, PSpice

PSpice

**F, G**

| 5 CIRCUIT THEOREMS | 6 THE OPERATIONAL AMPLIFIER | 7 ENERGY STORAGE ELEMENTS | 8 THE COMPLETE RESPONSE OF *RL* AND *RC* CIRCUITS |

14 LAPLACE TRANSFORM

16 FILTER CIRCUITS

| 13 FREQUENCY RESPONSE | 14 THE LAPLACE TRANSFORM | 15 FOURIER SERIES AND FOURIER TRANSFORM | 16 FILTER CIRCUITS | 17 TWO-PORT NETWORKS |

6 THE OPERATIONAL AMPLIFIER

17 TWO-PORT NETWORKS

Legend:

Appendix

Chapter

Primary flow

Optional flow

and MATLAB are used throughout the text to solve various circuit analysis and design problems. For example, PSpice is used in Chapter 5 to find a Thévenin equivalent circuit and in Chapter 15 to represent circuit inputs and outputs as Fourier series. MATLAB is frequently used to obtain plots of circuit inputs and outputs that help us to see what our equations are telling us. MALAB also helps us with some long and tedious arithmetic. For example, in Chapter 10, MATLAB helps us do the complex arithmetic that we must do in order to analyze ac circuits, and in Chapter 14, MATLAB helps with the partial fraction required to find inverse Laplace transforms.

Of course, there's more to using PSpice and MATLAB than simply running the programs. We pay particular attention to interpreting the output of these computer programs and checking it to make sure that it is correct. Frequently, this is done in the section called "How Can We Check . . . ?" that is included in every chapter. For example, Section 8.9 shows how to interpret and check a PSpice "Transient Response," and Section 13.7 shows how to interpret and check a frequency response produced using MATLAB or PSpice.

### Design Examples, a Problem-Solving Method, and "How Can We Check . . . ?" Sections

Each chapter concludes with a design example that uses the methods of that chapter to solve a design problem. A formal five-step problem-solving method is introduced in Chapter 1 and then used in each of the design examples. An important step in the problem-solving method requires you to check your results to verify that they are correct. Each chapter includes a section entitled "How Can We Check . . . ?" that illustrates how the kind of results obtained in that chapter can be checked to ensure correctness.

### Key Equations and Formulas

You will find that key equations, formulas, and important notes have been called out in a shaded box to help you pinpoint critical information.

### Summarizing Tables and Figures

The procedures and methods developed in this text have been summarized in certain key tables and figures. Students will find these to be an important problem-solving resource.

- Table 1.5-1. The passive convention.
- Figure 2.7-1 and Table 2.7-1. Dependent sources.
- Table 3.10-1. Series and parallel sources.
- Table 3.10-1. Series and parallel elements. Voltage and current division.
- Figure 4.2-3. Node voltages versus element currents and voltages.
- Figure 4.5-4. Mesh currents versus element currents and voltages.
- Figures 5.4-3 and 5.4-4. Thévenin equivalent circuits.
- Figure 6.3-1. The ideal op amp.
- Figure 6.5-1. A catalog of popular op amp circuits.
- Table 7.8-1. Capacitors and inductors.
- Table 7.13-2. Series and parallel capacitors and inductors.
- Table 8.11-1. First-order circuits.
- Tables 9.13-1, 2, and 3. Second-order circuits.
- Table 10.5-1. Voltage and current division for AC circuits.
- Table 10.16-1. AC circuits in the frequency domain (phasors and impedances).
- Table 11.5-1. Power formulas for AC circuits.
- Tables 11.13-1 and 11.13-2. Coupled inductors and ideal transformers.
- Table 13.4-1. Resonant circuits.
- Tables 14.2-1 and 14.2-2. Laplace transform tables.

- Table 14.7-1. s-domain models of circuit elements.
- Table 15.4-1. Fourier series of selected periodic waveforms.

## Introduction to Signal Processing

Signal processing is an important application of electric circuits. This book introduces signal processing in two ways. First, two sections (Sections 6.6 and 7.9) describe methods to design electric circuits that implement algebraic and differential equations. Second, numerous examples and problems throughout this book illustrate signal processing. The input and output signals of an electric circuit are explicitly identified in each of these examples and problems. These examples and problems investigate the relationship between the input and output signals that is imposed by the circuit.

## Interactive Examples and Exercises

Numerous examples throughout this book are labeled as interactive examples. This label indicates that computerized versions of that example are available at the textbook's companion site, www.wiley.com/svoboda. Figure 2 illustrates the relationship between the textbook example and the computerized example available on the Web site. Figure 2a shows an example from Chapter 3. The problem presented by the interactive example shown in Figure 2b is similar to the textbook example but different in several ways:

- The values of the circuit parameters have been randomized.
- The independent and dependent sources may be reversed.
- The reference direction of the measured voltage may be reversed.
- A different question is asked. Here, the student is asked to work the textbook problem backward, using the measured voltage to determine the value of a circuit parameter.

The interactive example poses a problem and then accepts and checks the user's answer. Students are provided with immediate feedback regarding the correctness of their work. The interactive example chooses parameter values somewhat randomly, providing a seemingly endless supply of problems. This pairing of a solution to a particular problem with an endless supply of similar problems is an effective aid for learning about electric circuits.

The interactive exercise shown in Figure 2c considers a similar, but different, circuit. Like the interactive example, the interactive exercise poses a problem and then accepts and checks the user's answer. Student learning is further supported by extensive help in the form of worked example problems, available from within the interactive exercise, using the Worked Example button.

Variations of this problem are obtained using the New Problem button. We can peek at the answer, using the Show Answer button. The interactive examples and exercises provide hundreds of additional practice problems with countless variations, all with answers that are checked immediately by the computer.

## *Supplements and Web Site Material*

The almost ubiquitous use of computers and the Web have provided an exciting opportunity to rethink supplementary material. The supplements available have been greatly enhanced.

### *Book Companion Site*

Additional student and instructor resources can be found on the John Wiley & Sons textbook companion site at www.wiley.com/college/svoboda.
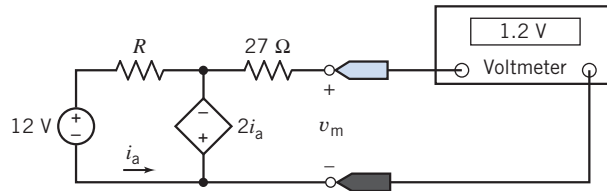
*(a)*
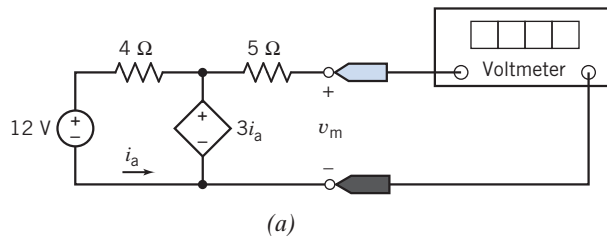


The voltmeter measures a voltage in volts.
What is the value of the resistance $R$ in $\Omega$?

*(b)*
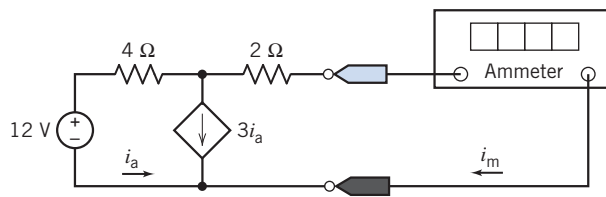


The ammeter measures a current in amps. What
is the value of the current measured by the ammeter?

*(c)*

**FIGURE 2** (*a*) The circuit considered Example 3.2-5. (*b*) A corresponding interactive example. (*c*) A corresponding interactive exercise.

## Student

• **Interactive Examples** The interactive examples and exercises are powerful support resources for students. They were created as tools to assist students in mastering skills and building their confidence. The examples selected from the text and included on the Web give students options for navigating through the problem. They can immediately request to see the solution or select a more gradual approach to help. Then they can try their hand at a similar problem by simply electing to change the values in the problem. By the time students attempt the homework, they have built the confidence and skills to complete their assignments successfully. It's a virtual homework helper.

- *PSpice for Linear Circuits*, available for purchase.
- *WileyPLUS* option.

### Instructor

- Solutions manual.
- PowerPoint slides.
- *WileyPLUS* option.

### *WileyPLUS*

***Pspice for Linear Circuits*** is a student supplement available for purchase. The *PSpice for Linear Circuits* manual describes in careful detail how to incorporate this valuable tool in solving problems. This manual emphasizes the need to verify the correctness of computer output. No example is finished until the simulation results have been checked to ensure that they are correct.

## A c k n o w l e d g m e n t s   a n d   C o m m i t m e n t   t o   A c c u r a c y

We are grateful to many people whose efforts have gone into the making of this textbook. We are especially grateful to our Executive Editor Daniel Sayre, Executive Marketing Manager Chris Ruel and Marketing Assistant Marissa Carroll for their support and enthusiasm. We are grateful to Tim Lindner and Kevin Holm of Wiley and Bruce Hobart of Laserwords Maine for their efforts in producing this textbook. We wish to thank Senior Product Designer Jenny Welter, Content Editor Wendy Ashenberg, and Editorial Assistant Jess Knecht for their significant contributions to this project.

We are particularly grateful to the team of reviewers who checked the problems and solutions to ensure their accuracy:

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# Contents

# CHAPTER 1 ⬮ *Electric Circuit Variables*

## IN THIS CHAPTER

## 1.1 Introduction

A circuit consists of electrical elements connected together. Engineers use electric circuits to solve problems that are important to modern society. In particular:

1.  Electric circuits are used in the generation, transmission, and consumption of electric power and energy.
2.  Electric circuits are used in the encoding, decoding, storage, retrieval, transmission, and processing of information.

In this chapter, we will do the following:

*   Represent the current and voltage of an electric circuit element, paying particular attention to the reference direction of the current and to the reference direction or polarity of the voltage.
*   Calculate the power and energy supplied or received by a circuit element.
*   Use the passive convention to determine whether the product of the current and voltage of a circuit element is the power supplied by that element or the power received by the element.
*   Use scientific notation to represent electrical quantities with a wide range of magnitudes.

## 1.2 Electric Circuits and Current

The outstanding characteristics of electricity when compared with other power sources are its mobility and flexibility. Electrical energy can be moved to any point along a couple of wires and, depending on the user's requirements, converted to light, heat, or motion.

An **electric circuit** or electric network is an interconnection of electrical elements linked together in a closed path so that an electric current may flow continuously.

Consider a simple circuit consisting of two well-known electrical elements, a battery and a resistor, as shown in Figure 1.2-1. Each element is represented by the two-terminal element shown in Figure 1.2-2. Elements are sometimes called devices, and terminals are sometimes called nodes.
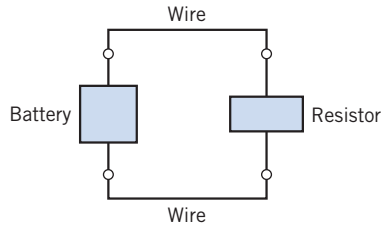


FIGURE 1.2-1 A simple circuit.



**FIGURE 1.2-2** A general two-terminal electrical element with terminals a and b.

Charge may flow in an electric circuit. *Current is the time rate of change of charge past a given point.* Charge is the intrinsic property of matter responsible for electric phenomena. The quantity of charge $q$ can be expressed in terms of the charge on one electron, which is $-1.602 \times 10^{-19}$ coulombs. Thus, $-1$ coulomb is the charge on $6.24 \times 10^{18}$ electrons. The current through a specified area is defined by the electric charge passing through the area per unit of time. Thus, $q$ is defined as the charge expressed in coulombs (C).

**Charge** is the quantity of electricity responsible for electric phenomena.

Then we can express current as

$$i = \frac{dq}{dt}$$  (1.2-1)

The unit of current is the ampere (A); an ampere is 1 coulomb per second.

**Current** is the time rate of flow of electric charge past a given point.

Note that throughout this chapter we use a lowercase letter, such as $q$, to denote a variable that is a function of time, $q(t)$. We use an uppercase letter, such as $Q$, to represent a constant.

The flow of current is conventionally represented as a flow of positive charges. This convention was initiated by Benjamin Franklin, the first great American electrical scientist. Of course, we now know that charge flow in metal conductors results from electrons with a negative charge. Nevertheless, we will conceive of current as the flow of positive charge, according to accepted convention.
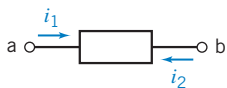


FIGURE 1.2-3 Current in a circuit element.

Figure 1.2-3 shows the notation that we use to describe a current. There are two parts to this notation: a value (perhaps represented by a variable name) and an assigned direction. As a matter of vocabulary, we say that a current exists *in* or *through* an element. Figure 1.2-3 shows that there are two ways to assign the direction of the current through an element. The current $i_1$ is the rate of flow of electric charge from terminal a to terminal b. On the other hand, the current $i_2$ is the flow of electric charge from terminal b to terminal a. The currents $i_1$ and $i_2$ are
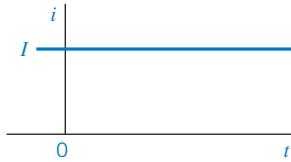
**FIGURE 1.2-4** A direct current of magnitude *I*.

similar but different. They are the same size but have different directions. Therefore, $i_2$ is the negative of $i_1$ and

$$i_1 = -i_2$$

We always associate an arrow with a current to denote its direction. A complete description of current requires both a value (which can be positive or negative) and a direction (indicated by an arrow).

If the current flowing through an element is constant, we represent it by the constant *I*, as shown in Figure 1.2-4. A constant current is called a *direct current* (dc).

A **direct current** (dc) is a current of constant magnitude.

A time-varying current $i(t)$ can take many forms, such as a ramp, a sinusoid, or an exponential, as shown in Figure 1.2-5. The sinusoidal current is called an *alternating current* (ac).
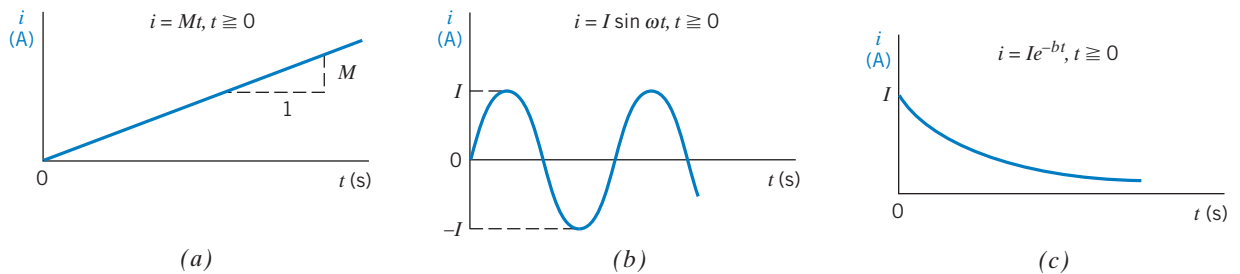


**FIGURE 1.2-5** (*a*) A ramp with a slope *M*. (*b*) A sinusoid. (*c*) An exponential. *I* is a constant. The current *i* is zero for $t < 0$.

If the charge *q* is known, the current *i* is readily found using Eq. 1.2-1. Alternatively, if the current *i* is known, the charge *q* is readily calculated. Note that from Eq. 1.2-1, we obtain

$$q = \int_{-\infty}^{t} i \, d\tau = \int_{0}^{t} i \, d\tau + q(0) \tag{1.2-2}$$

where $q(0)$ is the charge at $t = 0$.

**EXAMPLE 1.2-1   Current from Charge**

Find the current in an element when the charge entering the element is

$$q = 12t \text{ C}$$

where *t* is the time in seconds.

## Solution

Recall that the unit of charge is coulombs, C. Then the current, from Eq. 1.2-1, is

$$i = \frac{dq}{dt} = 12 \text{ A}$$

where the unit of current is amperes, A.

**EXAMPLE 1.2-2    Charge from Current**

Find the charge that has entered the terminal of an element from $t = 0$ s to $t = 3$ s when the current entering the element is as shown in Figure 1.2-6.
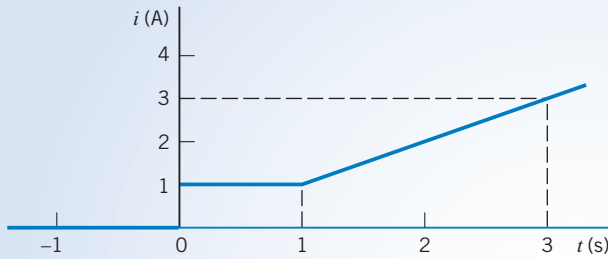


**FIGURE 1.2-6** Current waveform for Example 1.2-2.

## Solution

From Figure 1.2-6, we can describe $i(t)$ as

$$i(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t \leq 1 \\ t & t > 1 \end{cases}$$

Using Eq. 1.2-2, we have

$$q(3) - q(0) = \int_0^3 i(t)\,dt = \int_0^1 1\,dt + \int_1^3 t\,dt$$

$$= t\Big|_0^1 + \frac{t^2}{2}\Big|_1^3 = 1 + \frac{1}{2}(9-1) = 5 \text{ C}$$

Alternatively, we note that integration of $i(t)$ from $t = 0$ to $t = 3$ s simply requires the calculation of the area under the curve shown in Figure 1.2-6. Then, we have

$$q = 1 + 2 \times 2 = 5 \text{ C}$$

**EXERCISE 1.2-1** Find the charge that has entered an element by time $t$ when $i = 8t^2 - 4t$ A, $t \geq 0$. Assume $q(t) = 0$ for $t < 0$.

*Answer:* $q(t) = \frac{8}{3}t^3 - 2t^2$ C

**EXERCISE 1.2-2** The total charge that has entered a circuit element is $q(t) = 4 \sin 3t$ C when $t \geq 0$, and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t > 0$.

*Answer:* $i(t) = \frac{d}{dt} 4 \sin 3t = 12 \cos 3t$ A

## 1.3   *Systems of Units*

In representing a circuit and its elements, we must define a consistent system of units for the quantities occurring in the circuit. At the 1960 meeting of the General Conference of Weights and Measures, the representatives modernized the metric system and created the Système International d'Unites, commonly called SI units.

> **SI** is *Système International d'Unités* or the International System of Units.

The fundamental, or base, units of SI are shown in Table 1.3-1. Symbols for units that represent proper (persons') names are capitalized; the others are not. Periods are not used after the symbols, and the symbols do not take on plural forms. The derived units for other physical quantities are obtained by combining the fundamental units. Table 1.3-2 shows the more common derived units along with their formulas in terms of the fundamental units or preceding derived units. Symbols are shown for the units that have them.

**Table 1.3-1   SI Base Units**

| QUANTITY | SI UNIT | |
| --- | --- | --- |
| | NAME | SYMBOL |
| Length | meter | m |
| Mass | kilogram | kg |
| Time | second | s |
| Electric current | ampere | A |
| Thermodynamic temperature | kelvin | K |
| Amount of substance | mole | mol |
| Luminous intensity | candela | cd |

**Table 1.3-2   Derived Units in SI**

| QUANTITY | UNIT NAME | FORMULA | SYMBOL |
| --- | --- | --- | --- |
| Acceleration — linear | meter per second per second | $m/s^2$ | |
| Velocity — linear | meter per second | $m/s$ | |
| Frequency | hertz | $s^{-1}$ | Hz |
| Force | newton | $kg \cdot m/s^2$ | N |
| Pressure or stress | pascal | $N/m^2$ | Pa |
| Density | kilogram per cubic meter | $kg/m^3$ | |
| Energy or work | joule | $N \cdot m$ | J |
| Power | watt | $J/s$ | W |
| Electric charge | coulomb | $A \cdot s$ | C |
| Electric potential | volt | $W/A$ | V |
| Electric resistance | ohm | $V/A$ | $\Omega$ |
| Electric conductance | siemens | $A/V$ | S |
| Electric capacitance | farad | $C/V$ | F |
| Magnetic flux | weber | $V \cdot s$ | Wb |
| Inductance | henry | $Wb/A$ | H |

**Table 1.3-3  SI Prefixes**

| MULTIPLE | PREFIX | SYMBOL |
|----------|--------|--------|
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |

The basic units such as length in meters (m), time in seconds (s), and current in amperes (A) can be used to obtain the derived units. Then, for example, we have the unit for charge (C) derived from the product of current and time (A · s). The fundamental unit for energy is the joule (J), which is force times distance or N · m.

The great advantage of the SI system is that it incorporates a decimal system for relating larger or smaller quantities to the basic unit. The powers of 10 are represented by standard prefixes given in Table 1.3-3. An example of the common use of a prefix is the centimeter (cm), which is 0.01 meter.

The decimal multiplier must always accompany the appropriate units and is never written by itself. Thus, we may write 2500 W as 2.5 kW. Similarly, we write 0.012 A as 12 mA.

## E X A M P L E  1 . 3 - 1   SI Units

A mass of 150 grams experiences a force of 100 newtons. Find the energy or work expended if the mass moves 10 centimeters. Also, find the power if the mass completes its move in 1 millisecond.

### Solution
The energy is found as

$$\text{energy} = \text{force} \times \text{distance} = 100 \times 0.1 = 10 \text{ J}$$

Note that we used the distance in units of meters. The power is found from

$$\text{power} = \frac{\text{energy}}{\text{time period}}$$

where the time period is $10^{-3}$ s. Thus,

$$\text{power} = \frac{10}{10^{-3}} = 10^4 \text{ W} = 10 \text{ kW}$$

**EXERCISE 1.3-1**   Which of the three currents, $i_1 = 45 \ \mu\text{A}$, $i_2 = 0.03$ mA, and $i_3 = 25 \times 10^{-4}$ A, is largest?

*Answer:* $i_3$ is largest.

## 1.4  *Voltage*

The basic variables in an electrical circuit are current and voltage. These variables describe the flow of charge through the elements of a circuit and the energy required to cause charge to flow. Figure 1.4-1 shows the notation we use to describe a voltage. There are two parts to this notation: a value (perhaps represented by a variable name) and an assigned direction. The value of a voltage may be positive or negative. The direction of a voltage is given by its polarities $(+, -)$. As a matter of vocabulary, we say that a voltage exists *across* an element. Figure 1.4-1 shows that there are two ways to label the voltage across an element. The voltage $v_{ba}$ is proportional to the work required to move a positive charge from terminal a to terminal b. On the other hand, the voltage $v_{ab}$ is proportional to the work required to move a positive charge from terminal b to terminal a. We sometimes read $v_{ba}$ as "the voltage at terminal b with respect to terminal a." Similarly, $v_{ab}$ can be read as "the voltage at terminal a with respect to terminal b." Alternatively, we sometimes say that $v_{ba}$ is the voltage drop from terminal a to terminal b. The voltages $v_{ab}$ and $v_{ba}$ are similar but different. They have the same magnitude but different polarities. This means that

**FIGURE 1.4-1** Voltage across a circuit element.

$$v_{ab} = -v_{ba}$$

When considering $v_{ba}$, terminal b is called the "+ terminal" and terminal a is called the "- terminal." On the other hand, when talking about $v_{ab}$, terminal a is called the "+ terminal" and terminal b is called the "- terminal."

> The **voltage** across an element is the work (energy) required to move a unit positive charge from the $-$ terminal to the $+$ terminal. The unit of voltage is the volt, V.

The equation for the voltage across the element is

$$v = \frac{dw}{dq} \tag{1.4-1}$$

where $v$ is voltage, $w$ is energy (or work), and $q$ is charge. A charge of 1 coulomb delivers an energy of 1 joule as it moves through a voltage of 1 volt.

## 1.5  *Power and Energy*

The power and energy delivered to an element are of great importance. For example, the useful output of an electric lightbulb can be expressed in terms of power. We know that a 300-watt bulb delivers more light than a 100-watt bulb.

> **Power** is the time rate of supplying or receiving power.

Thus, we have the equation

$$p = \frac{dw}{dt} \tag{1.5-1}$$

(a)



(b)

**FIGURE 1.5-1** (a) The element voltage and current **adhere** to the passive convention. (b) The element voltage and current **do not adhere** to the passive convention.

where $p$ is power in watts, $w$ is energy in joules, and $t$ is time in seconds. The power associated with the current through an element is

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i \qquad (1.5\text{-}2)$$

From Eq. 1.5-2, we see that the power is simply the product of the voltage across an element times the current through the element. The power has units of watts.

Two circuit variables are assigned to each element of a circuit: a voltage and a current. Figure 1.5-1 shows that there are two different ways to arrange the direction of the current and the polarity of the voltage. In Figure 1.5-1a, the current is directed from the + toward the − of the voltage polarity. In contrast, in Figure 1.5-1b, the current is directed from the − toward the + of the voltage polarity.

First, consider Figure 1.5-1a. When the current enters the circuit element at the + terminal of the voltage and exits at the − terminal, the voltage and current are said to "adhere to the passive convention." In the passive convention, the voltage pushes a positive charge in the direction indicated by the current. Accordingly, the power calculated by multiplying the element voltage by the element current

$$p = vi$$

is the power **received** by the element. (This power is sometimes called "the power absorbed by the element" or "the power dissipated by the element.") The power received by an element can be either positive or negative. This will depend on the values of the element voltage and current.
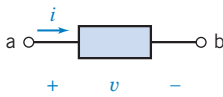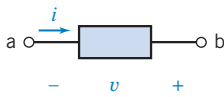
Next, consider Figure 1.5-1b. Here the passive convention has not been used. Instead, the current enters the circuit element at the − terminal of the voltage and exits at the + terminal. In this case, the voltage pushes a positive charge in the direction opposite to the direction indicated by the current. Accordingly, when the element voltage and current do not adhere to the passive convention, the power calculated by multiplying the element voltage by the element current is the power **supplied** by the element. The power supplied by an element can be either positive or negative, depending on the values of the element voltage and current.

The power received by an element and the power supplied by that same element are related by

$$\text{power received} = -\text{power supplied}$$

The rules for the passive convention are summarized in Table 1.5-1. When the element voltage and current adhere to the passive convention, the energy received by an element can be determined

**Table 1.5-1  Power Received or Supplied by an Element**

| POWER RECEIVED BY AN ELEMENT | POWER SUPPLIED BY AN ELEMENT |
| --- | --- |
|  |  |
| Because the reference directions of $v$ and $i$ adhere to the passive convention, the power | Because the reference directions of $v$ and $i$ do not adhere to the passive convention, the power |
| $$p = vi$$ | $$p = vi$$ |
| is the power received by the element. | is the power supplied by the element. |

from Eq. 1.5-1 by rewriting it as

$$dw = p \, dt \tag{1.5-3}$$

On integrating, we have

$$w = \int_{-\infty}^{t} p \, d\tau \tag{1.5-4}$$

If the element only receives power for $t \geq t_0$ and we let $t_0 = 0$, then we have

$$w = \int_{0}^{t} p \, d\tau \tag{1.5-5}$$

## EXAMPLE 1.5-1 Electrical Power and Energy
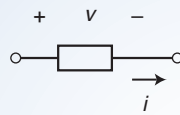


FIGURE 1.5-2 The element considered in Example 1.5-1.

Let us consider the element shown in Figure 1.5-2 when $v = 8$ V and $i = 25$ mA. Find the power received by the element and the energy received during a 10-ms interval.

### Solution

In Figure 1.5-2 the current $i$ and voltage $v$ adhere to the passive convention. Consequently the power

$$p = vi = 8 \, (0.025) = 0.2 \text{ W} = 200 \text{ mW}$$

is the power *received* by the circuit element. Next, the energy received by the element is

$$w = \int_{0}^{t} p \, dt = \int_{0}^{0.010} 0.2 \, dt = 0.2(0.010) = 0.002 \text{ J} = 2 \text{ mJ}$$

## EXAMPLE 1.5-2 Electrical Power and the Passive Convention
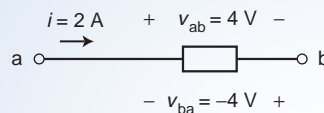


FIGURE 1.5-3 The element considered in Example 1.5-2.

Consider the element shown in Figure 1.5-3. The current $i$ and voltage $v_{ab}$ adhere to the passive convention, so

$$i \cdot v_{ab} = 2 \cdot (-4) = -8 \text{ W}$$

is the power *received* by this element. The current $i$ and voltage $v_{ba}$ do not adhere to the passive convention, so

$$i \cdot v_{ba} = 2 \cdot (4) = 8 \text{ W}$$

is the power *supplied* by this element. As expected

$$\text{power received} = -\text{power supplied}$$

### EXAMPLE 1.5-3 Power, Energy, and the Passive Convention

Consider the circuit shown in Figure 1.5-4 with $v(t) = 12e^{-8t}$ V and $i(t) = 5e^{-8t}$ A for $t \geq 0$. Both $v(t)$ and $i(t)$ are zero for $t < 0$. Find the power supplied by this element and the energy supplied by the element over the first 100 ms of operation.

$i(t)$    −    $v(t)$   +
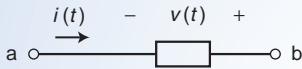
a            b    **FIGURE 1.5-4** The element considered in Example 1.5-3.

### Solution

The power

$$p(t) = v(i)\, i(t) = \left(12e^{-8t}\right)\left(5e^{-8t}\right) = 60e^{-16t} \text{ W}$$

is the power *supplied* by the element because $v(t)$ and $i(t)$ do not adhere to the passive convention. This element is supplying power to the charge flowing through it.

The energy supplied during the first 100 ms $= 0.1$ seconds is

$$w(0.1) = \int_0^{0.1} p\, dt = \int_0^{0.1} \left(60e^{-16t}\right) dt$$

$$= 60 \left. \frac{e^{-16t}}{-16} \right|_0^{0.1} = -\frac{60}{16}\left(e^{-1.6} - 1\right) = 3.75\left(1 - e^{-1.6}\right) = 2.99 \text{ J}$$

### EXAMPLE 1.5-4 Energy in a Thunderbolt

The average current in a typical lightning thunderbolt is $2 \times 10^4$ A, and its typical duration is 0.1 s (Williams, 1988). The voltage between the clouds and the ground is $5 \times 10^8$ V. Determine the total charge transmitted to the earth and the energy released.

### Solution

The total charge is

$$Q = \int_0^{0.1} i(t)\, dt = \int_0^{0.1} 2 \times 10^4\, dt = 2 \times 10^3 \text{ C}$$

The total energy released is

$$w = \int_0^{0.1} i(t) \times v(t)\, dt = \int_0^{0.1} \left(2 \times 10^4\right)\left(5 \times 10^8\right) dt = 10^{12} \text{ J} = 1 \text{ TJ}$$

**EXERCISE 1.5-1** Figure E 1.5-1 shows four circuit elements identified by the letters $A$, $B$, $C$, and $D$.

(a) Which of the devices supply 12 W?

(b) Which of the devices absorb 12 W?

(c)   What is the value of the power received by device *B*?

(d)   What is the value of the power delivered by device *B*?

(e)   What is the value of the power delivered by device *D*?



+   4 V   −          −   2 V   +          +   6 V   −          −   3 V   +

3 A                6 A                2 A                4 A

(A)                (B)                (C)                (D)
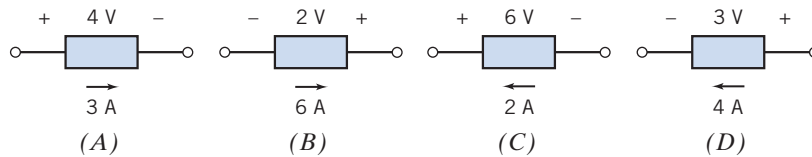
**FIGURE E 1.5-1**

*Answers:* **(a)** *B* and *C*, **(b)** *A* and *D*, **(c)** −12 W, **(d)** 12 W, **(e)** −12 W

## 1.6   *Circuit Analysis and Design*

The analysis and design of electric circuits are the primary activities described in this book and are key skills for an electrical engineer. The *analysis* of a circuit is concerned with the methodical study of a given circuit designed to obtain the magnitude and direction of one or more circuit variables, such as a current or voltage.

The analysis process begins with a statement of the problem and usually includes a given circuit model. The goal is to determine the magnitude and direction of one or more circuit variables, and the final task is to verify that the proposed solution is indeed correct. Usually, the engineer first identifies what is known and the principles that will be used to determine the unknown variable.

The problem-solving method that will be used throughout this book is shown in Figure 1.6-1. Generally, the problem statement is given. The analysis process then moves sequentially through the five steps shown in Figure 1.6-1. First, we describe the situation and the assumptions. We also record or review the circuit model that is provided. Second, we state the goals and requirements, and we



Problem          State the problem.

Situation        Describe the situation and the assumptions.

Goal             State the goals and requirements.

Plan             Generate a plan to obtain a solution of the problem.

Act              Act on the plan.

Verify           Verify that the proposed solution is indeed correct.

Incorrect        Correct

Solution         Communicate the solution.
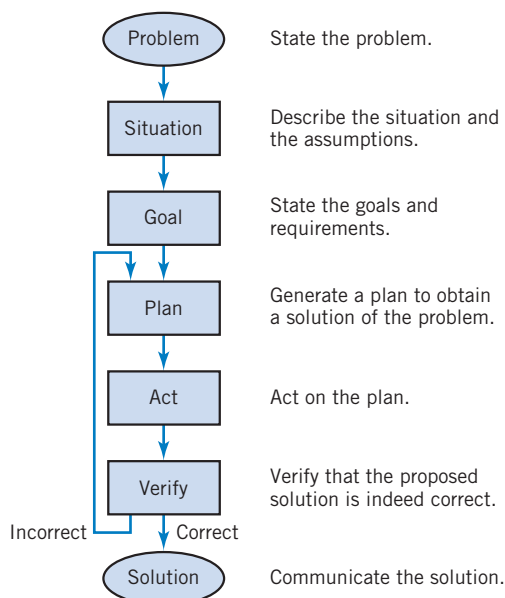
**FIGURE 1.6-1** The problem-solving method.

normally record the required circuit variable to be determined. The third step is to create a plan that will help obtain the solution of the problem. Typically, we record the principles and techniques that pertain to this problem. The fourth step is to act on the plan and carry out the steps described in the plan. The final step is to verify that the proposed solution is indeed correct. If it is correct, we communicate this solution by recording it in writing or by presenting it verbally. If the verification step indicates that the proposed solution is incorrect or inadequate, then we return to the plan steps, reformulate an improved plan, and repeat steps 4 and 5.

To illustrate this analytical method, we will consider an example. In Example 1.6-1, we use the steps described in the problem-solving method of Figure 1.6-1.

EXAMPLE **1.6-1**   The Formal Problem-Solving Method

An experimenter in a lab assumes that an element is absorbing power and uses a voltmeter and ammeter to measure the voltage and current as shown in Figure 1.6-2. The measurements indicate that the voltage is $v = +12$ V and the current is $i = -2$ A. Determine whether the experimenter's assumption is correct.

**Describe the Situation and the Assumptions:** Strictly speaking, the element *is* absorbing power. The value of the power absorbed by the element may be positive or zero or negative. When we say that someone "assumes that an element is absorbing power," we mean that someone assumes that the power absorbed by the element is positive.

The meters are ideal. These meters have been connected to the element in such a way as to measure the voltage labeled $v$ and the current labeled $i$. The values of the voltage and current are given by the meter readings.

**State the Goals:** Calculate the power absorbed by the element to determine whether the value of the power absorbed is positive.

**Generate a Plan:** Verify that the element voltage and current adhere to the passive convention. If so, the power absorbed by the device is $p = vi$. If not, the power absorbed by the device is $p = -vi$.

**Act on the Plan:** Referring to Table 1.5-1, we see that the element voltage and current do adhere to the passive convention. Therefore, power absorbed by the element is

$$p = vi = 12 \cdot (-2) = -24 \text{ W}$$

The value of the power absorbed is not positive.

**Verify the Proposed Solution:** Let's reverse the ammeter probes as shown in Figure 1.6-3. Now the ammeter measures the current $i_1$ rather than the current $i$, so $i_1 = 2$ A and $v = 12$ V. Because $i_1$ and $v$ do not adhere to the passive convention, $p = i_1 \cdot v = 24$ W is the power supplied by the element. Supplying 24 W is equivalent to absorbing $-24$ W, thus verifying the proposed solution.
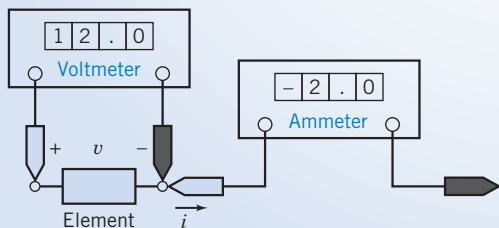


FIGURE 1.6-2 An element with a voltmeter and ammeter.
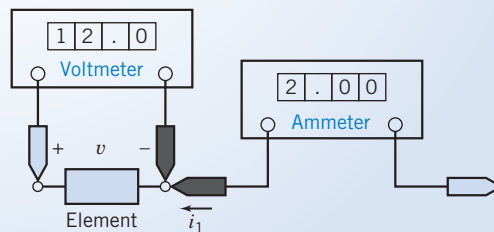


FIGURE 1.6-3 The circuit from Figure 1.6-2 with the ammeter probes reversed.

*Design* is a purposeful activity in which a designer visualizes a desired outcome. It is the process of originating circuits and predicting how these circuits will fulfill objectives. Engineering design is the process of producing a set of descriptions of a circuit that satisfy a set of performance requirements and constraints.

The design process may incorporate three phases: analysis, synthesis, and evaluation. The first task is to diagnose, define, and prepare—that is, to understand the problem and produce an explicit statement of goals; the second task involves finding plausible solutions; the third concerns judging the validity of solutions relative to the goals and selecting among alternatives. A cycle is implied in which the solution is revised and improved by reexamining the analysis. These three phases are part of a framework for planning, organizing, and evolving design projects.

> **Design** is the process of creating a circuit to satisfy a set of goals.

The problem-solving process shown in Figure 1.6-1 is used in Design Examples included in each chapter.

## 1.7  *How Can We Check . . . ?*

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able quickly to identify those solutions that need more work.

This text includes some examples that illustrate techniques useful for checking the solutions of the particular problems discussed in that chapter. At the end of each chapter, some problems are presented that provide an opportunity to practice these techniques.

---

**EXAMPLE 1.7-1**   How Can We Check Power and the Passive Convention?

A laboratory report states that the measured values of $v$ and $i$ for the circuit element shown in Figure 1.7-1 are $-5$ V and 2 A, respectively. The report also states that the power absorbed by the element is 10 W. **How can we check** the reported value of the power absorbed by this element?
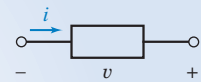


FIGURE 1.7-1 A circuit element with measured voltage and current.

### Solution

Does the circuit element absorb $-10$ W or $+10$ W? The voltage and current shown in Figure 1.7-1 do not adhere to the passive sign convention. Referring to Table 1.5-1, we see that the product of this voltage and current is the power supplied by the element rather than the power absorbed by the element.

Then the power supplied by the element is

$$p = vi = (-5)(2) = -10 \text{ W}$$

The power absorbed and the power supplied by an element have the same magnitude but the opposite sign. Thus, we have verified that the circuit element is indeed absorbing 10 W.

---

**1.8  DESIGN EXAMPLE**  Jet Valve Controller

---

A small, experimental space rocket uses a two-element circuit, as shown in Figure 1.8-1, to control a jet valve from point of liftoff at $t=0$ until expiration of the rocket after one minute. The energy that must be supplied by element 1 for the one-minute period is 40 mJ. Element 1 is a battery to be selected.

It is known that $i(t)=De^{-t/60}$ mA for $t \geq 0$, and the voltage across the second element is $v_2(t)=Be^{-t/60}$ V for $t \geq 0$. The maximum magnitude of the current, $D$, is limited to 1 mA. Determine the required constants $D$ and $B$ and describe the required battery.
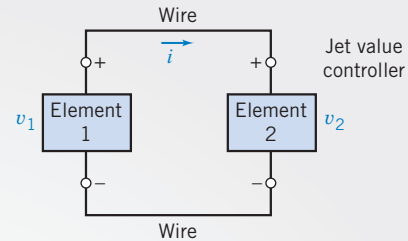


FIGURE 1.8-1 The circuit to control a jet valve for a space rocket.

### Describe the Situation and the Assumptions

1.  The current enters the plus terminal of the second element.
2.  The current leaves the plus terminal of the first element.
3.  The wires are perfect and have no effect on the circuit (they do not absorb energy).
4.  The model of the circuit, as shown in Figure 1.8-1, assumes that the voltage across the two elements is equal; that is, $v_1 = v_2$.
5.  The battery voltage $v_1$ is $v_1 = Be^{-t/60}$ V where $B$ is the initial voltage of the battery that will discharge exponentially as it supplies energy to the valve.
6.  The circuit operates from $t=0$ to $t=60$ s.
7.  The current is limited, so $D \leq 1$ mA.

### State the Goal

Determine the energy supplied by the first element for the one-minute period and then select the constants $D$ and $B$. Describe the battery selected.

### Generate a Plan

First, find $v_1(t)$ and $i(t)$ and then obtain the power, $p_1(t)$, supplied by the first element. Next, using $p_1(t)$, find the energy supplied for the first 60 s.

| GOAL | EQUATION | NEED | INFORMATION |
|------|----------|------|-------------|
| The energy $w_1$ for the first 60 s | $w_1 = \displaystyle\int_0^{60} p_1(t)\ dt$ | $p_1(t)$ | $v_1$ and $i$ known except for constants $D$ and $B$ |

### Act on the Plan

First, we need $p_1(t)$, so we first calculate

$$\begin{aligned} p_1(t) &= iv_1 = \left(De^{-t/60} \times 10^{-3} \text{ A}\right)\left(Be^{-t/60} \text{ V}\right) \\ &= DBe^{-t/30} \times 10^{-3} \text{ W} = DBe^{-t/30} \text{ mW} \end{aligned}$$

Second, we need to find $w_1$ for the first 60 s as

$$w_1 = \int_0^{60} \left( DBe^{-t/30} \times 10^{-3} \right) dt = \left. \frac{DB \times 10^{-3} e^{-t/30}}{-1/30} \right|_0^{60}$$
$$= -30DB \times 10^{-3}(e^{-2} - 1) = 25.9DB \times 10^{-3} \text{ J}$$

Because we require $w_1 \geq 40$ mJ,

$$40 \leq 25.9DB$$

Next, select the limiting value, $D = 1$, to get

$$B \geq \frac{40}{(25, .9)(1)} = 1.54 \text{ V}$$

Thus, we select a 2-V battery so that the magnitude of the current is less than 1 mA.

### Verify the Proposed Solution

We must verify that at least 40 mJ is supplied using the 2-V battery. Because $i = e^{-t/60}$ mA and $v_2 = 2e^{-t/60}$ V, the energy supplied by the battery is

$$w = \int_0^{60} \left( 2e^{-t/60} \right) \left( e^{-t/60} \times 10^{-3} \right) dt = \int_0^{60} 2e^{-t/30} \times 10^{-3} \, dt = 51.8 \text{ mJ}$$

Thus, we have verified the solution, and we communicate it by recording the requirement for a 2-V battery.

## 1.9   SUMMARY

○ Charge is the intrinsic property of matter responsible for electric phenomena. The current in a circuit element is the rate of movement of charge through the element. The voltage across an element indicates the energy available to cause charge to move through the element.

○ Given the current, $i$, and voltage, $v$, of a circuit element, the power, $p$, and energy, $w$, are given by

$$p = v \cdot i \quad \text{and} \quad w = \int_0^t p \, d\tau$$

○ Table 1.5-1 summarizes the use of the passive convention when calculating the power supplied or received by a circuit element.

○ The SI units (Table 1.3-1) are used by today's engineers and scientists. Using decimal prefixes (Table 1.3-3), we may simply express electrical quantities with a wide range of magnitudes.

## PROBLEMS

➕ Problem available in WileyPLUS at instructor's discretion.

### Section 1.2  Electric Circuits and Current

**P 1.2-1** ➕ The total charge that has entered a circuit element is $q(t) = 1.25(1 - e^{-5t})$ when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t \geq 0$.

*Answer:* $i(t) = 6.25e^{-5t}$ A

**P 1.2-2** ➕ The current in a circuit element is $i(t) = 4(1 - e^{-5t})$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

*Hint:* $q(0) = \int_{-\infty}^0 i(\tau) \, d\tau = \int_{-\infty}^0 0 \, d\tau = 0$

*Answer:* $q(t) = 4t + 0.8e^{-5t} - 0.8$ C for $t \geq 0$

**P 1.2-3** ➕ The current in a circuit element is $i(t) = 4 \sin 5t$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

*Hint:* $q(0) = \int_{-\infty}^0 i(\tau) \, d\tau = \int_{-\infty}^0 0 \, d\tau = 0$

**P 1.2-4**  The current in a circuit element is

$$i(t) = \begin{cases} 0 & t < 2 \\ 2 & 2 < t < 4 \\ -1 & 4 < t < 8 \\ 0 & 8 < t \end{cases}$$

where the units of current are A and the units of time are s. Determine the total charge that has entered a circuit element for $t \geq 0$.

*Answer:*

$$q(t) = \begin{cases} 0 & t < 2 \\ 2t - 4 & 2 < t < 4 \\ 8 - t & 4 < t < 8 \\ 0 & 8 < t \end{cases} \quad \text{where the units of}$$

charge are C.

**P 1.2-5**  ⊕  The total charge $q(t)$, in coulombs, that enters the terminal of an element is

$$q(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 \leq t \leq 2 \\ 3 + e^{-2(t-2)} & t > 2 \end{cases}$$

Find the current $i(t)$ and sketch its waveform for $t \geq 0$.

**P 1.2-6**  ⊕  An electroplating bath, as shown in Figure P 1.2-6, is used to plate silver uniformly onto objects such as kitchenware and plates. A current of 450 A flows for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams?
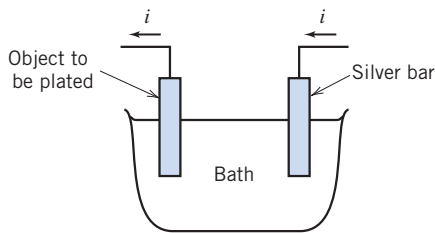
**Figure P 1.2-6** An electroplating bath.

**P 1.2-7**  Find the charge $q(t)$ and sketch its waveform when the current entering a terminal of an element is as shown in Figure P 1.2-7. Assume that $q(t) = 0$ for $t < 0$.
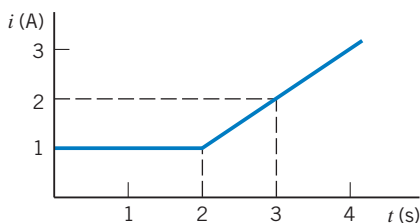
**Figure P 1.2-7**

**Section 1.3  Systems of Units**

**P 1.3-1**  ⊕  A constant current of 3.2 $\mu$A flows through an element. What is the charge that has passed through the element in the first millisecond?

*Answer:* 3.2 nC

**P 1.3-2**  ⊕  A charge of 45 nC passes through a circuit element during a particular interval of time that is 5 ms in duration. Determine the average current in this circuit element during that interval of time.

*Answer:* $i = 9 \mu$A

**P 1.3-3**  ⊕  Ten billion electrons per second pass through a particular circuit element. What is the average current in that circuit element?

*Answer:* $i = 1.602$ nA

**P 1.3-4**  The charge flowing in a wire is plotted in Figure P 1.3-4. Sketch the corresponding current.
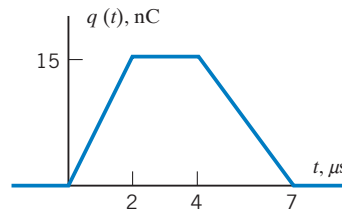
**Figure P 1.3-4**

**P 1.3-5**  The current in a circuit element is plotted in Figure P 1.3-5. Sketch the corresponding charge flowing through the element for $t > 0$.
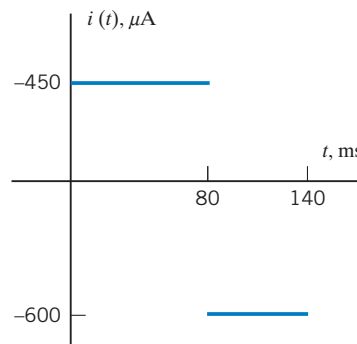
**Figure P 1.3-5**

**P 1.3-6**  The current in a circuit element is plotted in Figure P 1.3-6. Determine the total charge that flows through the circuit element between 300 and 1200 $\mu$s.
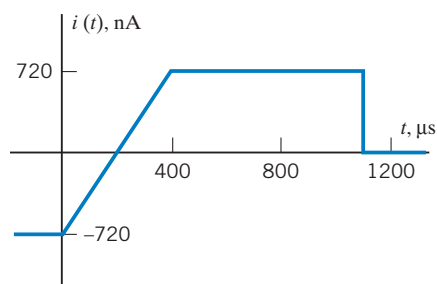
**Figure P 1.3-6**

## Section 1.5  Power and Energy

**P 1.5-1** ⊕  Figure P 1.5-1 shows four circuit elements identified by the letters A, B, C, and D.

**(a)** Which of the devices supply 30 mW?
**(b)** Which of the devices absorb 0.03 W?
**(c)** What is the value of the power received by device B?
**(d)** What is the value of the power delivered by device B?
**(e)** What is the value of the power delivered by device C?
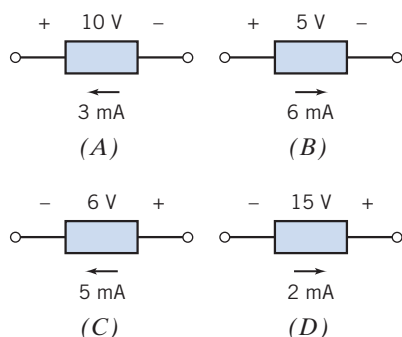


**Figure P 1.5-1**

**P 1.5-2** ⊕  An electric range has a constant current of 10 A entering the positive voltage terminal with a voltage of 110 V. The range is operated for two hours. (a) Find the charge in coulombs that passes through the range. (b) Find the power absorbed by the range. (c) If electric energy costs 12 cents per kilowatt-hour, determine the cost of operating the range for two hours.

**P 1.5-3** A walker's cassette tape player uses four AA batteries in series to provide 6 V to the player circuit. The four alkaline battery cells store a total of 200 watt-seconds of energy. If the cassette player is drawing a constant 10 mA from the battery pack, how long will the cassette operate at normal power?

**P 1.5-4** The current through and voltage across an element vary with time as shown in Figure P 1.5-4. Sketch the power delivered to the element for $t > 0$. What is the total energy delivered to the element between $t = 0$ and $t = 25$ s? The element voltage and current adhere to the passive convention.
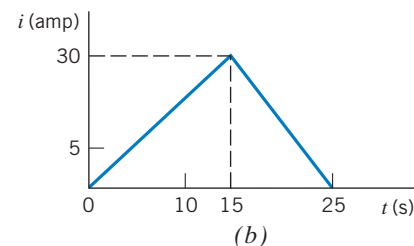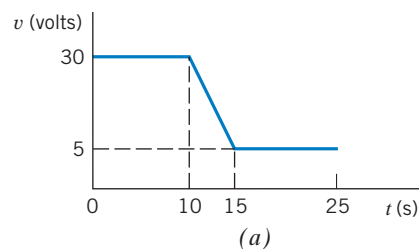


**Figure P 1.5-4** (a) Voltage v(t) and (b) current i(t) for an element.

**P 1.5-5** ⊕  An automobile battery is charged with a constant current of 2 A for five hours. The terminal voltage of the battery is $v = 11 + 0.5t$ V for $t > 0$, where $t$ is in hours. (a) Find the energy delivered to the battery during the five hours. (b) If electric energy costs 15 cents/kWh, find the cost of charging the battery for five hours.

*Answer:* (b) 1.84 cents

**P 1.5-6** ⊕  Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 4 \cos 3t$ V and $i(t) = \dfrac{\sin 3t}{12}$ A. Evaluate $p(t)$ at $t = 0.5$ s and at $t = 1$ s. Observe that the power supplied by this element has a positive value at some times and a negative value at other times.

*Hint:* $(\sin at)(\cos bt) = \dfrac{1}{2}(\sin(a+b)t + \sin(a-b)t)$

*Answer:*

$$p(t) = \frac{1}{6}\sin 6t \text{ W}, \quad p(0.5) = 0.0235 \text{ W}, \quad p(1) = -0.0466 \text{ W}$$
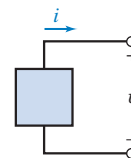


**Figure P 1.5-6** An element.

**P 1.5-7** ⊕  Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 8 \sin 3t$ V and $i(t) = 2 \sin 3t$ A.

*Hint:* $(\sin at)(\sin bt) = \dfrac{1}{2}(\cos(a-b)t - \cos(a+b)t)$

*Answer:* $p(t) = 8 - 8\cos 6t$ W

**P 1.5-8** ➕ Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6. The element voltage is represented as $v(t) = 4(1-e^{-2t})$V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The element current is represented as $i(t) = 2e^{-2t}$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$.

*Answer:* $p(t) = 8(1 - e^{-2t})e^{-2t}$ W

**P 1.5-9** ➕ The battery of a flashlight develops 3 V, and the current through the bulb is 200 mA. What power is absorbed by the bulb? Find the energy absorbed by the bulb in a five-minute period.

**P 1.5-10** Medical researchers studying hypertension often use a technique called "2D gel electrophoresis" to analyze the protein content of a tissue sample. An image of a typical "gel" is shown in Figure P1.5-10a.

The procedure for preparing the gel uses the electric circuit illustrated in Figure 1.5-10b. The sample consists of a gel and a filter paper containing ionized proteins. A voltage source causes a large, constant voltage, 500 V, across the sample. The large, constant voltage moves the ionized proteins from the filter paper to the gel. The current in the sample is given by

$$i(t) = 2 + 30e^{-at} \text{ mA}$$

where $t$ is the time elapsed since the beginning of the procedure and the value of the constant $a$ is

$$a = 0.85 \frac{1}{hr}$$

Determine the energy supplied by the voltage source when the gel preparation procedure lasts 3 hours.
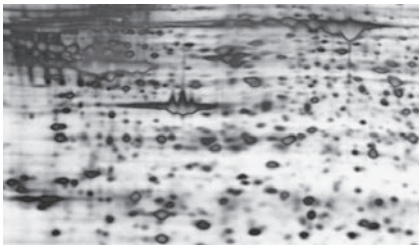


Devon Svoboda, Queen's University
*(a)*



*(b)*
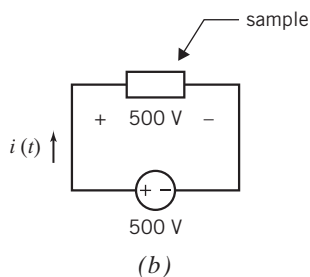
**Figure P 1.5-10** (*a*) An image of a gel and (*b*) the electric circuit used to prepare gel.

## Section 1.7  How Can We Check . . . ?

**P 1.7-1** ➕ Conservation of energy requires that the sum of the power received by all of the elements in a circuit be zero. Figure P 1.7-1 shows a circuit. All of the element voltages and

currents are specified. Are these voltage and currents correct? Justify your answer.

*Hint:* Calculate the power received by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.
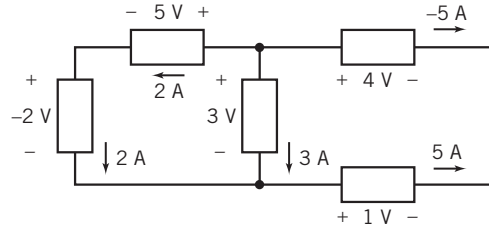


**Figure P 1.7-1**

**P 1.7-2** ➕ Conservation of energy requires that the sum of the power received by all of the elements in a circuit be zero. Figure P 1.7-2 shows a circuit. All of the element voltages and currents are specified. Are these voltage and currents correct? Justify your answer.

*Hint:* Calculate the power received by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.
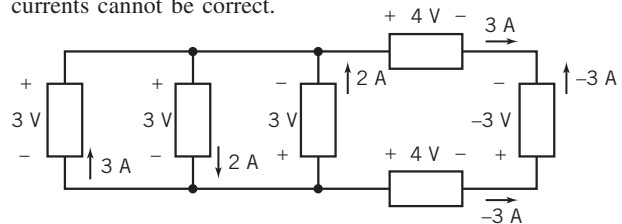


**Figure P 1.7-2**

**P 1.7-3** ➕ The element currents and voltages shown in Figure P 1.7-3 are correct with one exception: the reference direction of exactly one of the element currents is reversed. Determine which reference direction has been reversed.
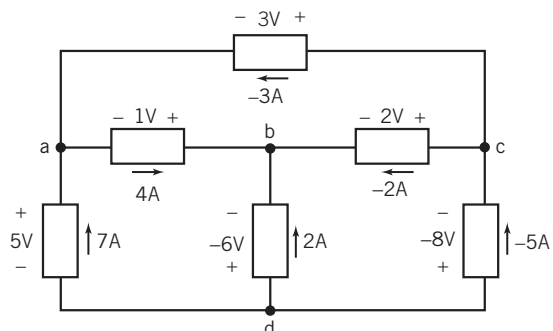


**Figure P 1.7-3**

# Design Problems

**DP 1-1** A particular circuit element is available in three grades. Grade A guarantees that the element can safely absorb $1/2$ W continuously. Similarly, Grade B guarantees that $1/4$ W can be absorbed safely, and Grade C guarantees that $1/8$ W can be absorbed safely. As a rule, elements that can safely absorb more power are also more expensive and bulkier.

The voltage across an element is expected to be about 20 V, and the current in the element is expected to be about 8 mA. Both estimates are accurate to within 25 percent. The voltage and current reference adhere to the passive convention.

Specify the grade of this element. Safety is the most important consideration, but don't specify an element that is more expensive than necessary.

**DP 1-2** The voltage across a circuit element is $v(t) = 20\,(1-e^{-8t})$ V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The current in this element is $i(t) = 30e^{-8t}$ mA when $t \geq 0$ and $i(t) = 0$ when $t < 0$. The element current and voltage adhere to the passive convention. Specify the power that this device must be able to absorb safely.

*Hint:* Use MATLAB, or a similar program, to plot the power.

# CHAPTER 2 ⬤ *Circuit Elements*

## IN THIS CHAPTER

## 2.1    *Introduction*

Not surprisingly, the behavior of an electric circuit depends on the behaviors of the individual circuit elements that comprise the circuit. Of course, different types of circuit elements behave differently. The equations that describe the behaviors of the various types of circuit elements are called the constitutive equations. Frequently, the constitutive equations describe a relationship between the current and voltage of the element. Ohm's law is a well-known example of a constitutive equation.

In this chapter, we will investigate the behavior of several common types of circuit element:

- Resistors.
- Independent voltage and current sources.
- Open circuits and short circuits.
- Voltmeters and ammeters.
- Dependent sources.
- Transducers.
- Switches.

## 2.2    *Engineering and Linear Models*

The art of engineering is to take a bright idea and, using money, materials, knowledgeable people, and a regard for the environment, produce something the buyer wants at an affordable price.

Engineers use *models* to represent the elements of an electric circuit. A model is a description of those properties of a device that we think are important. Frequently, the model will consist of an equation relating the element voltage and current. Though the model is different from the electric device, the model can be used in pencil-and-paper calculations that will predict how a circuit composed of actual devices will operate. Engineers frequently face a trade-off when selecting a model for a device. Simple models are easy to work with but may not be accurate. Accurate models are usually more complicated and harder to use. The conventional wisdom suggests that simple models be used first. The results obtained using the models must be checked to verify that use of these simple models is appropriate. More accurate models are used when necessary.

The idealized models of electric devices are precisely defined. It is important to distinguish between actual devices and their idealized models, which we call circuit elements. The goal of circuit analysis is to predict the quantitative electrical behavior of physical circuits. Its aim is to predict and to explain the terminal voltages and terminal currents of the circuit elements and thus the overall operation of the circuit.

Models of circuit elements can be categorized in a variety of ways. For example, it is important to distinguish linear models from nonlinear models because circuits that consist entirely of linear circuit elements are easier to analyze than circuits that contain some nonlinear elements.

An element or circuit is *linear* if the element's excitation and response satisfy certain properties. Consider the element shown in Figure 2.2-1. Suppose that the excitation is the current $i$ and the response is the voltage $v$. When the element is subjected to a current $i_1$, it provides a response $v_1$. Furthermore, when the element is subjected to a current $i_2$, it provides a response $v_2$. For a linear element, it is necessary that the excitation $i_1 + i_2$ result in a response $v_1 + v_2$. This is usually called the *principle of superposition.*

Also, multiplying the input of a linear device by a constant must have the consequence of multiplying the output by the same constant. For example, doubling the size of the input causes the size of the output to double. This is called the *property of homogeneity.* An element is linear if, and only if, the properties of superposition and homogeneity are satisfied for all excitations and responses.
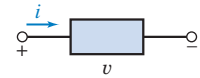
**FIGURE 2.2-1**

An element with an excitation current $i$ and a response $v$.

A **linear element** satisfies the properties of both superposition and homogeneity.

Let us restate mathematically the two required properties of a linear circuit, using the arrow notation to imply the transition from excitation to response:

$$i \rightarrow v$$

Then we may state the two properties required as follows.
*Superposition:*

$$i_1 \rightarrow v_1$$
$$i_2 \rightarrow v_2$$

then

$$i_1 + i_2 \rightarrow v_1 + v_2 \tag{2.2-1}$$

*Homogeneity:*

$$i \rightarrow v$$

then

$$ki \rightarrow kv \tag{2.2-2}$$

A device that does not satisfy either the superposition or the homogeneity principle is said to be nonlinear.

**E XAMPLE 2.2-1**    A Linear Device

Consider the element represented by the relationship between current and voltage as

$$v = Ri$$

Determine whether this device is linear.

## Solution

The response to a current $i_1$ is
$$v_1 = Ri_1$$

The response to a current $i_2$ is
$$v_2 = Ri_2$$

The sum of these responses is

$$v_1 + v_2 = Ri_1 + Ri_2 = R(i_1 + i_2)$$

Because the sum of the responses to $i_1$ and $i_2$ is equal to the response to $i_1 + i_2$, the principle of superposition is satisfied. Next, consider the principle of homogeneity. Because

$$v_1 = Ri_1$$

we have for an excitation $i_2 = ki_1$

$$v_2 = Ri_2 = Rki_1$$

Therefore,
$$v_2 = kv_1$$

satisfies the principle of homogeneity. Because the element satisfies the properties of both superposition and homogeneity, it is linear.

---

### EXAMPLE 2.2-2    A Nonlinear Device

Now let us consider an element represented by the relationship between current and voltage:
$$v = i^2$$

Determine whether this device is linear.

## Solution

The response to a current $i_1$ is
$$v_1 = i_1^2$$

The response to a current $i_2$ is
$$v_2 = i_1^2$$

The sum of these responses is

$$v_1 + v_2 = i_1^2 + i_1^2$$

The response to $i_1 + i_2$ is

$$(i_1 + i_2)^2 = i_1^2 + 2i_1i_2 + i_1^2$$

Because

$$i_1^2 + i_1^2 \neq (i_1 + i_2)^2$$

the principle of superposition is not satisfied. Therefore, the device is nonlinear.

EXAMPLE 2.2-3   A Model of a Linear Device

A linear element has voltage $v$ and current $i$ as shown in Figure 2.2-2a. Values of the current $i$ and corresponding voltage $v$ have been tabulated as shown in Figure 2.2-2b. Represent the element by an equation that expresses $v$ as a function of $i$. This equation is a model of the element. Use the model to predict the value of $v$ corresponding to a current of $i = 100\,mA$ and the value of $i$ corresponding to a voltage of $v = 18\,V$.

| $v$, V | $i$, mA |
|--------|---------|
| 4.5    | 10      |
| 11.25  | 25      |
| 22.5   | 50      |

(a)            (b)
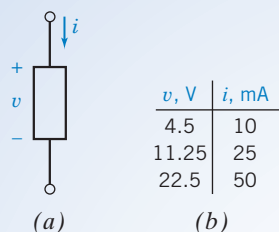
**FIGURE 2.2-2** (a) A linear circuit element and (b) a tabulation of corresponding values of its voltage and current.
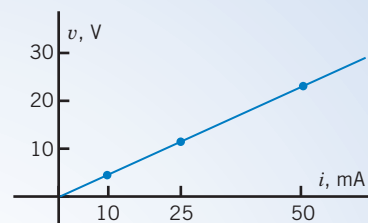
**FIGURE 2.2-3** A plot of voltage versus current for the linear element from Figure 2.2-2.

## Solution

Figure 2.2-3 is a plot of the voltage $v$ versus the current $i$. The points marked by dots represent corresponding values of $v$ and $i$ from the rows of the table in Figure 2.2-2b. Because the circuit element is linear, we expect these points to lie on a straight line, and indeed they do. We can represent the straight line by the equation

$$v = mi + b$$

where $m$ is the slope and $b$ is the $v$-intercept. Noticing that the straight line passes through the origin, $v = 0$ when $i = 0$, we see that $b = 0$. We are left with

$$v = mi$$

The slope $m$ can be calculated from the data in any two rows of the table in Figure 2.2-2b. For example:

$$\frac{11.25 - 4.5}{25 - 10} = 0.45\ \frac{V}{mA}, \frac{22.5 - 11.25}{50 - 25} = 0.45\ \frac{V}{mA}, \text{and } \frac{22.5 - 4.5}{50 - 10} = 0.45\ \frac{V}{mA}$$

Consequently,

$$m = 0.45\ \frac{V}{mA} = 450\ \frac{V}{A}$$

and

$$v = 450i$$

This equation is a model of the linear element. It predicts that the voltage $v = 450(0.1) = 45\,V$ corresponds to the current $i = 100\,mA = 0.1\,A$ and that the current $i = 18/450 = 0.04\,A = 40\,mA$ corresponds to the voltage $v = 18\,V$.

## 2.3    Active and Passive Circuit Elements

We may classify circuit elements in two categories, *passive* and *active*, by determining whether they absorb energy or supply energy. An element is said to be passive if the total energy delivered to it from the rest of the circuit is always nonnegative (zero or positive). Then for a passive element, with the current flowing into the + terminal as shown in Figure 2.3-1a, this means that

$$w = \int_{-\infty}^{t} vi\, d\tau \geq 0 \qquad\qquad (2.3\text{-}1)$$

for all values of $t$.

A **passive element** absorbs energy.

Entry node        Exit node



Exit node        Entry node

(a)        (b)
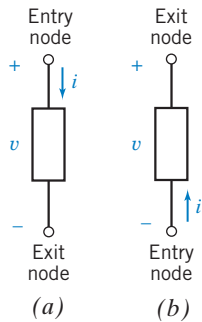
**FIGURE 2.3-1** (a) The entry node of the current $i$ is the positive node of the voltage $v$; (b) the entry node of the current $i$ is the negative node of the voltage $v$. The current flows from the entry node to the exit node.

An element is said to be *active* if it is capable of delivering energy. Thus, an active element violates Eq. 2.3-1 when it is represented by Figure 2.3-1a. In other words, an active element is one that is capable of generating energy. Active elements are potential sources of energy, whereas passive elements are sinks or absorbers of energy. Examples of active elements include batteries and generators. Consider the element shown in Figure 2.3-1b. Note that the current flows into the negative terminal and out of the positive terminal. This element is said to be active if

$$w = \int_{-\infty}^{t} vi\, d\tau \geq 0 \qquad\qquad (2.3\text{-}2)$$

for at least one value of $t$.

An **active element** is capable of supplying energy.

**E XAMPLE 2.3-1** An Active Circuit Element

A circuit has an element represented by Figure 2.3-1b where the current is a constant 5 A and the voltage is a constant 6 V. Find the energy supplied over the time interval 0 to $T$.

**Solution**
Because the current enters the negative terminal, the energy *supplied* by the element is given by

$$w = \int_{0}^{T} (6)(5)d\tau = 30T \text{ J}$$

Thus, the device is a generator or an active element, in this case a dc battery.

## 2.4  *Resistors*

The ability of a material to resist the flow of charge is called its *resistivity*, $\rho$. Materials that are good electrical insulators have a high value of resistivity. Materials that are good conductors of electric current have low values of resistivity. Resistivity values for selected materials are given in Table 2.4-1. Copper is commonly used for wires because it permits current to flow relatively unimpeded. Silicon is commonly used to provide resistance in semiconductor electric circuits. Polystyrene is used as an insulator.

| Table 2.4-1  Resistivities of Selected Materials | |
| --- | --- |
| MATERIAL | RESISTIVITY $\rho$ (OHM.CM) |
| Polystyrene | $1 \times 10^{18}$ |
| Silicon | $2.3 \times 10^{5}$ |
| Carbon | $4 \times 10^{-3}$ |
| Aluminum | $2.7 \times 10^{-6}$ |
| Copper | $1.7 \times 10^{-6}$ |

**Resistance** is the physical property of an element or device that impedes the flow of current; it is represented by the symbol $R$.

Georg Simon Ohm was able to show that the current in a circuit composed of a battery and a conducting wire of uniform cross-section could be expressed as

$$i = \frac{Av}{\rho L} \tag{2.4-1}$$

where $A$ is the cross-sectional area, $\rho$ the resistivity, $L$ the length, and $v$ the voltage across the wire element. Ohm, who is shown in Figure 2.4-1, defined the constant resistance $R$ as

$$R = \frac{\rho L}{A} \tag{2.4-2}$$

Ohm's law, which related the voltage and current, was published in 1827 as

$$v = Ri \tag{2.4-3}$$

The unit of resistance $R$ was named the ohm in honor of Ohm and is usually abbreviated by the $\Omega$ (capital omega) symbol, where $1\ \Omega = 1$ V/A. The resistance of a 10-m length of common TV cable is 2 m$\Omega$.

An element that has a resistance $R$ is called a *resistor*. A resistor is represented by the two-terminal symbol shown in Figure 2.4-2. Ohm's law, Eq. 2.4-3, requires that the *i*-versus-*v* relationship be linear. As shown in Figure 2.4-3, a resistor may become nonlinear outside its normal rated range of operation. We will assume that a resistor is linear unless stated otherwise. Thus, we will use a linear model of the resistor as represented by Ohm's law.

In Figure 2.4-4, the element current and element voltage of a resistor are labeled. The relationship between the directions of this current and voltage is important. The voltage direction marks one resistor terminal $+$ and the other $-$. The current $i_a$ flows from the terminal marked $+$ to the terminal marked $-$. This relationship between the current and voltage reference directions is a convention called the passive convention. Ohm's law states that when the element voltage and the element current adhere to the passive convention, then

$$v = Ri_a \tag{2.4-4}$$

Photo by Hulton Archive/ Getty Images

**FIGURE 2.4-1** Georg Simon Ohm (1787–1854), who determined Ohm's law in 1827. The ohm was chosen as the unit of electrical resistance in his honor.
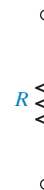


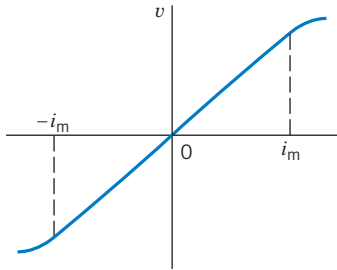**FIGURE 2.4-2** Symbol for a resistor having a resistance of $R$ ohms.

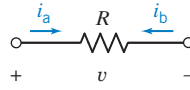**FIGURE 2.4-3** A resistor operating within its specified current range, $\pm i_m$, can be modeled by Ohm's law.



**FIGURE 2.4-4** A resistor with element current and element voltage.



*(a)*      *(b)*

Courtesy of Vishay Intertechnology, Inc.

**FIGURE 2.4-5** (*a*) Wirewound resistor with an adjustable center tap. (*b*) Wirewound resistor with a fixed tap.

Consider Figure 2.4-4. The element currents $i_a$ and $i_b$ are the same except for the assigned direction, so

$$i_a = -i_b$$

The element current $i_a$ and the element voltage $v$ adhere to the passive convention,

$$v = Ri_a$$

Replacing $i_a$ by $-i_b$ gives

$$v = -Ri_b$$

There is a minus sign in this equation because the element current $i_b$ and the element voltage $v$ do not adhere to the passive convention. We must pay attention to the current direction so that we don't overlook this minus sign.

Ohm's law, Eq. 2.4-3, can also be written as

$$i = Gv \tag{2.4-5}$$

where $G$ denotes the *conductance* in siemens (S) and is the reciprocal of $R$; that is, $G = 1/R$. Many engineers denote the units of conductance as mhos with the ℧ symbol, which is an inverted omega (mho is *ohm* spelled backward). However, we will use SI units and retain siemens as the units for conductance.

Most discrete resistors fall into one of four basic categories: carbon composition, carbon film, metal film, or wirewound. Carbon composition resistors have been in use for nearly 100 years and are still popular. Carbon film resistors have supplanted carbon composition resistors for many general-purpose uses because of their lower cost and better tolerances. Two wirewound resistors are shown in Figure 2.4-5.

Carbon composition resistors, as shown in Figure 2.4-6, are used in circuits because of their low cost and small size. General-purpose resistors are available in standard values for tolerances of 2, 5, 10, and 20 percent. Carbon composition resistors and some wirewounds have a color code with three to five bands. A color code is a system of standard colors adopted for identification of the resistance of resistors. Figure 2.4-7 shows a metal film resistor with its color bands. This is a 1/4-watt resistor, implying that it should be operated at or below 1/4 watt of power delivered to it. The normal range of resistors is from less than 1 ohm to 10 megohms. Typical values of some commercially available resistors are given in Appendix D.

Courtesy of Hifi Collective.
**FIGURE 2.4-6** Carbon composition resistors.
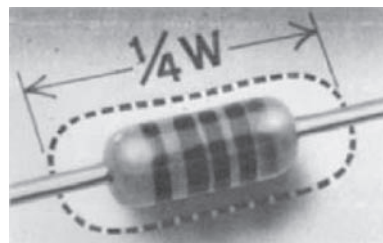


Courtesy of Vishay Intertechnology, Inc.
**FIGURE 2.4-7** A 1/4-watt metal film resistor. The body of the resistor is 6 mm long.

The power delivered to a resistor (when the passive convention is used) is

$$p = vi = v\left(\frac{v}{R}\right) = \frac{v^2}{R} \tag{2.4-6}$$

Alternatively, because $v = iR$, we can write the equation for power as

$$p = vi = (iR)i = i^2R \tag{2.4-7}$$

Thus, the power is expressed as a nonlinear function of the current $i$ through the resistor or of the voltage $v$ across it.

---

### EXAMPLE 2.4-1  Power Dissipated by a Resistor

Let us devise a model for a car battery when the lights are left on and the engine is off. We have all experienced or seen a car parked with its lights on. If we leave the car for a period, the battery will run down or go dead. An auto battery is a 12-V constant-voltage source, and the lightbulb can be modeled by a resistor of 6 ohms. The circuit is shown in Figure 2.4-8. Let us find the current $i$, the power $p$, and the energy supplied by the battery for a four-hour period.



**FIGURE 2.4-8** Model of a car battery and the headlight lamp.

#### Solution
According to Ohm's law, Eq. 2.4-3, we have

$$v = Ri$$

Because $v = 12$ V and $R = 6$ $\Omega$, we have $i = 2$ A.

To find the power delivered by the battery, we use

$$p = vi = 12(2) = 24 \text{ W}$$

Finally, the energy delivered in the four-hour period is

$$w = \int_0^t p \, d\tau = 24t = 24(60 \times 60 \times 4) = 3.46 \times 10^5 \text{ J}$$

Because the battery has a finite amount of stored energy, it will deliver this energy and eventually be unable to deliver further energy without recharging. We then say the battery is run down or dead until recharged. A typical auto battery may store $10^6$ J in a fully charged condition.
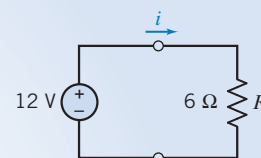
**EXERCISE 2.4-1**   Find the power absorbed by a 100-ohm resistor when it is connected directly across a constant 10-V source.

*Answer:* 1-W

**EXERCISE 2.4-2**   A voltage source $v = 10 \cos t$ V is connected across a resistor of 10 ohms. Find the power delivered to the resistor.

*Answer:* $10 \cos^2 t$ W

## 2.5   Independent Sources

Some devices are intended to supply energy to a circuit. These devices are called sources. Sources are categorized as being one of two types: voltage sources and current sources. Figure 2.5-1*a* shows the symbol that is used to represent a voltage source. The voltage of a voltage source is specified, but the current is determined by the rest of the circuit. A voltage source is described by specifying the function $v(t)$, for example,

$$v(t) = 12 \cos 1000t \quad \text{or} \quad v(t) = 9 \quad \text{or} \quad v(t) = 12 - 2t$$

An active two-terminal element that supplies energy to a circuit is a *source* of energy. An *independent voltage source* provides a specified voltage independent of the current through it and is independent of any other circuit variable.
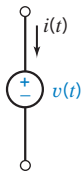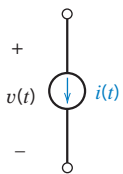
> A **source** is a voltage or current generator capable of supplying energy to a circuit.

An *independent current source* provides a current independent of the voltage across the source element and is independent of any other circuit variable. Thus, when we say a source is independent, we mean it is independent of any other voltage or current in the circuit.

> An **independent source** is a voltage or current generator not dependent on other circuit variables.

Suppose the voltage source is a battery and

$$v(t) = 9 \text{ volts}$$

The voltage of this battery is known to be 9 volts regardless of the circuit in which the battery is used. In contrast, the current of the voltage source is not known and depends on the circuit in which the source is used. The current could be 6 amps when the voltage source is connected to one circuit and 6 milliamps when the voltage source is connected to another circuit.

Figure 2.5-1*b* shows the symbol that is used to represent a current source. The current of a current source is specified, but the voltage is determined by the rest of the circuit. A current source is described by specifying the function $i(t)$, for example,

$$i(t) = 6 \sin 500t \quad \text{or} \quad i(t) = -0.25 \quad \text{or} \quad i(t) = t + 8$$

A current source specified by $i(t) = -0.25$ milliamps will have a current of $-0.25$ milliamps in any circuit in which it is used. The voltage across this current source will depend on the particular circuit.

The preceding paragraphs have ignored some complexities to give a simple description of the way sources work. The voltage across a 9-volt battery may not actually be 9 volts. This voltage depends on the age of the battery, the temperature, variations in manufacturing, and the battery

**FIGURE 2.5-1**
(*a*) Voltage source.
(*b*) Current source.

current. It is useful to make a distinction between real sources, such as batteries, and the simple voltage and current sources described in the preceding paragraphs. It would be *ideal* if the real sources worked like these simple sources. Indeed, the word *ideal* is used to make this distinction. The simple sources described in the previous paragraph are called the *ideal voltage source* and the *ideal current source*.

> The voltage of an **ideal voltage source** is given to be a specified function, say $v(t)$. The current is determined by the rest of the circuit.
>
> The current of an **ideal current source** is given to be a specified function, say $i(t)$. The voltage is determined by the rest of the circuit.
>
> An **ideal source** is a voltage or a current generator independent of the current through the voltage source or the voltage across the current source.

Engineers frequently face a trade-off when selecting a model for a device. Simple models are easy to work with but may not be accurate. Accurate models are usually more complicated and harder to use. The conventional wisdom suggests that simple models be used first. The results obtained using the models must be checked to verify that use of these simple models is appropriate. More accurate models are used when necessary.

## EXAMPLE 2.5-1  A Battery Modeled as a Voltage Source

Consider the plight of the engineer who needs to analyze a circuit containing a 9-volt battery. Is it really necessary for this engineer to include the dependence of battery voltage on the age of the battery, the temperature, variations in manufacturing, and the battery current in this analysis? Hopefully not. We expect the battery to act enough like an ideal 9-volt voltage source that the differences can be ignored. In this case, it is said that the battery is *modeled* as an ideal voltage source.

To be specific, consider a battery specified by the plot of voltage versus current shown in Figure 2.5-2a. This plot indicates that the battery voltage will be $v = 9$ volts when $i \leq 10$ milliamps. As the current increases above 10 milliamps, the voltage decreases from 9 volts. When $i \leq 10$ milliamps, the dependence of the battery voltage on the battery current can be ignored and the battery can be modeled as an ideal voltage source.
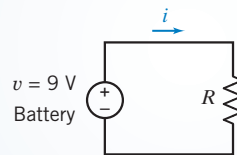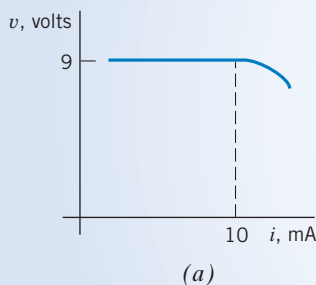


FIGURE 2.5-2 (*a*) A plot of battery voltage versus battery current. (*b*) The battery is modeled as an independent voltage source.

Suppose a resistor is connected across the terminals of the battery as shown in Figure 2.5-2b. The battery current will be

$$i = \frac{v}{R} \tag{2.5-1}$$

The relationship between $v$ and $i$ shown in Figure 2.5-2a complicates this equation. This complication can be safely ignored when $i \leq 10$ milliamps. When the battery is modeled as an ideal 9-volt voltage source, the voltage source current is given by

$$i = \frac{9}{R} \tag{2.5-2}$$

The distinction between these two equations is important. Eq. 2.5-1, involving the $v-i$ relationship shown in Figure 2.5-2a, is more accurate but also more complicated. Equation 2.5-2 is simpler but may be inaccurate.

Suppose that $R = 1000$ ohms. Equation 2.5-2 gives the current of the ideal voltage source:

$$i = \frac{9}{1000} = 9 \text{ mA} \tag{2.5-3}$$

Because this current is less than 10 milliamps, the ideal voltage source is a good model for the battery, and it is reasonable to expect that the battery current is 9 milliamps.

Suppose, instead, that $R = 600$ ohms. Once again, Eq. 2.5-2 gives the current of the ideal voltage source:

$$i = \frac{9}{600} = 15 \text{ mA} \tag{2.5-4}$$

Because this current is greater than 10 milliamps, the ideal voltage source is not a good model for the battery. In this case, it is reasonable to expect that the battery current is different from the current for the ideal voltage source.
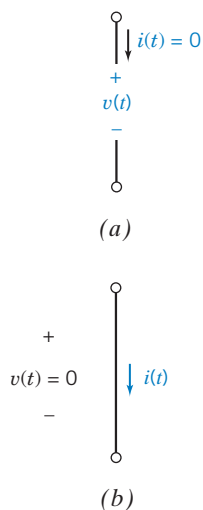
The short circuit and open circuit are special cases of ideal sources. A *short circuit* is an ideal voltage source having $v(t) = 0$. The current in a short circuit is determined by the rest of the circuit. An *open circuit* is an ideal current source having $i(t) = 0$. The voltage across an open circuit is determined by the rest of the circuit. Figure 2.5-3 shows the symbols used to represent the short circuit and the open circuit. Notice that the power absorbed by each of these devices is zero.

Open and short circuits can be added to a circuit without disturbing the branch currents and voltages of all the other devices in the circuit. Figure 2.6-3 shows how this can be done. Figure 2.6-3a shows an example circuit. In Figure 2.6-3b an open circuit and a short circuit have been added to this example circuit. The open circuit was connected between two nodes of the original circuit. In contrast, the short circuit was added by cutting a wire and inserting the short circuit. Adding open circuits and short circuits to a network in this way does not change the network.

Open circuits and short circuits can also be described as special cases of resistors. A resistor with resistance $R = 0$ $(G = \infty)$ is a short circuit. A resistor with conductance $G = 0$ $(R = \infty)$ is an open circuit.

**FIGURE 2.5-3**
(a) Open circuit.
(b) Short circuit.

## 2.6 *Voltmeters and Ammeters*

Measurements of dc current and voltage are made with direct-reading (analog) or digital meters, as shown in Figure 2.6-1. A direct-reading meter has an indicating pointer whose angular deflection depends on the magnitude of the variable it is measuring. A digital meter displays a set of digits indicating the measured variable value.

To measure a voltage or current, a meter is connected to a circuit, using terminals called probes. These probes are color coded to indicate the reference direction of the variable being measured. Frequently, meter probes are colored red and black. An ideal voltmeter measures the voltage from the red to the black probe. The red terminal is the positive terminal, and the black terminal is the negative terminal (see Figure 2.6-2b).

An ideal ammeter measures the current flowing through its terminals, as shown in Figure 2.6-2a and has zero voltage, $v_m$, across its terminals. An ideal voltmeter measures the voltage across its terminals, as shown in Figure 2.6-2b, and has terminal current, $i_m$, equal to zero. Practical measuring