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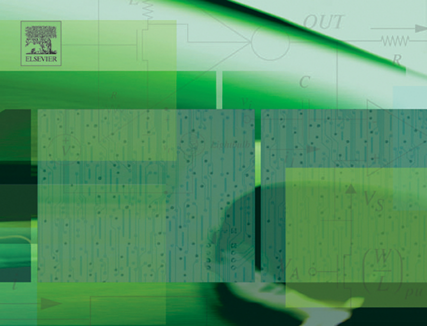

Foundations of Analog and Digital Electronic Circuits

ANANT AGARWAL AND JEFFREY H. LANG

MK MORGAN KAUFMANN

In Praise of *Foundations of Analog and Digital Electronic Circuits*

"This book, crafted and tested with MIT sophomores in electrical engineering and computer science over a period of more than six years, provides a comprehensive treatment of both circuit analysis and basic electronic circuits. Examples such as digital and analog circuit applications, field-effect transistors, and operational amplifiers provide the platform for modeling of active devices, including large-signal, small-signal (incremental), nonlinear and piecewise-linear models. The treatment of circuits with energy-storage elements in transient and sinusoidal-steady-state circumstances is thorough and accessible. Having taught from drafts of this book five times, I believe that it is an improvement over the traditional approach to circuits and electronics, in which the focus is on analog circuits alone."

-PAUL E. GRAY, Massachusetts Institute of Technology

"My overall reaction to this book is overwhelmingly favorable. Well-written and pedagogically sound, the book provides a good balance between theory and practical application. I think that combining circuits and electronics is a very good idea. Most introductory circuit theory texts focus primarily on the analysis of lumped element networks without putting these networks into a practical electronics context. However, it is becoming more critical for our electrical and computer engineering students to understand and appreciate the common ground from which both fields originate."

-GARY MAY, Georgia Institute of Technology

"Without a doubt, students in engineering today want to quickly relate what they learn from courses to what they experience in the electronics-filled world they live in. Understanding today's digital world requires a strong background in analog circuit principles as well as a keen intuition about their impact on electronics. In Foundations… Agarwal and Lang present a unique and powerful approach for an exciting first course introducing engineers to the world of analog and digital systems."

-RAVI SUBRAMANIAN, Berkeley Design Automation

"Finally, an introductory circuit analysis book has been written that truly unifies the treatment of traditional circuit analysis and electronics. Agarwal and Lang skillfully combine the fundamentals of circuit analysis with the fundamentals of modern analog and digital integrated circuits. I applaud their decision to eliminate from their book the usual mandatory chapter on Laplace transforms, a tool no longer in use by modern circuit designers. I expect this book to establish a new trend in the way introductory circuit analysis is taught to electrical and computer engineers."

-TIM TRICK, University of Illinois at Urbana-Champaign

Foundations of Analog and Digital Electronic Circuits

ABOUT THE AUTHORS

Anant Agarwal is Professor of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology. He joined the faculty in 1988, teaching courses in circuits and electronics, VLSI, digital logic and computer architecture. Between 1999 and 2003, he served as an associate director of the Laboratory for Computer Science. He holds a Ph.D. and an M.S. in Electrical Engineering from Stanford University, and a bachelor's degree in Electrical Engineering from IIT Madras. Agarwal led a group that developed Sparcle (1992), a multithreaded microprocessor, and the MIT Alewife (1994), a scalable shared-memory multiprocessor. He also led the VirtualWires project at MIT and was a founder of Virtual Machine Works, Inc., which took the VirtualWires logic emulation technology to market in 1993. Currently Agarwal leads the Raw project at MIT, which developed a new kind of reconfigurable computing chip. He and his team were awarded a Guinness world record in 2004 for LOUD, the largest microphone array in the world, which can pinpoint, track and amplify individual voices in a crowd. Co-founder of Engim, Inc., which develops multi-channel wireless mixed-signal chipsets, Agarwal also won the Maurice Wilkes prize for computer architecture in 2001, and the Presidential Young Investigator award in 1991.

Jeffrey H. Lang is Professor of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology. He joined the faculty in 1980 after receiving his SB (1975), SM (1977) and Ph.D. (1980) degrees from the Department of Electrical Engineering and Computer Science. He served as the Associate Director of the MIT Laboratory for Electromagnetic and Electronic Systems between 1991 and 2003, and as an Associate Editor of "Sensors and Actuators" between 1991 and 1994. Professor Lang's research and teaching interests focus on the analysis, design and control of electromechanical systems with an emphasis on rotating machinery, micro-scale sensors and actuators, and flexible structures. He has also taught courses in circuits and electronics at MIT. He has written over 170 papers and holds 10 patents in the areas of electromechanics, power electronics and applied control, and has been awarded four best-paper prizes from IEEE societies. Professor Lang is a Fellow of the IEEE, and a former Hertz Foundation Fellow.

Agarwal and Lang have been working together for the past eight years on a fresh approach to teaching circuits. For several decades, MIT had offered a traditional course in circuits designed as the first core undergraduate course in EE. But by the mid-'90s, vast advances in semiconductor technology, coupled with dramatic changes in students' backgrounds evolving from a ham radio to computer culture, had rendered this traditional course poorly motivated, and many parts of it were virtually obsolete. Agarwal and Lang decided to revamp and broaden this first course for EE, ECE or EECS by establishing a strong connection between the contemporary worlds of digital and analog systems, and by unifying the treatment of circuits and basic MOS electronics. As they developed the course, they solicited comments and received guidance from a large number of colleagues from MIT and other universities, students, and alumni, as well as industry leaders.

Unable to find a suitable text for their new introductory course, Agarwal and Lang wrote this book to follow the lecture schedule used in their course. "Circuits and Electronics" is taught in both the spring and fall semesters at MIT, and serves as a prerequisite for courses in signals and systems, digital/computer design, and advanced electronics. The course material is available worldwide on MIT's OpenCourseWare website, http://ocw.mit.edu/OcwWeb/index.htm.

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To Anu, Akash, and Anisha
—Anant Agarwal

To Marija, Chris, John, Matt
—Jeffrey Lang

CONTENTS

Material marked with **WWW** appears on the Internet (please see Preface for details).

PREFACE

APPROACH

This book is designed to serve as a first course in an electrical engineering or an electrical engineering and computer science curriculum, providing students at the sophomore level a transition from the world of physics to the world of electronics and computation. The book attempts to satisfy two goals: Combine circuits and electronics into a single, unified treatment, and establish a strong connection with the contemporary worlds of both digital and analog systems.

These goals arise from the observation that the approach to introducing electrical engineering through a course in traditional circuit analysis is fast becoming obsolete. Our world has gone digital. A large fraction of the student population in electrical engineering is destined for industry or graduate study in digital electronics or computer systems. Even those students who remain in core electrical engineering are heavily influenced by the digital domain.

Because of this elevated focus on the digital domain, basic electrical engineering education must change in two ways: First, the traditional approach to teaching circuits and electronics without regard to the digital domain must be replaced by one that stresses the circuits foundations common to both the digital and analog domains. Because most of the fundamental concepts in circuits and electronics are equally applicable to both the digital and the analog domains, this means that, primarily, we must change the way in which we motivate circuits and electronics to emphasize their broader impact on digital systems. For example, although the traditional way of discussing the dynamics of first-order RC circuits appears unmotivated to the student headed into digital systems, the same pedagogy is exciting when motivated by the switching behavior of a switch and resistor inverter driving a non-ideal capacitive wire. Similarly, we motivate the study of the step response of a second-order RLC circuit by observing the behavior of a MOS inverter when pin parasitics are included.

Second, given the additional demands of computer engineering, many departments can ill-afford the luxury of separate courses on circuits and on electronics. Rather, they might be combined into one course.[1] Circuits courses

1. In his paper, "Teaching Circuits and Electronics to First-Year Students," in *Int. Symp. Circuits and Systems (ISCAS)*, 1998, Yannis Tsividis makes an excellent case for teaching an integrated course in circuits and electronics.

treat networks of passive elements such as resistors, sources, capacitors, and inductors. Electronics courses treat networks of both passive elements and active elements such as MOS transistors. Although this book offers a unified treatment for circuits and electronics, we have taken some pains to allow the crafting of a two-semester sequence — one focused on circuits and another on electronics — from the same basic content in the book.

Using the concept of "abstraction," the book attempts to form a bridge between the world of physics and the world of large computer systems. In particular, it attempts to unify electrical engineering and computer science as the art of creating and exploiting successive abstractions to manage the complexity of building useful electrical systems. Computer systems are simply one type of electrical system.

In crafting a single text for both circuits and electronics, the book takes the approach of covering a few important topics in depth, choosing more contemporary devices when possible. For example, it uses the MOSFET as the basic active device, and relegates discussions of other devices such as bipolar transistors to the exercises and examples. Furthermore, to allow students to understand basic circuit concepts without the trappings of specific devices, it introduces several abstract devices as examples and exercises. We believe this approach will allow students to tackle designs with many other extant devices and those that are yet to be invented.

Finally, the following are some additional differences from other books in this field:

- The book draws a clear connection between electrical engineering and physics by showing clearly how the lumped circuit abstraction directly derives from Maxwell's Equations and a set of simplifying assumptions.
- The concept of abstraction is used throughout the book to unify the set of engineering simplifications made in both analog and digital design.
- The book elevates the focus of the digital domain to that of analog. However, our treatment of digital systems emphasizes their analog aspects. We start with switches, sources, resistors, and MOSFETs, and apply KVL, KCL, and so on. The book shows that digital versus analog behavior is obtained by focusing on particular regions of device behavior.
- The MOSFET device is introduced using a progression of models of increased refinement — the S model, the SR model, the SCS model, and the SU model.
- The book shows how significant amounts of insight into the static and dynamic operation of digital circuits can be obtained with very simple models of MOSFETs.

- Various properties of devices, for example, the memory property of capacitors, or the gain property of amplifiers, are related to both their use in analog circuits and digital circuits.
- The state variable viewpoint of transient problems is emphasized for its intuitive appeal and since it motivates computer solutions of both linear or nonlinear network problems.
- Issues of energy and power are discussed in the context of both analog and digital circuits.
- A large number of examples are picked from the digital domain emphasizing VLSI concepts to emphasize the power and generality of traditional circuit analysis concepts.

With these features, we believe this book offers the needed foundation for students headed towards either the core electrical engineering majors — including digital and RF circuits, communication, controls, signal processing, devices, and fabrication — or the computer engineering majors — including digital design, architecture, operating systems, compilers, and languages.

MIT has a unified electrical engineering and computer science department. This book is being used in MIT's introductory course on circuits and electronics. This course is offered each semester and is taken by about 500 students a year.

OVERVIEW

Chapter 1 discusses the concept of abstraction and introduces the lumped circuit abstraction. It discusses how the lumped circuit abstraction derives from Maxwell's Equations and provides the basic method by which electrical engineering simplifies the analysis of complicated systems. It then introduces several ideal, lumped elements including resistors, voltage sources, and current sources.

This chapter also discusses two major motivations of studying electronic circuits — modeling physical systems and information processing. It introduces the concept of a model and discusses how physical elements can be modeled using ideal resistors and sources. It also discusses information processing and signal representation.

Chapter 2 introduces KVL and KCL and discusses their relationship to Maxwell's Equations. It then uses KVL and KCL to analyze simple resistive networks. This chapter also introduces another useful element called the dependent source.

Chapter 3 presents more sophisticated methods for network analysis.

Chapter 4 introduces the analysis of simple, nonlinear circuits.

Chapter 5 introduces the digital abstraction, and discusses the second major simplification by which electrical engineers manage the complexity of building large systems.[2]

Chapter 6 introduces the switch element and describes how digital logic elements are constructed. It also describes the implementation of switches using MOS transistors. Chapter 6 introduces the S (switch) and the SR (switch-resistor) models of the MOSFET and analyzes simple switch circuits using the network analysis methods presented earlier. Chapter 6 also discusses the relationship between amplification and noise margins in digital systems.

Chapter 7 discusses the concept of amplification. It presents the SCS (switch-current-source) model of the MOSFET and builds a MOSFET amplifier.

Chapter 8 continues with small signal amplifiers.

Chapter 9 introduces storage elements, namely, capacitors and inductors, and discusses why the modeling of capacitances and inductances is necessary in high-speed design.

Chapter 10 discusses first order transients in networks. This chapter also introduces several major applications of first-order networks, including digital memory.

Chapter 11 discusses energy and power issues in digital systems and introduces CMOS logic.

Chapter 12 analyzes second order transients in networks. It also discusses the resonance properties of RLC circuits from a time-domain point of view.

Chapter 13 discusses sinusoidal steady state analysis as an alternative to the time-domain transient analysis. The chapter also introduces the concepts of impedance and frequency response. This chapter presents the design of filters as a major motivating application.

Chapter 14 analyzes resonant circuits from a frequency point of view.

Chapter 15 introduces the operational amplifier as a key example of the application of abstraction in analog design.

Chapter 16 discusses diodes and simple diode circuits.

The book also contains appendices on trignometric functions, complex numbers, and simultaneous linear equations to help readers who need a quick refresher on these topics or to enable a quick lookup of results.

2. The point at which to introduce the digital abstraction in this book and in a corresponding curriculum was arguably the topic over which we agonized the most. We believe that introducing the digital abstraction at this point in the course balances (a) the need for introducing digital systems as early as possible in the curriculum to excite and motivate students (especially with laboratory experiments), with (b) the need for providing students with enough of a toolchest to be able to analyze interesting digital building blocks such as combinational logic. Note that we recommend introduction of digital systems a lot sooner than suggested by Tsividis in his 1998 ISCAS paper, although we completely agree his position on the need to include some digital design.

COURSE ORGANIZATION

The sequence of chapters has been organized to suit a one or two semester integrated course on circuits and electronics. First and second order circuits are introduced as late as possible to allow the students to attain a higher level of mathematical sophistication in situations in which they are taking a course on differential equations at the same time. The digital abstraction is introduced as early as possible to provide early motivation for the students.

Alternatively, the following chapter sequences can be selected to organize the course around a circuits sequence followed by an electronics sequence. The circuits sequence would include the following: Chapter 1 (lumped circuit abstraction), Chapter 2 (KVL and KCL), Chapter 3 (network analysis), Chapter 5 (digital abstraction), Chapter 6 (S and SR MOS models), Chapter 9 (capacitors and inductors), Chapter 10 (first-order transients), Chapter 11 (energy and power, and CMOS), Chapter 12 (second-order transients), Chapter 13 (sinusoidal steady state), Chapter 14 (frequency analysis of resonant circuits), and Chapter 15 (operational amplifier abstraction — optional).

The electronics sequence would include the following: Chapter 4 (nonlinear circuits), Chapter 7 (amplifiers, the SCS MOSFET model), Chapter 8 (small-signal amplifiers), Chapter 13 (sinusoidal steady state and filters), Chapter 15 (operational amplifier abstraction), and Chapter 16 (diodes and power circuits).

WEB SUPPLEMENTS

We have gathered a great deal of material to help students and instructors using this book. This information can be accessed from the Morgan Kaufmann website:

www.mkp.com/companions/1558607358

The site contains:

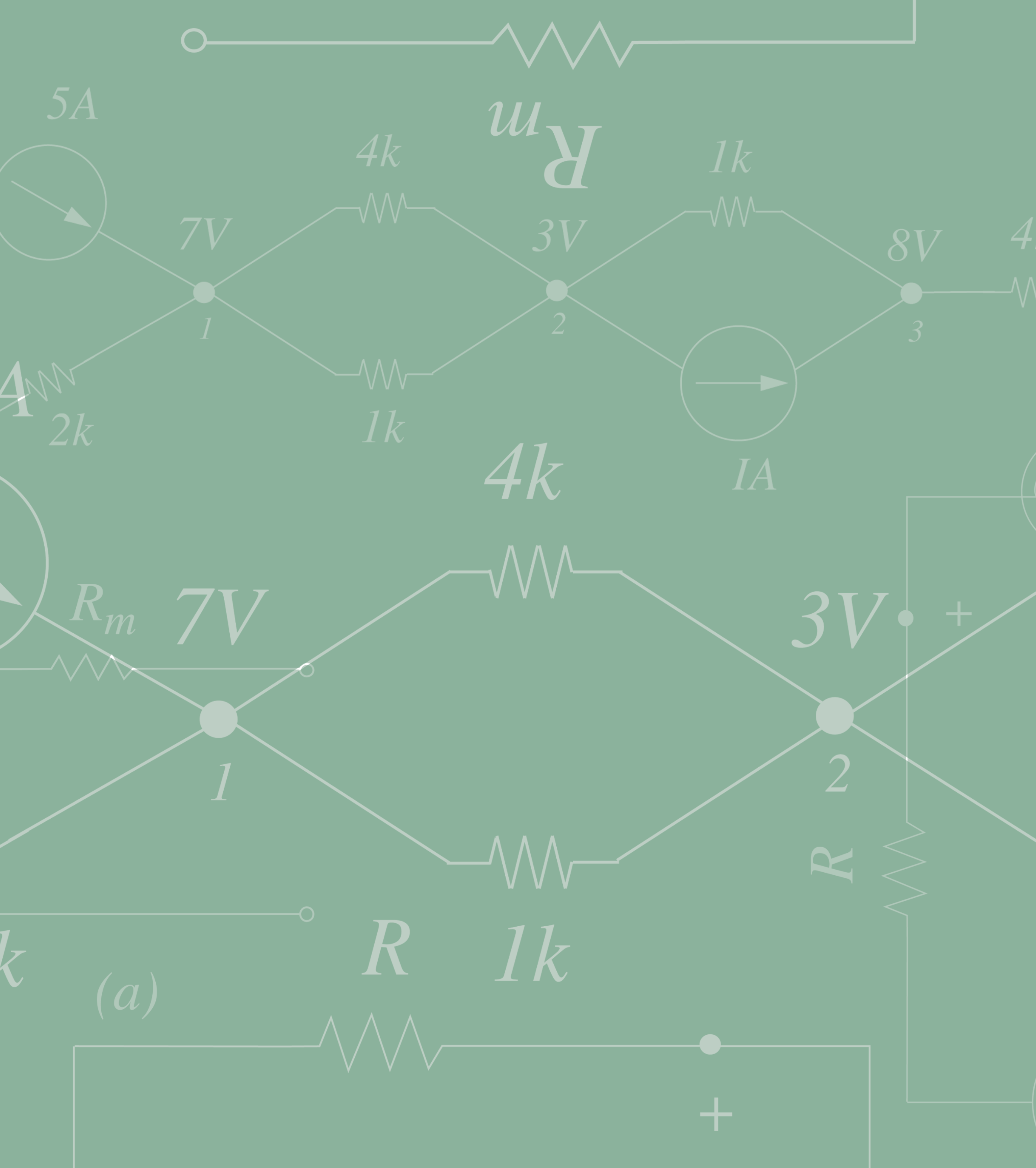
- Supplementary sections and examples. We have used the icon **WWW** in the text to identify sections or examples.
- Instructor's manual
- A link to the MIT OpenCourseWare website for the authors' course, 6.002 Circuits and Electronics. On this site you will find:
  - Syllabus. A summary of the objectives and learning outcomes for course 6.002.
  - Readings. Reading assignments based on Foundations of Analog and Digital Electronic Circuits.
  - Lecture Notes. Complete set of lecture notes, accompanying video lectures, and descriptions of the demonstrations made by the instructor during class.

- Labs. A collection of four labs: Thevenin/Norton Equivalents and Logic Gates, MOSFET Inverting Amplifiers and First-Order Circuits, Second-Order Networks, and Audio Playback System. Includes an equipment handout and lab tutorial. Labs include pre-lab exercises, in-lab exercises, and post-lab exercises.
- Assignments. A collection of eleven weekly homework assignments.
- Exams. Two quizzes and a Final Exam.
- Related Resources. Online exercises in Circuits and Electronics for demonstration and self-study.

ACKNOWLEDGMENTS

These notes evolved out of an initial set of notes written by Campbell Searle for 6.002 in 1991. The notes were also influenced by several who taught 6.002 at various times including Steve Senturia and Gerry Sussman. The notes have also benefited from the insights of Steve Ward, Tom Knight, Chris Terman, Ron Parker, Dimitri Antoniadis, Steve Umans, David Perreault, Karl Berggren, Gerry Wilson, Paul Gray, Keith Carver, Mark Horowitz, Yannis Tsividis, Cliff Pollock, Denise Penrose, Greg Schaffer, and Steve Senturia. We are also grateful to our reviewers including Timothy Trick, Barry Farbrother, John Pinkston, Stephane Lafortune, Gary May, Art Davis, Jeff Schowalter, John Uyemura, Mark Jupina, Barry Benedict, Barry Farbrother, and Ward Helms for their feedback. The help of Michael Zhang, Thit Minn, and Patrick Maurer in fleshing out problems and examples; that of Jose Oscar Mur-Miranda, Levente Jakab, Vishal Kapur, Matt Howland, Tom Kotwal, Michael Jura, Stephen Hou, Shelley Duvall, Amanda Wang, Ali Shoeb, Jason Kim, Charvak Karpe and Michael Jura in creating an answer key; that of Rob Geary, Yu Xinjie, Akash Agarwal, Chris Lang, and many of our students and colleagues in proofreading; and that of Anne McCarthy, Cornelia Colyer, and Jennifer Tucker in figure creation is also gratefully acknowledged. We gratefully acknowledge Maxim for their support of this book, and Ron Koo for making that support possible, as well as for capturing and providing us with numerous images of electronic components and chips. Ron Koo is also responsible for encouraging us to think about capturing and articulating the quick, intuitive process by which seasoned electrical engineers analyze circuits — our numerous sections on intuitive analysis are a direct result of his encouragement. We also thank Adam Brand and Intel Corp. for providing us with the images of the Pentium IV.

CHAPTER 1



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1 THE CIRCUIT ABSTRACTION

"Engineering is the purposeful use of science."

STEVE SENTURIA

1.1 THE POWER OF ABSTRACTION

Engineering is the purposeful use of science. Science provides an understanding of natural phenomena. Scientific study involves experiment, and scientific laws are concise statements or equations that explain the experimental data. The laws of physics can be viewed as a layer of abstraction between the experimental data and the practitioners who want to use specific phenomena to achieve their goals, without having to worry about the specifics of the experiments and the data that inspired the laws. Abstractions are constructed with a particular set of goals in mind, and they apply when appropriate constraints are met. For example, Newton's laws of motion are simple statements that relate the dynamics of rigid bodies to their masses and external forces. They apply under certain constraints, for example, when the velocities are much smaller than the speed of light. Scientific abstractions, or laws such as Newton's, are simple and easy to use, and enable us to harness and use the properties of nature.

Electrical engineering and computer science, or electrical engineering for short, is one of many engineering disciplines. Electrical engineering is the purposeful use of Maxwell's Equations (or Abstractions) for electromagnetic phenomena. To facilitate our use of electromagnetic phenomena, electrical engineering creates a new abstraction layer on top of Maxwell's Equations called the lumped circuit abstraction. By treating the lumped circuit abstraction layer, this book provides the connection between physics and electrical engineering. It unifies electrical engineering and computer science as the art of creating and exploiting successive abstractions to manage the complexity of building useful electrical systems. Computer systems are simply one type of electrical system.

The abstraction mechanism is very powerful because it can make the task of building complex systems tractable. As an example, consider the force equation:

$$F = ma. \tag{1.1}$$

The force equation enables us to calculate the acceleration of a particle with a given mass for an applied force. This simple force abstraction allows us to disregard many properties of objects such as their size, shape, density, and temperature, that are immaterial to the calculation of the object's acceleration. It also allows us to ignore the myriad details of the experiments and observations that led to the force equation, and accept it as a given. Thus, scientific laws and abstractions allow us to leverage and build upon past experience and work. (Without the force abstraction, consider the pain we would have to go through to perform experiments to achieve the same result.)

Over the past century, electrical engineering and computer science have developed a set of abstractions that enable us to transition from the physical sciences to engineering and thereby to build useful, complex systems.

The set of abstractions that transition from science to engineering and insulate the engineer from scientific minutiae are often derived through the *discretization discipline*. Discretization is also referred to as *lumping*. A discipline is a self-imposed constraint. The discipline of discretization states that we choose to deal with discrete elements or ranges and ascribe a single value to each discrete element or range. Consequently, the discretization discipline requires us to ignore the distribution of values within a discrete element. Of course, this discipline requires that systems built on this principle operate within appropriate constraints so that the single-value assumptions hold. As we will see shortly, the lumped circuit abstraction that is fundamental to electrical engineering and computer science is based on lumping or discretizing matter.[1] Digital systems use the digital abstraction, which is based on discretizing signal values. Clocked digital systems are based on discretizing both signals and time, and digital systolic arrays are based on discretizing signals, time *and* space.

Building upon the set of abstractions that define the transition from physics to electrical engineering, electrical engineering creates further abstractions to manage the complexity of building large systems. A lumped circuit element is often used as an abstract representation or a model of a piece of material with complicated internal behavior. Similarly, a circuit often serves as an abstract representation of interrelated physical phenomena. The operational amplifier composed of primitive discrete elements is a powerful abstraction that simplifies the building of bigger analog systems. The logic gate, the digital memory, the digital finite-state machine, and the microprocessor are themselves a succession of abstractions developed to facilitate building complex computer and control systems. Similarly, the art of computer programming involves the mastery of creating successively higher-level abstractions from lower-level primitives.

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1. Notice that Newton's laws of physics are themselves based on discretizing matter. Newton's laws describe the dynamics of discrete bodies of matter by treating them as point masses. The spatial distribution of properties within the discrete elements are ignored.

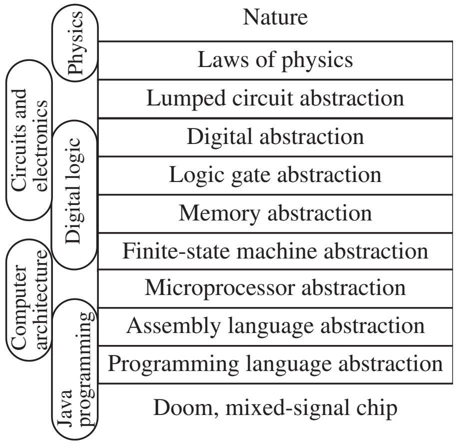

FIGURE 1.1 Sequence of courses and the abstraction layers introduced in a possible EECS course sequence that ultimately results in the ability to create the computer game "Doom," or a mixed-signal (containing both analog and digital components) microprocessor supervisory circuit such as that shown in Figure 1.2.

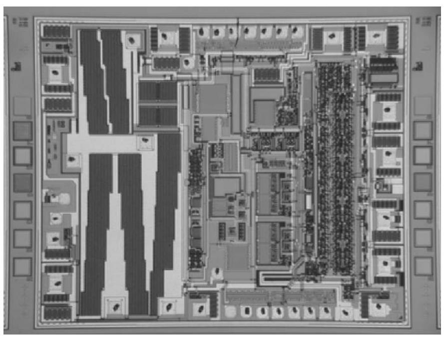

FIGURE 1.2 A photograph of the MAX807L microprocessor supervisory circuit from Maxim Integrated Products. The chip is roughly 2.5 mm by 3 mm. Analog circuits are to the left and center of the chip, while digital circuits are to the right. (Photograph Courtesy of Maxim Integrated Products.)

Figures 1.1 and 1.3 show possible course sequences that students might encounter in an EECS (Electrical Engineering and Computer Science) or an EE (Electrical Engineering) curriculum, respectively, to illustrate how each of the courses introduces several abstraction layers to simplify the building of useful electronic systems. This sequence of courses also illustrates how a circuits and electronics course using this book might fit within a general EE or EECS course framework.

1.2 THE LUMPED CIRCUIT ABSTRACTION

Consider the familiar lightbulb. When it is connected by a pair of cables to a battery, as shown in Figure 1.4a, it lights up. Suppose we are interested in finding out the amount of current flowing through the bulb. We might go about this by employing Maxwell's equations and deriving the amount of current by

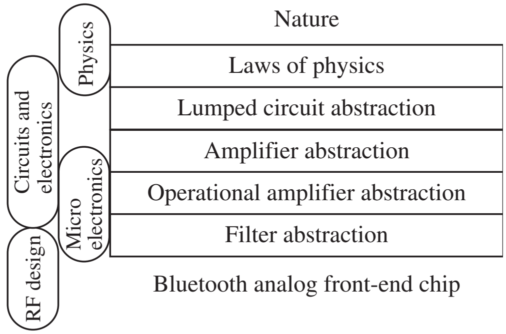

FIGURE 1.3 Sequence of courses and the abstraction layers that they introduce in a possible EE course sequence that ultimately results in the ability to create a wireless Bluetooth analog front-end chip.

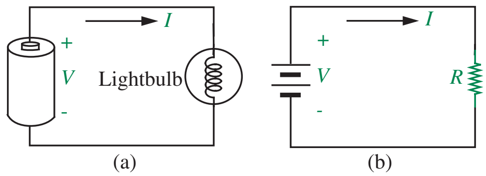

FIGURE 1.4 (a) A simple lightbulb circuit. (b) The lumped circuit representation.

a careful analysis of the physical properties of the bulb, the battery, and the cables. This is a horrendously complicated process.

As electrical engineers we are often interested in such computations in order to design more complex circuits, perhaps involving multiple bulbs and batteries. So how do we simplify our task? We observe that if we discipline ourselves to asking only simple questions, such as what is the net current flowing through the bulb, we can ignore the internal properties of the bulb and represent the bulb as a discrete element. Further, for the purpose of computing the current, we can create a discrete element known as a resistor and replace the bulb with it.[2] We define the resistance of the bulb R to be the ratio of the voltage applied to the bulb and the resulting current through it. In other words,

$$R = V/I.$$

Notice that the actual shape and physical properties of the bulb are irrelevant provided it offers the resistance R. We were able to ignore the internal properties and distribution of values inside the bulb simply by disciplining ourselves not to ask questions about those internal properties. In other words, when asking about the current, we were able to discretize the bulb into a single lumped element whose single relevant property was its resistance. This situation is

2. We note that the relationship between the voltage and the current for a bulb is generally much more complicated.

analogous to the point mass simplification that resulted in the force relation in Equation 1.1, where the single relevant property of the object is its mass.

As illustrated in Figure 1.5, a lumped element can be idealized to the point where it can be treated as a black box accessible through a few terminals. The behavior at the terminals is more important than the details of the behavior internal to the black box. That is, what happens at the terminals is more important than how it happens inside the black box. Said another way, the black box is a layer of abstraction between the user of the bulb and the internal structure of the bulb.

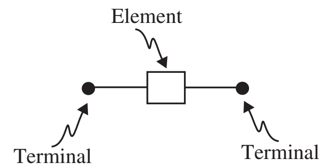

FIGURE 1.5 A lumped element.

The resistance is the property of the bulb of interest to us. Likewise, the voltage is the property of the battery that we most care about. Ignoring, for now, any internal resistance of the battery, we can lump the battery into a discrete element called by the same name supplying a constant voltage V, as shown in Figure 1.4b. Again, we can do this if we work within certain constraints to be discussed shortly, and provided we are not concerned with the internal properties of the battery, such as the distribution of the electrical field. In fact, the electric field within a real-life battery is horrendously difficult to chart accurately. Together, the collection of constraints that underlie the lumped circuit abstraction result in a marvelous simplification that allows us to focus on specifically those properties that are relevant to us.

Notice also that the orientation and shape of the wires are not relevant to our computation. We could even twist them or knot them in any way. Assuming for now that the wires are ideal conductors and offer zero resistance,[3] we can rewrite the bulb circuit as shown in Figure 1.4b using lumped circuit equivalents for the battery and the bulb resistance, which are connected by ideal wires. Accordingly, Figure 1.4b is called the lumped circuit abstraction of the lightbulb circuit. If the battery supplies a constant voltage V and has zero internal resistance, and if the resistance of the bulb is R, we can use simple algebra to compute the current flowing through the bulb as

$$I = V/R.$$

Lumped elements in circuits must have a voltage V and a current I defined for their terminals.[4] In general, the ratio of V and I need not be a constant. The ratio is a constant (called the resistance R) only for lumped elements that

3. If the wires offer nonzero resistance, then, as described in Section 1.6, we can separate each wire into an ideal wire connected in series with a resistor.

4. In general, the voltage and current can be time varying and can be represented in a more general form as $V(t)$ and $I(t)$. For devices with more than two terminals, the voltages are defined for any terminal with respect to any other reference terminal, and the currents are defined flowing into each of the terminals.

obey Ohm's law.[5] The circuit comprising a set of lumped elements must also have a voltage defined between any pair of points, and a current defined into any terminal. Furthermore, the elements must not interact with each other except through their terminal currents and voltages. That is, the internal physical phenomena that make an element function must interact with external electrical phenomena only at the electrical terminals of that element. As we will see in Section 1.3, lumped elements and the circuits formed using these elements must adhere to a set of constraints for these definitions and terminal interactions to exist. We name this set of constraints *the lumped matter discipline.*

The lumped circuit abstraction Capped a set of lumped elements that obey the lumped matter discipline using ideal wires to form an assembly that performs a specific function results in the lumped circuit abstraction.

Notice that the lumped circuit simplification is analogous to the point-mass simplification in Newton's laws. The lumped circuit abstraction represents the relevant properties of lumped elements using algebraic symbols. For example, we use R for the resistance of a resistor. Other values of interest, such as currents I and voltages V, are related through simple functions. The ease of using algebraic equations in place of Maxwell's equations to design and analyze complicated circuits will become much clearer in the following chapters.

The process of discretization can also be viewed as a way of modeling physical systems. The resistor is a model for a lightbulb if we are interested in finding the current flowing through the lightbulb for a given applied voltage. It can even tell us the power consumed by the lightbulb. Similarly, as we will see in Section 1.6, a constant voltage source is a good model for the battery when its internal resistance is zero. Thus, Figure 1.4b is also called the lumped circuit model of the lightbulb circuit. Models must be used only in the domain in which they are applicable. For example, the resistor model for a lightbulb tells us nothing about its cost or its expected lifetime.

The primitive circuit elements, the means for combining them, and the means of abstraction form the graphical language of circuits. Circuit theory is a well established discipline. With maturity has come widespread utility. The language of circuits has become universal for problem-solving in many disciplines. Mechanical, chemical, metallurgical, biological, thermal, and even economic processes are often represented in circuit theory terms, because the mathematics for analysis of linear and nonlinear circuits is both powerful and well-developed. For this reason electronic circuit models are often used as analogs in the study of many physical processes. Readers whose main focus is on some area of electrical engineering other than electronics should therefore view the material in this

5. Observe that Ohm's law itself is an abstraction for the electrical behavior of resistive material that allows us to replace tables of experimental data relating V and I by a simple equation.

book from the broad perspective of an introduction to the modeling of dynamic systems.

1.3 THE LUMPED MATTER DISCIPLINE

The scope of these equations is remarkable, including as it does the fundamental operating principles of all large-scale electromagnetic devices such as motors, cyclotrons, electronic computers, television, and microwave radar.

—HALLIDAY AND RESNICK ON MAXWELL'S EQUATIONS

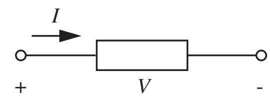

FIGURE 1.6 A lumped circuit element.

Lumped circuits comprise lumped elements (or discrete elements) connected by ideal wires. A lumped element has the property that a unique terminal voltage $V(t)$ and terminal current $I(t)$ can be defined for it. As depicted in Figure 1.6, for a two-terminal element, V is the voltage across the terminals of the element,[6] and I is the current through the element.[7] Furthermore, for lumped resistive elements, we can define a single property called the resistance R that relates the voltage across the terminals to the current through the terminals.

The voltage, the current, and the resistance are defined for an element only under certain constraints that we collectively call the *lumped matter discipline* (LMD). Once we adhere to the lumped matter discipline, we can make several simplifications in our circuit analysis and work with the lumped circuit abstraction. Thus the lumped matter discipline provides the foundation for the lumped circuit abstraction, and is the fundamental mechanism by which we are able to move from the domain of physics to the domain of electrical engineering. We will simply state these constraints here, but relegate the development of the constraints of the lumped matter discipline to Section A.1 in Appendix A. Section A.2 further shows how the lumped matter discipline results in the simplification of Maxwell's equations into the algebraic equations of the lumped circuit abstraction.

The lumped matter discipline imposes three constraints on how we choose lumped circuit elements:

1. Choose lumped element boundaries such that the rate of change of magnetic flux linked with any closed loop outside an element must be zero for all time. In other words, choose element boundaries such that

$$\frac{\partial \Phi_B}{\partial t} = 0$$

through any closed path outside the element.

6. The *voltage* across the terminals of an element is defined as the work done in moving a unit charge (one coulomb) from one terminal to the other through the element against the electrical field. Voltages are measured *in volts* (V), where one volt is one joule per coulomb.

7. The *current* is defined as the rate of flow of charge from one terminal to the other through the element. Current is measured in *amperes* (A) , where one ampere is one coulomb per second.

2. Choose lumped element boundaries so that there is no total time varying charge within the element for all time. In other words, choose element boundaries such that

$$\frac{\partial q}{\partial t} = 0$$

where q is the total charge within the element.

3. Operate in the regime in which signal timescales of interest are much larger than the propagation delay of electromagnetic waves across the lumped elements.

The intuition behind the first constraint is as follows. The definition of the voltage (or the potential difference) between a pair of points across an element is the work required to move a particle with unit charge from one point to the other *along some path* against the force due to the electrical field. For the lumped abstraction to hold, we require that this voltage be unique, and therefore the voltage value must not depend on the path taken. We can make this true by selecting element boundaries such that there is no time-varying magnetic flux outside the element.

If the first constraint allowed us to define a unique voltage across the terminals of an element, the second constraint results from our desire to define a unique value for the current entering and exiting the terminals of the element. A unique value for the current can be defined if we do not have charge buildup or depletion inside the element over time.

Under the first two constraints, elements do not interact with each other except through their terminal currents and voltages. Notice that the first two constraints require that the rate of change of magnetic flux outside the elements and net charge within the elements is zero *for all time*.[8] It directly follows that the magnetic flux and the electric fields outside the elements are also zero. Thus there are no fields related to one element that can exert influence on the other elements. This permits the behavior of each element to be analyzed independently.[9] The results of this analysis are then summarized by the

8. As discussed in Appendix A, assuming that the rate of change is zero for all time ensures that voltages and currents can be arbitrary functions of time.

9. The elements in most circuits will satisfy the restriction of non-interaction, but occasionally they will not. As will be seen later in this text, the magnetic fields from two inductors in close proximity might extend beyond the material boundaries of the respective inductors inducing significant electric fields in each other. In this case, the two inductors could not be treated as independent circuit elements. However, they could perhaps be treated together as a single element, called a transformer, if their distributed coupling could be modeled appropriately. A dependent source is yet another example of a circuit element that we will introduce later in this text in which interacting circuit elements are treated together as a single element.

relation between the terminal current and voltage of that element, for example, $V = IR$. More examples of such relations, or element laws, will be presented in Section 1.6.2. Further, when the restriction of non-interaction is satisfied, the focus of circuit operation becomes the terminal currents and voltages, and not the electromagnetic fields within the elements. Thus, these currents and voltages become the fundamental signals within the circuit. Such signals are discussed further in Section 1.8.

Let us dwell for a little longer on the third constraint. The lumped element approximation requires that we be able to define a voltage V between a pair of element terminals (for example, the two ends of a bulb filament) and a current through the terminal pair. Defining a current through the element means that the current in must equal the current out. Now consider the following thought experiment. Apply a current pulse at one terminal of the filament at time instant t and observe both the current into this terminal and the current out of the other terminal at a time instant $t + dt$ very close to t. If the filament were long enough, or if dt were small enough, the finite speed of electromagnetic waves might result in our measuring different values for the current in and the current out.

We cannot make this problem go away by postulating constant currents and voltages, since we are very much interested in situations such as those depicted in Figure 1.7, in which a time-varying voltage source drives a circuit.

Instead, we fix the problem created by the finite propagation speeds of electromagnetic waves by adding the third constraint, namely, that the timescale of interest in our problem be much larger than electromagnetic propagation delays through our elements. Put another way, the size of our lumped elements must be much smaller than the wavelength associated with the V and I signals.[10]

Under these speed constraints, electromagnetic waves can be treated as if they propagated instantly through a lumped element. By neglecting propagation

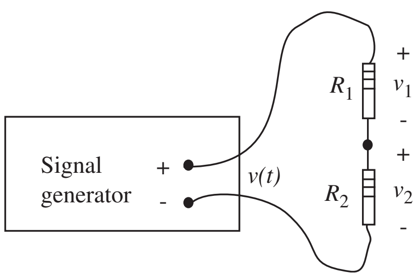

FIGURE 1.7 Resistor circuit connected to a signal generator.

10. More precisely, the wavelength that we are referring to is that wavelength of the electromagnetic wave launched by the signals.

effects, the lumped element approximation becomes analogous to the point-mass simplification, in which we are able to ignore many physical properties of elements such as their length, shape, size, and location.

Thus far, our discussion focused on the constraints that allowed us to treat individual elements as being lumped. We can now turn our attention to circuits. As defined earlier, circuits are sets of lumped elements connected by ideal wires. Currents outside the lumped elements are confined to the wires. An ideal wire does not develop a voltage across its terminals, irrespective of the amount of current it carries. Furthermore, we choose the wires such that they obey the lumped matter discipline, so the wires themselves are also lumped elements.

For their voltages and currents to be meaningful, the constraints that apply to lumped elements apply to entire circuits as well. In other words, for voltages between any pair of points in the circuit and for currents through wires to be defined, any segment of the circuit must obey a set of constraints similar to those imposed on each of the lumped elements.

Accordingly, the lumped matter discipline for circuits can be stated as

1. The rate of change of magnetic flux linked with any portion of the circuit must be zero for all time.
2. The rate of change of the charge at any node in the circuit must be zero for all time. A node is any point in the circuit at which two or more element terminals are connected using wires.
3. The signal timescales must be much larger than the propagation delay of electromagnetic waves through the circuit.

Notice that the first two constraints follow directly from the corresponding constraints applied to lumped elements. (Recall that wires are themselves lumped elements.) So, the first two constraints do not imply any new restrictions beyond those already assumed for lumped elements.[11]

The third constraint for circuits, however, imposes a stronger restriction on signal timescales than for elements, because a circuit can have a much larger physical extent than a single element. The third constraint says that the circuit must be much smaller in all its dimensions than the wavelength of light at the highest operating frequency of interest. If this requirement is satisfied, then wave phenomena are not important to the operation of the circuit. The circuit operates quasistatically, and information propagates instantaneously across it. For example, circuits operating in vacuum or air at 1 kHz, 1 MHz, and 1 GHz would have to be much smaller than 300 km, 300 m, and 300 mm, respectively.

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11. As we shall see in Chapter 9, it turns out that voltages and currents in circuits result in electric and magnetic fields, thus appearing to violate the set of constraints to which we promised to adhere. In most cases these are negligible. However, when their effects cannot be ignored, we explicitly model them using elements called capacitors and inductors.

Most circuits satisfy such a restriction. But, interestingly, an uninterrupted 5000-km power grid operating at 60 Hz, and a 30-cm computer motherboard operating at 1 GHz, would not. Both systems are approximately one wavelength in size so wave phenomena are very important to their operation and they must be analyzed accordingly. Wave phenomena are now becoming important to microprocessors as well. We will address this issue in more detail in Section 1.4.

When a circuit meets these three constraints, the circuit can itself be abstracted as a lumped element with external terminals for which voltages and currents can be defined. Circuits that adhere to the lumped matter discipline yield additional simplifications in circuit analysis. Specifically, we will show in Chapter 2 that the voltages and currents across the collection of lumped circuits obey simple algebraic relationships stated as two laws: Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL).

1.4 LIMITATIONS OF THE LUMPED CIRCUIT ABSTRACTION

We used the lumped circuit abstraction to represent the circuit pictured in Figure 1.4a by the schematic diagram of Figure 1.4b. We stated that it was permissible to ignore the physical extent and topology of the wires connecting the elements and define voltages and currents for the elements provided they met the lumped matter discipline.

The third postulate of the lumped matter discipline requires us to limit ourselves to signal speeds that are significantly lower than the speed of electromagnetic waves. As technology advances, propagation effects are becoming harder to ignore. In particular, as computer speeds pass the gigahertz range, increasing signal speeds and fixed system dimensions tend to break our abstractions, so that engineers working on the forefront of technology must constantly revisit the disciplines upon which abstractions are based and prepare to resort to fundamental physics if the constraints are violated.

As an example, let us work out the numbers for a microprocessor. In a microprocessor, the conductors are typically encased in insulators such as silicon dioxide. These insulators have dielectric constants nearly four times that of free space, and so electromagnetic waves travel only half as fast through them. Electromagnetic waves travel at the speed of approximately 1 foot or 30 cm per nanosecond in vacuum, so they travel at roughly 6 inches or 15 cm per nanosecond in the insulators. Since modern microprocessors (for example, the Alpha microprocessor from Digital/Compaq) can approach 2.5 cm in size, the propagation delay of electromagnetic waves across the chip is on the order of 1/6 ns. These microprocessors are approaching a clock rate of 2 GHz in 2001. Taking the reciprocal, this translates to a clock cycle time of 1/2 ns. Thus, the wave propagation delay across the chip is about 33% of a clock cycle. Although techniques such as pipelining attempt to reduce the number of elements (and therefore distance) a signal traverses in a clock cycle, certain clock or power lines in microprocessors can travel the full extent of the chip, and will suffer this large delay. Here, wave phenomena must be modeled explicitly.

In contrast, slower chips built in earlier times satisfied our lumped matter discipline more easily. For example, the MIPS microprocessor built in 1984 was implemented on a chip that was 1 cm on a side. It ran at a speed of 20 MHz, which translates to a cycle time of 50 ns. The wave propagation delay across the chip was 1/15 ns, which was significantly smaller then the chip cycle time.

As another example, a Pentium II chip built in 1998 clocked at 400 MHz, but used a chip size that was more or less the same as that of the MIPS chip — namely, about 1 cm on a side. As calculated earlier, the wave propagation delay across a 1-cm chip is about 1/15 ns. Clearly the 2.5-ns cycle time of the Pentium II chip is still significantly larger than the wave propagation delay across the chip.

Now consider a Pentium IV chip built in 2004 that clocked at 3.4 GHz, and was roughly 1 cm on a side. The 0.29-ns cycle time is only four times the wave propagation delay across the chip!

If we are interested in signal speeds that are comparable to the speed of electromagnetic waves, then the lumped matter discipline is violated, and therefore we cannot use the lumped circuit abstraction. Instead, we must resort to distributed circuit models based on elements such as transmission lines and waveguides.[12] In these distributed elements, the voltages and currents at any instant of time are a function of the location within the elements. The treatment of distributed elements are beyond the scope of this book.

The lumped circuit abstraction encounters other problems with time-varying signals even when signal frequencies are small enough that propagation effects can be neglected. Let us revisit the circuit pictured in Figure 1.7 in which a signal generator drives a resistor circuit. It turns out that under certain conditions the *frequency* of the oscillator and the lengths and layout of the wires may have a profound effect on the voltages. If the oscillator is generating a sine wave at some low frequency, such as 256 Hz (Middle C in musical terms), then the voltage divider relation developed in Chapter 2 (Equation 2.138) could be used to calculate with some accuracy the voltage across R_2. But if the frequency of the sine wave were 100 MHz (1×10^8 Hertz), then we have a problem. As we will see later, capacitive and inductive effects in the resistors and the wires (resulting from electric fields and magnetic fluxes generated by the signal) will

12. In case you are wondering how the Pentium IV and similar chips get away with high clock speeds, the key lies in designing circuits and laying them out on the chip in a way that most signals traverse a relatively small fraction of the chip in a clock cycle. To enable succeeding generations of the chip to be clocked faster, signals must traverse progressively shorter distances. A technique called pipelining is the key enabling mechanism that accomplishes this. The few circuits in which signals travel the length of the chip must be designed with extreme care using transmission line analysis.

seriously affect the circuit behavior, and these are not currently represented in our model. In Chapter 9, we will separate these effects into new lumped elements called capacitors and inductors so our lumped circuit abstraction holds at high frequencies as well.

All circuit model discussions in this book are predicated on the assumption that the frequencies involved are low enough that the effects of the fields can be adequately modeled by lumped elements. In Chapters 1 through 8, we assume that the frequencies involved are even lower so we can ignore all capacitive and inductive effects as well.

Are there other additional practical considerations in addition to the constraints imposed by the lumped matter discipline? For example, are we justified in neglecting contact potentials, and lumping all battery effects in V? Can we neglect all resistance associated with the wires, and lump all the resistive effects in a series connected resistor? Does the voltage V change when the resistors are connected and current flows? Some of these issues will be addressed in Sections 1.6 and 1.7.

1.5 PRACTICAL TWO-TERMINAL ELEMENTS

Resistors and batteries are two of our most familiar lumped elements. Such lumped elements are the primitive building blocks of electronic circuits. Electronic access to an element is made through its *terminals*. At times, terminals are paired together in a natural way to form *ports*. These ports offer an alternative view of how electronic access is made to an element. An example of an arbitrary element with two terminals and one port is shown in Figure 1.8. Other elements may have three or more terminals, and two or more ports.

Most circuit analyses are effectively carried out on circuits containing only two-terminal elements. This is due in part to the common use of two-terminal elements, and in part to the fact that most, if not all, elements having more than two terminals are usually modeled using combinations of two-terminal elements. Thus, two-terminal elements appear prominently in all electronic

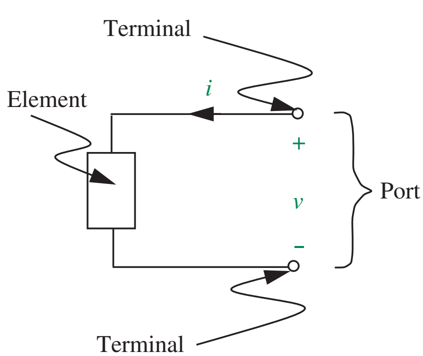

FIGURE 1.8 An arbitrary two-terminal circuit element.

circuit analyses. In this section, we discuss a couple of familiar examples of two-terminal elements — resistors and batteries.

1.5.1 BATTERIES

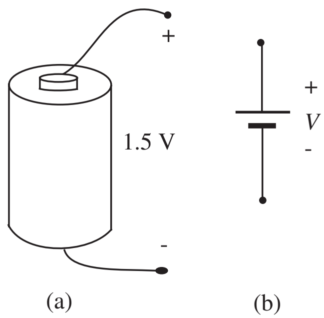

FIGURE 1.9 Symbol for battery.

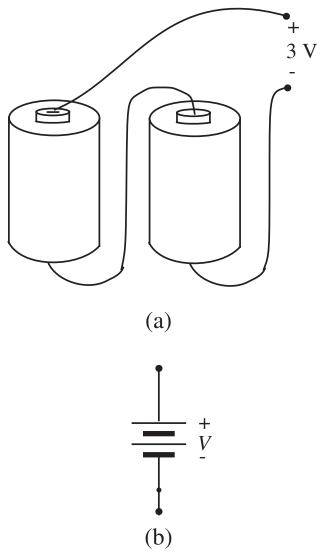

FIGURE 1.10 Cells in series.

Cell phone batteries, laptop batteries, flashlight batteries, watch batteries, car batteries, calculator batteries, are all common devices in our culture. All are sources of energy, derived in each case from an internal chemical reaction.

The important specifications for a battery are its nominal voltage, its total store of energy, and its internal resistance. In this section, we will assume that the internal resistance of a battery is zero. The voltage measured at the terminals of a single cell is fundamentally related to the chemical reaction releasing the energy. In a flashlight battery, for example, the carbon central rod is approximately 1.5 V positive with respect to the zinc case, as noted in Figure 1.9a. In a circuit diagram, such a single-cell battery is usually represented schematically by the symbol shown in Figure 1.9b. Of course, to obtain a larger voltage, several cells can be connected in *series*: the positive terminal of the first cell connected to the negative terminal of the second cell, and so forth, as suggested pictorially in Figure 1.10. Multiple-cell batteries are usually represented by the symbol in Figure 1.10b, (with no particular correspondence between the number of lines and the actual number of cells in series).

The second important parameter of a battery is the total amount of energy it can store, often measured in *joules*. However, if you pick up a camcorder or flashlight battery, you might notice the ratings of *ampere-hours* or *watt-hours*. Let us reconcile these ratings. When a battery is connected across a resistive load in a circuit, it delivers power. The lightbulb in Figure 1.4a is an example of a resistive load.

The power delivered by the battery is the product of the voltage and the current:

$$p = VI. \tag{1.2}$$

Power is *delivered* by the battery when the current I flowing out of the positive voltage terminal of the battery is positive. Power is measured in *watts*. A battery delivers one watt of power when V is one volt and I is one ampere.

Power is the rate of delivery of energy. Thus the amount of energy w delivered by the battery is the time integral of the power.

If a constant amount of power p is delivered over an interval T, the energy w supplied is

$$w = pT. \tag{1.3}$$

The battery delivers one joule of energy if it supplies one watt of power over one second. Thus, joules and watt-seconds are equivalent units. Similarly, if a battery delivers one watt for an hour, then we say that it has supplied one watt-hour (3600 joules) of energy.

Assuming that the battery terminal voltage is constant at V, because the power delivered by the battery is the product of the voltage and the current, an equivalent indication of the power delivered is the amount of current being supplied. Similarly, the product of current and the length of time the battery will sustain that current is an indication of the energy capacity of the battery. A car battery, for instance, might be rated at 12 V and 50 A-hours. This means that the battery can provide a 1-A current for 50 hours, or a 100-A current for 30 minutes. The amount of energy stored in such a battery is

$$\text{Energy} = 12 \times 50 = 600 \text{ watt-hours} = 600 \times 3600 = 2.16 \times 10^6 \text{ joules.}$$

EXAMPLE 1.1 A LITHIUM-ION BATTERY A Lithium-Ion (Li-Ion) battery pack for a camcorder is rated as 7.2 V and 5 W-hours. What are its equivalent ratings in mA-hours and joules?

Since a joule(J) is equivalent to a W-second, 5 W-hours is the same as $5 \times 3600 = 18000$ J.

Since the battery has a voltage of 7.2 V, the battery rating in ampere-hours is $5/7.2 = 0.69$. Equivalently, its rating in mA-hours is 690.

EXAMPLE 1.2 ENERGY COMPARISON Does a Nickel-Cadmium (Ni-Cad) battery pack rated at 6 V and 950 mA-hours store more or less energy than a Li-Ion battery pack rated at 7.2 V and 900 mA-hours?

We can directly compare the two by converting their respective energies into joules. The Ni-Cad battery pack stores $6 \times 950 \times 3600/1000 = 20520$ J, while the Li-Ion battery pack stores $7.2 \times 900 \times 3600/1000 = 23328$ J. Thus the Li-Ion battery pack stores more energy.

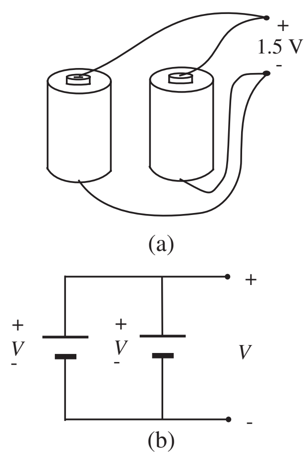

FIGURE 1.11 Cells in parallel.

When a battery is connected across a resistor, as illustrated in Figure 1.4, we saw that the battery delivers energy at some rate. The power was the rate of delivery of energy. Where does this energy go? Energy is dissipated by the resistor, through heat, and sometimes even light and sound if the resistor overheats and explodes! We will discuss resistors and power dissipation in Section 1.5.2.

If one wishes to increase the current capacity of a battery without increasing the voltage at the terminals, individual cells can be connected in *parallel*, as shown in Figure 1.11. It is important that cells to be connected in parallel be nearly identical in voltage to prevent one cell from destroying another. For example, a 2-V lead-acid cell connected in parallel with a 1.5 V flashlight cell will surely destroy the flashlight cell by driving a huge current through it.

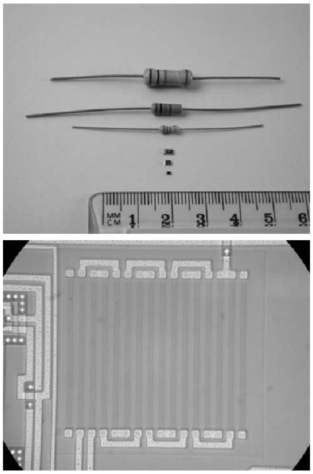

FIGURE 1.12 Discrete resistors (above), and Deposits integrated-circuit resistors (below). The image on the bottom shows a small region of the MAX807L microprocessor supervisory circuit from Maxim Integrated Products, and depicts an array of silicon-chromium thin-film resistors, each with 6 μm width and 217.5 μm length, and nominal resistance 50 kΩ. (Photograph Courtesy of Maxim Integrated Products.)

FIGURE 1.13 Symbol for resistor.

The corresponding constraint for cells connected in series is that the nominal current capacity be nearly the same for all cells. The total energy stored in a multicell battery is the same for series, parallel, or series-parallel interconnections.

1.5.2 LINEAR RESISTORS

Resistors come in many forms (see Figure 1.12), ranging from lengths of nichrome wire used in toasters and electric stoves and planar layers of polysilicon in highly complex computer chips, to small rods of carbon particles encased in Bakelite commonly found in electronic equipment. The symbol for resistors in common usage is shown in Figure 1.13.

Over some limited range of voltage and current, carbon, wire and polysilicon resistors obey Ohm's law:

$$v = iR \tag{1.4}$$

that is, the voltage measured across the terminals of a resistor is linearly proportional to the current flowing through the resistor. The constant of proportionality is called the *resistance*. As we show shortly, the resistance of a piece of material is proportional to its length and inversely proportional to its cross-sectional area.

In our example of Figure 1.4b, suppose that the battery is rated at 1.5 V. Further assume that the resistance of the bulb is $R = 10\ \Omega$. Assume that the internal resistance of the battery is zero. Then, a current of $i = v/R = 150$ mA will flow through the bulb.

EXAMPLE 1.3 MORE ON RESISTANCE In the circuit in Figure 1.4b, suppose that the battery is rated at 1.5 V. Suppose we observe through some means a current of 500 mA through the resistor. What is the resistance of the resistor?

For a resistor, we know from Equation 1.4 that

$$R = \frac{v}{i}$$

Since the voltage v across the resistor is 1.5 V and the current i through the resistor is 500 mA, the resistance of the resistor is 3 Ω.

The resistance of a piece of material depends on its geometry. As illustrated in Figure 1.14, assume the resistor has a conducting channel with cross-sectional area a, length l, and resistivity ρ. This channel is terminated at its extremes by two conducting plates that extend to form the two terminals of the resistor. If this cylindrical piece of material satisfies the lumped matter

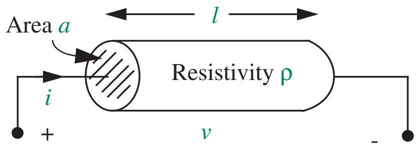

FIGURE 1.14 A cylindrical-wire shaped resistor.

discipline and obey's Ohm's law, we can write[13]

$$R = \rho \frac{l}{a} \tag{1.5}$$

Equation 1.5 shows that the resistance of a piece of material is proportional to its length and inversely proportional to its cross-sectional area.

Similarly, the resistance of a cuboid shaped resistor with length l, width w, and height h is given by

$$R = \rho \frac{l}{wh} \tag{1.6}$$

when the terminals are taken at the pair of surfaces with area wh.

EXAMPLE 1.4 RESISTANCE OF A CUBE Determine the resistance of a cube with sides of length 1 cm and resistivity 10 ohm-cms, when a pair of opposite surfaces are chosen as the terminals.

Substituting $\rho = 10\ \Omega$-cm, $l = 1$ cm, $w = 1$ cm, and $h = 1$ cm in Equation 1.6, we get $R = 10\ \Omega$.

EXAMPLE 1.5 RESISTANCE OF A CYLINDER By what factor is the resistance of a wire with cross-sectional radius r greater than the resistance of a wire with cross-sectional radius $2r$?

A wire is cylindrical in shape. Equation 1.5 relates the resistance of a cylinder to its cross-sectional area. Rewriting Equation 1.5 in terms of the cross-sectional radius r we have

$$R = \rho \frac{l}{\pi r^2}.$$

From this equation it is clear that the resistance of a wire with radius r is four times greater than that of a wire with cross-sectional radius $2r$.

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13. See Appendix A.3 for a derivation.

EXAMPLE 1.6 CARBON-CORE RESISTORS The resistance of small carbon-core resistors can range from 1 Ω to 10^6 Ω. Assuming that the core of these resistors is 1 mm in diameter and 5 mm long, what must be the range of resistivity of the carbon cores?

Given a 1-mm diameter, the cross-sectional area of the core is $A \approx 7.9 \times 10^{-7}$ m^2. Further, its length is $l = 5 \times 10^{-3}$m. Thus, $A/l \approx 1.6 \times 10^{-4}$ m.

Finally, using Equation 1.5, with $1\ \Omega \leq R \leq 10^6\ \Omega$, it follows that the approximate range of its resistivity is $1.6 \times 10^{-4}\ \Omega\text{m} \leq \rho \leq 1.6 \times 10^{2}\Omega\text{m}$.

EXAMPLE 1.7 POLY-CRYSTALLINE SILICON RESISTOR A thin poly-crystalline silicon resistor is 1 μm thick, 10 μm wide, and 100 μm long, where 1 μm is 10^{-6} m. If the resistivity of its poly-crystalline silicon ranges from 10^{-6} Ωm to 10^2 Ωm, what is the range of its resistance?

The cross-sectional area of the resistor is $A = 10^{-11}$ m, and its length is $l = 10^{-4}$ m. Thus $l/A = 10^7$ m^{-1}. Using Equation 1.5, and the given range of resistivity, ρ, the resistance satisfies $10\ \Omega \leq R \leq 10^9\ \Omega$.

EXAMPLE 1.8 RESISTANCE OF PLANAR MATERIALS ON A CHIP Figure 1.15 shows several pieces of material with varying geometries. *Assume all the pieces have the same thickness.* In other words, the pieces of material are *planar*. Let us determine the resistance of these pieces between the pairs of terminals shown. For a given thickness, remember that the resistance of a piece of material in the shape of a cuboid is determined by the ratio of the length to the width of the piece of material (Equation 1.6). Assuming that R_o is the resistance of a piece of planar material

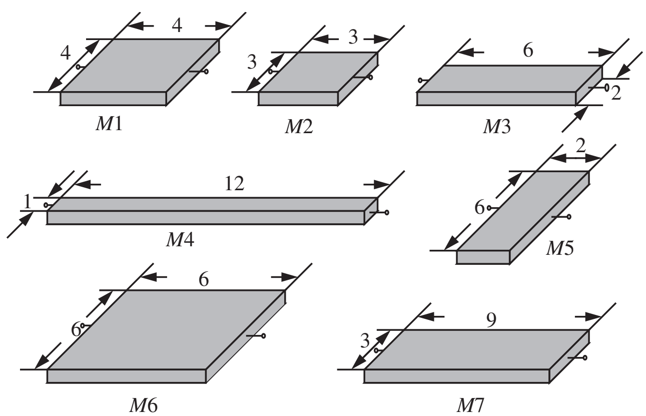

FIGURE 1.15 Resistors of various shapes.

with unit length and width, show that the resistance of a piece of planar material with length L and width W is $(L/W)R_o$.

From Equation 1.6, the resistance of a cuboid shaped material with length L, width W, height H, and resistivity ρ is

$$R = \rho \frac{L}{WH}. \tag{1.7}$$

We are given that the resistance of a piece of the same material with $L = 1$ and $W = 1$ is R_o. In other words,

$$R_o = \rho \frac{1}{H}. \tag{1.8}$$

Substituting $R_o = \rho/H$ in Equation 1.7, we get

$$R = \frac{L}{W} R_o. \tag{1.9}$$

Now, assume $R_o = 2\ \mathrm{k}\Omega$ for our material. Recall that Ohms are the unit of resistance and are written as Ω. We denote a 1000-Ω value as kilo-Ω or kΩ. Assuming that the dimensions of the pieces of material shown in Figure 1.15 are in μ-m, or micrometers, what are their resistances?

First, observe that pieces $M1$, $M2$, and $M6$ must have the same resistance of 2 kΩ because they are squares (in Equation 1.9, notice that $L/W = 1$ for a square).

Second, $M3$ and $M7$ must have the same resistance because both have the same ratio $L/W = 3$. Therefore, both have a resistance of $3 \times 2 = 6$ kΩ. Among them, $M4$ has the biggest L/W ratio of 12. Therefore it has the largest resistance of 24 kΩ. $M5$ has the smallest L/W ratio of 1/3, and accordingly has the smallest resistance of 2/3 kΩ.

Because all square pieces made out of a given material have the same resistance (provided, of course, the pieces have the same thickness), we often characterize the resistivity of planar material of a given thickness with

$$R_\square = R_o, \tag{1.10}$$

where R_o is the resistance of a piece of the same material with unit length and width. Pronounced "R square," $R_\square$ is the resistance of a square piece of material.

EXAMPLE 1.9 MORE ON PLANAR RESISTANCES Referring back to Figure 1.15, suppose an error in the material fabrication process results in each dimension (L and W) increasing by a fraction e. By what amount will the resistances of each of the pieces of material change?

Recall that the resistance R of a planar rectangular piece of material is proportional to L/W. If each dimension increases by a fraction e, the new length becomes $L(1+e)$ and the new width becomes $W(1+e)$. Notice that the resistance given by

$$R = \frac{L(1+e)}{W(1+e)} R_o = \frac{L}{W} R_o$$

is unchanged.

EXAMPLE 1.10 RATIO OF RESISTANCES Referring again to Figure 1.15, suppose the material fabrication process undergoes a "shrink" to decrease each dimension (this time around, increasing the thickness H in addition to L and W) by a fraction α (e.g., $\alpha = 0.8$). Assume further, that the resistivity ρ changes by some other fraction to ρ'. Now consider a pair of resistors with resistances R_1 and R_2, and original dimensions L_1, W_1 and L_2, W_2 respectively, and the same thickness H. By what fraction does the ratio of the resistance values change after the process shrink?

From Equation 1.7, the ratio of the original resistance values is given by

$$\frac{R_1}{R_2} = \frac{\rho L_1/(W_1 H)}{\rho L_2/(W_2 H)} = \frac{L_1/W_1}{L_2/W_2}.$$

Let the resistance values after the process shrink be R_1' and R_2'. Since each dimension shrinks by the fraction α, each new dimension will be α times the original value. Thus, for example, the length L_1 will change to αL_1. Using Equation 1.7, the ratio of the new resistance values is given by

$$\frac{R_1'}{R_2'} = \frac{\rho' \alpha L_1/(\alpha W_1 \alpha H)}{\rho' \alpha L_2/(\alpha W_2 \alpha H)} = \frac{L_1/W_1}{L_2/W_2}$$

In other words, the ratio of the resistance values is unchanged by the process shrink.

The ratio property of planar resistance — that is that the ratio of the resistances of rectangular pieces of material with a given thickness and resistivity is independent of the actual values of the length and the width provided the ratio of the length and the width is fixed — enables us to perform process shrinks (for example, from a 0.25-μm process to a 0.18-μm process) without needing to change the chip layout. Process shrinks are performed by scaling the dimensions of the chip and its components by the same factor, thereby resulting in a smaller chip. The chip is designed such that relevant

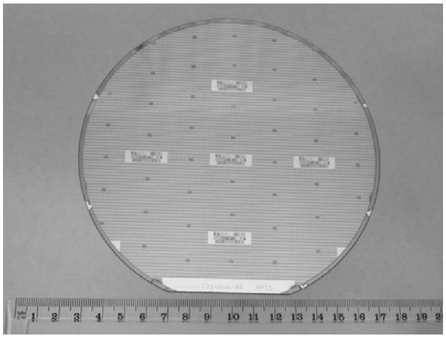

FIGURE 1.16 A silicon wafer. (Photograph Courtesy of Maxim Integrated Products.)

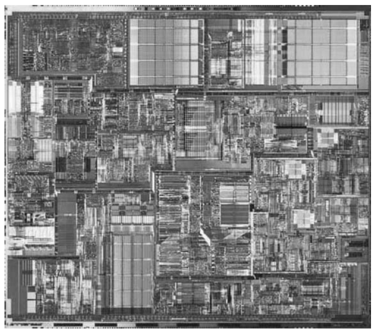

FIGURE 1.17 A chip photo of Intel's 2-GHz Pentium IV processor implemented in 0.18μm-technology. The chip is roughly 1 cm on a side. (Photograph courtesy of Intel Corp.)

signal values are derived as a function of resistance ratios,[14] thereby ensuring that the chip manufactured after a process shrink continues to function as before.

VLSI stands for "Very Large Scale Integration." Silicon-based VLSI is the technology behind most of today's computer chips. In this technology, lumped planar elements such as wires, resistors, and a host of others that we will soon encounter, are fabricated on the surface of a planar piece of silicon called a *wafer* (for example, see Figures 1.15 and 1.12). A wafer has roughly the shape and size of a Mexican tortilla or an Indian chapati (see Figure 1.16). The planar elements are connected together using planar wires to form circuits. After fabrication, each wafer is diced into several hundred chips or "dies," typically, each the size of a thumbnail. A Pentium chip, for example, contains hundreds of millions of planar elements (see Figure 1.17). Chips are attached, or bonded, to packages (for example, see Figure 12.3.4), which are in turn mounted on a printed-circuit board along with other discrete components such as resistors and capacitors (for example, see Figure 1.18) and wired together.

14. We will study many such examples in the ensuing sections, including the voltage divider in Section 2.3.4 and the inverter in Section 6.8.

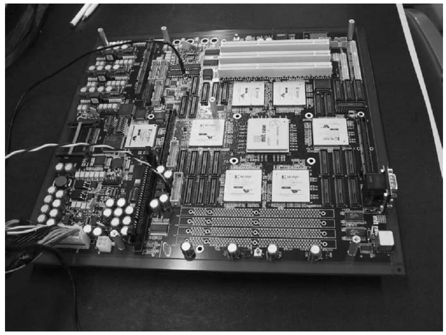

FIGURE 1.18 A printed-circuit board containing several interconnected chip packages and discrete components such as resistors (tiny box-like objects) and capacitors (tall cylindrical objects). (Photograph Courtesy of Anant Agarwal, the Raw Group.)

As better processes become available, VLSI fabrication processes undergo periodic shrinks to reduce the size of chips without needing significant design changes. The Pentium III, for example, initially appeared in the 0.25-μm process, and later in the 0.18-μm process. The Pentium IV chip shown in Figure 1.17 initially appeared in a 0.18-μm process in the year 2000, and later in 0.13-μm and 0.09-μm processes in 2001 and 2004, respectively.

There are two important limiting cases of the linear resistor: *open circuits* and *short circuits*. An open circuit is an element through which no current flows, regardless of its terminal voltage. It behaves like a linear resistor in the limit $R \rightarrow \infty$.

A short circuit is at the opposite extreme. It is an element across which no voltage can appear regardless of the current through it. It behaves like a linear resistor in the limit $R \rightarrow 0$. Observe that the short circuit element is the same as an ideal wire. Note that neither the open circuit nor the short circuit dissipate power since the product of their terminal variables (v and i) is identically zero.

Most often, resistances are thought of as time-invariant parameters. But if the temperature of a resistor changes, then so too can its resistance. Thus, a linear resistor can be a time-varying element.

The linear resistor is but one example of a larger class of resistive elements. In particular, resistors need not be linear; they can be nonlinear as well. In general, *a two-terminal resistor is any two-terminal element that has an algebraic relation between its instantaneous terminal current and its instantaneous terminal voltage*. Such a resistor could be linear or nonlinear, time-invariant or time-varying. For example, elements characterized by the following element relationships are all general resistors:

$$\text{Linear resistor:} \quad v(t) = i(t)R(t)$$

$$\text{Linear, time-invariant resistor:} \quad v(t) = i(t)R$$

$$\text{Nonlinear resistor:} \quad v(t) = Ki(t)^3$$

However, as introduced in Chapter 9, elements characterized by these relationships are not general resistors:

$$v(t) = L\frac{di(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t')dt'$$

What is important about the general resistor is that its terminal current and voltage depend only on the instantaneous values of each other. For our convenience, however, an unqualified reference to a resistor in this book means a linear, time-invariant resistor.

1.5.3 ASSOCIATED VARIABLES CONVENTION

Equation 1.4 implies a specific relation between reference directions chosen for voltage and current. This relation is shown explicitly in Figure 1.19: the arrow that defines the positive flow of current (flow of positive charge) is directed *in* at the resistor terminal assigned to be positive in voltage. This convention, referred to as *associated variables*, is generalized to an arbitrary element in Figure 1.20 and will be followed whenever possible in this text. The variables v and i are called the *terminal variables* for the element. Note that the values of each of these variables may be positive or negative depending on the actual direction of current flow or the actual polarity of the voltage.

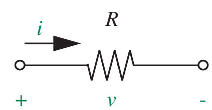

FIGURE 1.19 Definition of terminal variables v and i for the resistor.

Associated Variables Convention Define current to flow *in* at the device terminal assigned to be *positive* in voltage.

When the voltage v and current i for an element are defined under the associated variables convention, the power *into* the element is positive when both v and i are positive. In other words, energy is pumped into an element when a positive current i is directed *into* the voltage terminal marked positive. Depending on the type of element, the energy is either dissipated or stored. Conversely, power is supplied by an element when a positive current i is directed *out* of the voltage terminal marked positive. When the terminal variables for a resistor are defined according to associated variables, the power dissipated in the resistor is a *positive* quantity, an intuitively satisfying result.

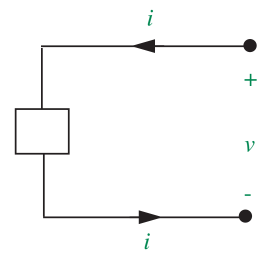

FIGURE 1.20 Definition of the terminal variables v and i for a two-terminal element under the associated variables convention.

While Figure 1.20 is quite simple, it nonetheless makes several important points. First, the two terminals of the element in Figure 1.20 form a single port through which the element is addressed. Second, the current i circulates through that port. That is, the current that enters one terminal is instantaneously equal to the current that exits the other terminal. Thus, according to the lumped matter discipline, net charge cannot accumulate within the element. Third, the voltage v of the element is defined across the port. Thus, the element is assumed to respond only to the difference of the electrical potentials at its two terminals,