DESIGN OF MACHINERY

AN INTRODUCTION TO THE SYNTHESIS AND ANALYSIS OF MECHANISMS AND MACHINES

Second Edition

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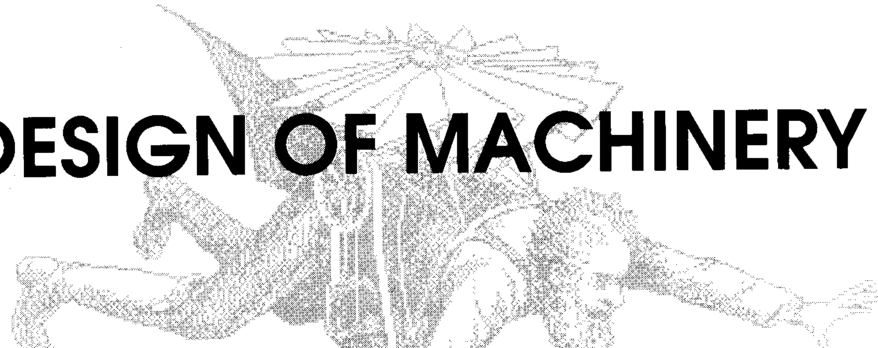

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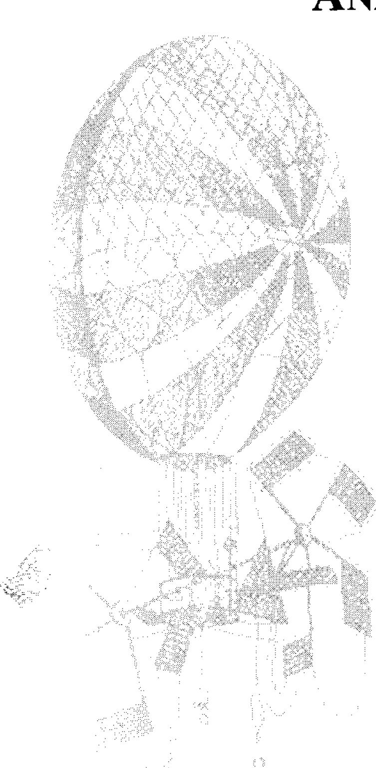

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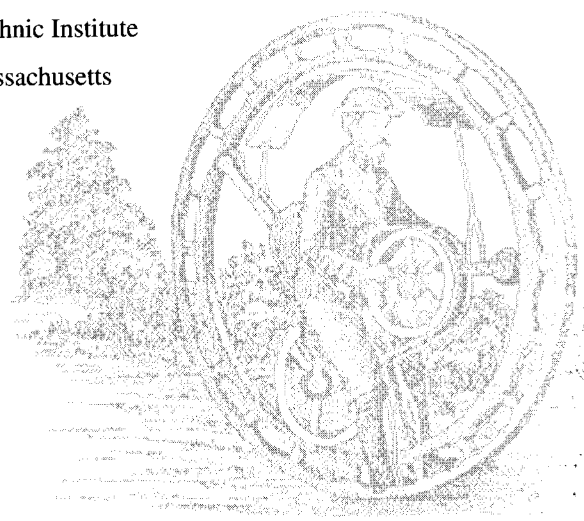

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Robert L. Norton earned undergraduate degrees in both mechanical engineering and industrial technology at Northeastern University and an MS in engineering design at Tufts University. He is a registered professional engineer in Massachusetts and New Hampshire. He has extensive industrial experience in engineering design and manufacturing and many years experience teaching mechanical engineering, engineering design, computer science, and related subjects at Northeastern University, Tufts University, and Worcester Polytechnic Institute. At Polaroid Corporation for ten years, he designed cameras, related mechanisms, and high-speed automated machinery. He spent three years at Jet Spray Cooler Inc., Waltham, Mass., designing food-handling machinery and products. For five years he helped develop artificial-heart and noninvasive assisted-circulation (counterpulsation) devices at the Tufts New England Medical Center and Boston City Hospital. Since leaving industry to join academia, he has continued as an independent consultant on engineering projects ranging from disposable medical products to high-speed production machinery. He holds 13 U.S. patents.

Norton has been on the faculty of Worcester Polytechnic Institute since 1981 and is currently professor of mechanical engineering and head of the design group in that department. He teaches undergraduate and graduate courses in mechanical engineering with emphasis on design, kinematics, and dynamics of machinery. He is the author of numerous technical papers and journal articles covering kinematics, dynamics of machinery, carn design and manufacturing, computers in education, and engineering education and of the text *Machine Design: An Integrated Approach.* He is a Fellow of the American Society of Mechanical Engineers and a member of the Society of Automotive Engineers. Rumors about the transplantation of a Pentium microprocessor into his brain are decidedly untrue (though he could use some additional RAM). As for the unobtainium* ring, well, that's another story.

* See Index.

This book is dedicated to the memory of my father,

Harry J. Norton, Sr.

who sparked a young boy's interest in engineering;

to the memory of my mother,

Kathryn W Norton

who made it all possible;

to my wife,

Nancy Norton

who provides unflagging patience and support;

and to my children,

Robert, Mary, and Thomas,

who make it all worthwhile.

CONTENTS

PREFACE

to the Second Edition

Why is it we never have time to do it right the first time, but always seem to have time to do it over?
ANONYMOUS

The second edition has been revised based on feedback from a large number of users of the book. **In** general, the material in many chapters has been updated to reflect the latest research findings in the literature. Over 250 problem sets have been added, more than doubling the total number of problems. Some design projects have been added also. All the illustrations have been redrawn, enhanced, and improved.

Coverage of the design process in Chapter 1 has been expanded. The discussions of the Grashof condition and rotatability criteria in Chapter 2 have been strengthened and that of electric motors expanded. A section on the optimum design of approximate straight line linkages has been added to Chapter 3. A discussion of circuits and branches in linkages and a section on the Newton-Raphson method of solution have been added to Chapter 4. A discussion of other methods for analytical and computational solutions to the position synthesis problem has been added to Chapter 5. This reflects the latest publications on this subject and is accompanied by an extensive bibliography.

The chapters formerly devoted to explanations of the accompanying software (old Chapters 8 and 16) have been eliminated. Instead, a new Appendix A has been added to describe the programs FOURBAR, FIVEBAR, SrXBAR, SLIDER, DYNACAM, ENGINE, and MATRIX that are on the attached CD-ROM. These programs have been completely rewritten as *Windows* applications and are much improved. A student version of the simulation program *Working Model* by *Knowledge Revolution,* compatible with both *Macintosh* and *Windows* computers, is also included on CD-ROM along with 20 models of mechanisms from the book done in that package. A user's manual for *Working Model* is also on the CD-ROM.

Chapter 8 on cam design (formerly 9) has been shortened without reducing the scope of its coverage. Chapter 9 on gear trains (formerly 10) has been significantly expanded and enhanced, especially in respect to the design of compound and epicyclic trains and their efficiency. Chapter 10 on dynamics fundamentals has been augmented with material formerly in Chapter 17 to give a more coherent treatment of dynamic modeling. Chapter 12 on balancing (formerly 13) has been expanded to include discussion of moment balancing of linkages.

The author would like to express his appreciation to all the users and reviewers who have made suggestions for improvement and pointed out errors, especially those who responded to the survey about the first edition. There are too many to list here, so rather than risk offense by omitting anyone, let me simply extend my sincerest thanks to you all for your efforts.

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PREFACE

to the First Edition

When I hear, I forget
When I see, I remember
When I do, I understand
ANCIENT CHINESE PROVERB

This text is intended for the kinematics and dynamics of machinery topics which are often given as a single course, or two-course sequence, in the junior year of most mechanical engineering programs. The usual prerequisites are first courses in statics, dynamics and calculus. Usually, the first semester, or portion, is devoted to kinematics, and the second to dynamics of machinery. These courses are ideal vehicles for introducing the mechanical engineering student to the process of design, since mechanisms tend to be intuitive for the typical mechanical engineering student to visualize and create. While this text attempts to be thorough and complete on the topics of analysis, it also emphasizes the synthesis and design aspects of the subject to a greater degree than most texts in print on these subjects. Also, it emphasizes the use of computer-aided engineering as an approach to the design and analysis of this class of problems by providing software that can enhance student understanding. While the mathematical level of this text is aimed at second- or third-year university students, it is presented *de novo* and should be understandable to the technical school student as well.

Part I of this text is suitable for a one-semester or one-term course in kinematics. Part II is suitable for a one-semester or one-term course in dynamics of machinery. Alternatively, both topic areas can be covered in one semester with less emphasis on some of the topics covered in the text.

The writing and style of presentation in the text is designed to be clear, informal, and easy to read. Many example problems and solution techniques are presented and spelled out in detail, both verbally and graphically. All the illustrations are done with computer-drawing or drafting programs. Some scanned photographic images are also included. The entire text, including equations and artwork, is printed directly from computer disk by laser typesetting for maximum clarity and quality. Many suggested readings are provided in the bibliography. Short problems, and where appropriate, many longer, unstructured design project assignments are provided at the ends of chapters. These projects provide an opportunity for the students *to do and understand.*

The author's approach to these courses and this text is based on over 35 years' experience in mechanical engineering design, both in industry and as a consultant. He has taught these subjects since 1967, both in evening school to practicing engineers and in day school to younger students. His approach to the course has evolved a great deal in that time, from a traditional approach, emphasizing graphical analysis of many structured problems, through emphasis on algebraic methods as computers became available, through requiring students to write their own computer programs, to the current state described above.

The one constant throughout has been the attempt to convey the art of the design process to the students in order to prepare them to cope with *real* engineering problems in practice. Thus, the author has always promoted design within these courses. Only recently, however, has technology provided a means to more effectively accomplish this goal, in the form of the graphics microcomputer. This text attempts to be an improvement over those currently available by providing up-to-date methods and techniques for analysis and synthesis which take full advantage of the graphics microcomputer, and by emphasizing design as well as analysis. The text also provides a more complete, modem, and thorough treatment of cam design than existing texts in print on the subject.

The author has written several interactive, student-friendly computer programs for the design and analysis of mechanisms and machines. These programs are designed to enhance the student's understanding of the basic concepts in these courses while simultaneously allowing more comprehensive and realistic problem and project assignments to be done in the limited time available, than could ever be done with manual solution techniques, whether graphical or algebraic. Unstructured, realistic design problems which have many valid solutions are assigned. Synthesis and analysis are equally emphasized. The analysis methods presented are up to date, using vector equations and matrix techniques wherever applicable. Manual graphical analysis methods are de-emphasized. The graphics output from the computer programs allows the student to see the results of variation of parameters rapidly and accurately and reinforces learning.

These computer programs are distributed, on CD-ROM, with this book which also contains instructions for their use on any IBM compatible, Windows 3.1 or Windows 95/NT capable computer. The earlier DOS versions of these programs are also included for those without access to Windows. Programs SLIDER, FOURBAR, FIVEBAR and SIXBAR analyze the kinematics of those types of linkages. Program FOURBAR also does a complete dynamic analysis of the fourbar linkage in addition to its kinematics. Program DYNACAM allows the design and dynamic analysis of cam-follower systems. Program ENGINE analyzes the slider-crank linkage as used in the internal combustion engine and provides a complete dynamic analysis of single and multicylinder engine configurations, allowing the mechanical dynamic design of engines to be done. Program MATRIX is a general purpose linear equation system solver. All these programs, except MATRIX, provide dynamic, graphical animation of the designed devices. The reader is strongly urged to make use of these programs in order to investigate the results of variation of parameters in these kinematic devices. The programs are designed to enhance and augment the text rather than be a substitute for it. The converse is also true. Many solutions to the book's examples and to the problem sets are provided on the CD-ROM as files to be read into these programs. Many of these solutions can be animated on the computer screen for a better demonstration of the concept than is possible on the printed page. The instructor and students are both encouraged to take advantage of the computer programs provided. Instructions for their use are in Appendix A.

The author's intention is that synthesis topics be introduced first to allow the students to work on some simple design tasks early in the term while still mastering the analysis topics. Though this is not the "traditional" approach to the teaching of this material, the author believes that it is a superior method to that of initial concentration on detailed analysis of mechanisms for which the student has no concept of origin or purpose. Chapters 1 and 2 are introductory. Those instructors wishing to teach analysis before synthesis can leave Chapters 3 and 5 on linkage synthesis for later consumption. Chapters 4, 6, and 7 on position, velocity, and acceleration analysis are sequential and build upon each other. In fact, some of the problem sets are common among these three chapters so that students can use their position solutions to find velocities and then later use both to find the accelerations in the same linkages. Chapter 8 on cams is more extensive and complete than that of other kinematics texts and takes a design approach. Chapter 9 on gear trains is introductory. The dynamic force treatment in Part II uses matrix methods for the solution of the system simultaneous equations. Graphical force analysis is not emphasized. Chapter 10 presents an introduction to dynamic systems modelling. Chapter **11** deals with force analysis oflinkages. Balancing of rotating machinery and linkages is covered in Chapter 12. Chapters 13 and 14 use the internal combustion engine as an example to pull together many dynamic concepts in a design context. Chapter 15 presents an introduction to dynamic systems modelling and uses the cam-follower system as the example. Chapters 3, 8, 11, 13, and 14 provide open ended project problems as well as structured problem sets. The assignment and execution of unstructured project problems can greatly enhance the student's understanding of the concepts as described by the proverb in the epigraph to this preface.

ACKNOWLEDGMENTS The sources of photographs and other nonoriginal art used in the text are acknowledged in the captions and opposite the title page, but the author would also like to express his thanks for the cooperation of all those individuals and companies who generously made these items available. The author would also like to thank those who have reviewed various sections of the first edition of the text and who made many useful suggestions for improvement. Mr. John Titus of the University of Minnesota reviewed Chapter 5 on analytical synthesis and Mr. Dennis Klipp of Klipp Engineering, Waterville, Maine, reviewed Chapter 8 on cam design. Professor William J. Crochetiere and Mr. Homer Eckhardt of Tufts University, Medford, Mass., reviewed Chapter 15. Mr. Eckhardt and Professor Crochetiere of Tufts, and Professor Charles Warren of the University of Alabama taught from and reviewed Part I. Professor Holly K. Ault of Worcester Polytechnic Institute thoroughly reviewed the entire text while teaching from the pre-publication, class-test versions of the complete book. Professor Michael Keefe of the University of Delaware provided many helpful comments. Sincere thanks also go to the large number of undergraduate students and graduate teaching assistants who caught many typos and errors in the text and in the programs while using the pre-publication versions. Since the book's first printing, Profs. D. Cronin, K. Gupta, P. Jensen, and Mr. R. Jantz have written to point out errors or make suggestions which I have incorporated and for which I thank them. The author takes full responsibility for any errors that may remain and invites from all readers their criticisms, suggestions for improvement, and identification of errors in the text or programs, so that both can be improved in future versions.

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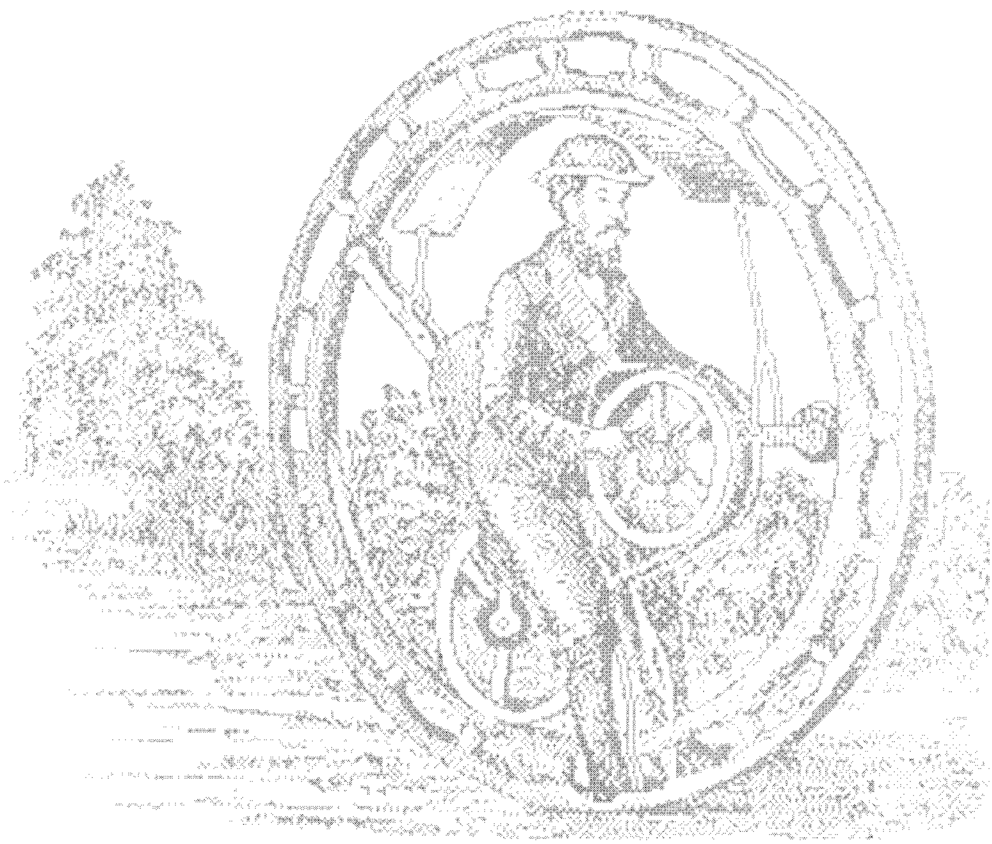
Take to Kinematics. It will repay you. It is more fecund than geometry; it adds a fourth dimension to space.

CHEBYSCHEV TO SYLVESTER, 1873

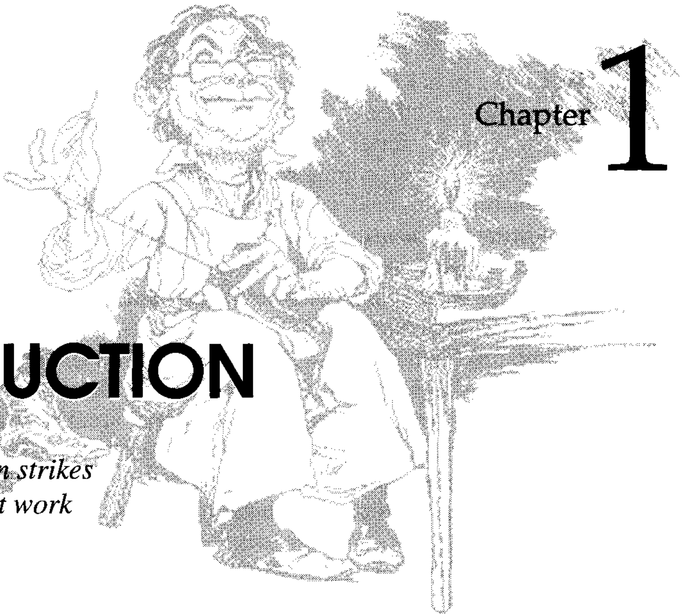
PART I



KINEMATICS OF MECHANISMS

Chapter 1

INTRODUCTION



Inspiration most often strikes
those who are hard at work
ANONYMOUS

1.0 PURPOSE

In this text we will explore the topics of kinematics and dynamics of machinery in respect to the synthesis of mechanisms in order to accomplish desired motions or tasks, and also the analysis of mechanisms in order to determine their rigid-body dynamic behavior. These topics are fundamental to the broader subject of machine design. On the premise that we cannot analyze anything until it has been synthesized into existence, we will first explore the topic of synthesis of mechanisms. Then we will investigate techniques of analysis of mechanisms. All this will be directed toward developing your ability to design viable mechanism solutions to real, unstructured engineering problems by using a design process. We will begin with careful definitions of the terms used in these topics.

1.1 KINEMATICS AND KINETICS

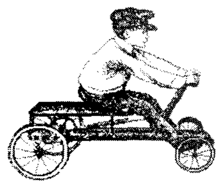
KINEMATICS *The study of motion without regard to forces.*

KINETIcs *The study of forces on systems in motion.*

These two concepts are really *not* physically separable. We arbitrarily separate them for instructional reasons in engineering education. It is also valid in engineering design practice to first consider the desired kinematic motions and their consequences, and then subsequently investigate the kinetic forces associated with those motions. The student should realize that the division between kinematics and kinetics is quite arbitrary and is done largely for convenience. One cannot design most dynamic mechanical systems without taking both topics into thorough consideration. It is quite logical to consider them in the order listed since, from Newton's second law, $F = ma$, one typically needs to know the accelerations (a) in order to compute the dynamic forces (F) due to the motion of the system's mass *(m).* There are also many situations in which the applied forces are known and the resultant accelerations are to be found.

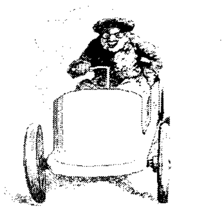
One principal aim of kinematics is to create (design) the desired motions of the subject mechanical parts and then mathematically compute the positions, velocities, and accelerations which those motions will create on the parts. Since, for most earthbound mechanical systems, the mass remains essentially constant with time, defining the accelerations as a function of time then also defines the dynamic forces as a function of time. Stresses, in turn, will be a function of both applied and inertial *(ma)* forces. Since engineering design is charged with creating systems which will not fail during their expected service life, the goal is to keep stresses within acceptable limits for the materials chosen and the environmental conditions encountered. This obviously requires that all system forces be defined and kept within desired limits. In machinery which moves (the only interesting kind), the largest forces encountered are often those due to the dynamics of the machine itself. These dynamic forces are proportional to acceleration, which brings us back to kinematics, the foundation of mechanical design. Very basic and early decisions in the design process involving kinematic principles can be crucial to the success of any mechanical design. A design which has poor kinematics will prove troublesome and perform badly.

1.2 MECHANISMS AND MACHINES



A mechanism

A mechanism is a device which transforms motion to some desirable pattern and typically develops very low forces and transmits little power. A machine typically contains mechanisms which are designed to provide significant forces and transmit significant powerJI] Some examples of common mechanisms are a pencil sharpener, a camera shutter, an analog clock, a folding chair, an adjustable desk lamp, and an umbrella. Some examples of machines which possess motions similar to the mechanisms listed above are a food blender, a bank vault door, an automobile transmission, a bulldozer, a robot, and an amusement park ride. There is no clear-cut dividing line between mechanisms and machines. They differ in degree rather than in kind. If the forces or energy levels within the device are significant, it is considered a machine; if not, it is considered a mechanism. A useful working definition of a mechanism is *A system of elements arranged to transmit* motion *in a predetermined fashion.* This can be converted to a definition of a machine by adding the words and energy after motion.

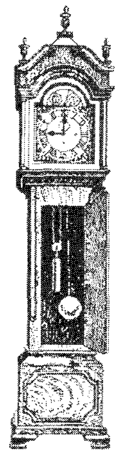


A machine

Mechanisms, if lightly loaded and run at slow speeds, can sometimes be treated strictly as kinematic devices; that is, they can be analyzed kinematically without regard to forces. Machines (and mechanisms running at higher speeds), on the other hand, must first be treated as mechanisms, a kinematic analysis of their velocities and accelerations must be done, and then they must be subsequently analyzed as dynamic systems in which their static and dynamic forces due to those accelerations are analyzed using the principles of kinetics. Part I of this text deals with Kinematics of Mechanisms, and Part II with Dynamics of Machinery. The techniques of mechanism synthesis presented in Part I are applicable to the design of both mechanisms and machines, since in each case some collection of moveable members must be created to provide and control the desired motions and geometry.

1.3 A BRIEFHISTORY OF KINEMATICS

Machines and mechanisms have been devised by people since the dawn of history. The ancient Egyptians devised primitive machines to accomplish the building of the pyramids and other monuments. Though the wheel and pulley (on an axle) were not known to the Old Kingdom Egyptians, they made use of the lever, the inclined plane (or wedge), and probably the log roller. The origin of the wheel and axle is not definitively known. Its first appearance seems to have been in Mesopotamia about 3000 to 4000 B.C.



A great deal of design effort was spent from early times on the problem of timekeeping as more sophisticated clockworks were devised. Much early machine design was directed toward military applications (catapults, wall scaling apparatus, etc.). The term civil engineering was later coined to differentiate civilian from military applications of technology. Mechanical engineering had its beginnings in machine design as the inventions of the industrial revolution required more complicated and sophisticated solutions to motion control problems. James Watt (1736-1819) probably deserves the title of first kinematician for his synthesis of a straight-line linkage (see Figure 3-29a on p. 121) to guide the very long stroke pistons in the then new steam engines. Since the planer was yet to be invented (in 1817), no means then existed to machine a long, straight guide to serve as a crosshead in the steam engine. Watt was certainly the first on record to recognize the value of the motions of the coupler link in the fourbar linkage. Oliver Evans (1755-1819) an early American inventor, also designed a straight-line linkage for a steam engine. Euler (1707-1783) was a contemporary of Watt, though they apparently never met. Euler presented an analytical treatment of mechanisms in his *Mechanica sive Motus Scienta Analytice Exposita* (1736-1742), which included the concept that planar motion is composed of two independent components, namely, translation of a point and rotation of the body about that point. Euler also suggested the separation of the problem of dynamic analysis into the "geometrical" and the "mechanical" in order to simplify the determination of the system's dynamics. Two of his contemporaries, d' Alembert and Kant, also proposed similar ideas. This is the origin of our division of the topic into kinematics and kinetics as described above.

In the early 1800s, L'Ecole Polytechnic in Paris, France, was the repository of engineering expertise. Lagrange and Fourier were among its faculty. One of its founders was Gaspard Monge (1746-1818), inventor of descriptive geometry (which incidentally was kept as a military secret by the French government for 30 years because of its value in planning fortifications). Monge created a course in elements of machines and set about the task of classifying all mechanisms and machines known to mankind! His colleague, Hachette, completed the work in 1806 and published it as what was probably the first mechanism text in 1811. Andre Marie Ampere (1775-1836), also a professor at L'Ecole Polytechnic, set about the formidable task of classifying "all human knowledge." In his *Essai sur la Philosophie des Sciences,* he was the first to use the term "einematique," from the Greek word for motion,* to describe the study of motion without regard to forces, and suggested that "this science ought to include all that can be said with respect to motion in its different kinds, independently of the forces by which it is produced." His term was later anglicized to *kinematics* and germanized to *kinematik.*

Robert Willis (1800-1875) wrote the text *Principles of Mechanism* in 1841 while a professor of natural philosophy at the University of Cambridge, England. He attempted to systematize the task of mechanism synthesis. He counted five ways of obtaining rel-

* Ampere is quoted as writing "(The science of mechanisms) must therefore not define a machine, as has usually been done, as an instrument by the help of which the direction and intensity of a given *force* can be altered, but as an instrument by the help of which the direction and *velocity* of a given motion can be altered. To this science ... I have given the name Kinematics from KtVIl<x-motion." in Maunder, L. (1979). "Theory and Practice." *Proc. 5th World Congo on Theory of Mechanisms and Machines,* Montreal, p. I.

ative motion between input and output links: rolling contact, sliding contact, linkages, wrapping connectors (belts, chains), and tackle (rope or chain hoists). Franz Reuleaux (1829-1905), published *Theoretische Kinematik* in 1875. Many of his ideas are still current and useful. Alexander Kennedy (1847-1928) translated Reuleaux into English in 1876. This text became the foundation of modem kinematics and is still in print! (See bibliography at end of chapter.) He provided us with the concept of a kinematic pair (joint), whose shape and interaction define the type of motion transmitted between elements in the mechanism. Reuleaux defined six basic mechanical components: the link, the wheel, the cam, the screw, the ratchet, and the belt. He also defined "higher" and "lower" pairs, higher having line or point contact (as in a roller or ball bearing) and lower having surface contact (as in pin joints). Reuleaux is generally considered the father of modem kinematics and is responsible for the symbolic notation of skeletal, generic linkages used in all modem kinematics texts.

In this century, prior to World War II, most theoretical work in kinematics was done in Europe, especially in Germany. Few research results were available in English. In the United States, kinematics was largely ignored until the 1940s, when A. E. R. DeJonge wrote "What Is Wrong with 'Kinematics' and 'Mechanisms'?,"[2] which called upon the U.S. mechanical engineering education establishment to pay attention to the European accomplishments in this field. Since then, much new work has been done, especially in kinematic synthesis, by American and European engineers and researchers such as J. Denavit, A. Erdman, F. Freudenstein, A. S. Hall, R. Hartenberg, R. Kaufman, B. Roth, G. Sandor, andA. Soni, (all of the U.S.) and K. Hain (of Germany). Since the fall of the "iron curtain" much original work done by Soviet Russian kinematicians has become available in the United States, such as that by Artobolevsky.[3] Many U.S. researchers have applied the computer to solve previously intractable problems, both of analysis and synthesis, making practical use of many of the theories of their predecessors.[4] This text will make much use of the availability of computers to allow more efficient analysis and synthesis of solutions to machine design problems. Several computer programs are included with this book for your use.

1.4 APPLICATIONS OF KINEMATICS

One of the first tasks in solving any machine design problem is to determine the kinematic configuration(s) needed to provide the desired motions. Force and stress analyses typically cannot be done until the kinematic issues have been resolved. This text addresses the design of kinematic devices such as linkages, cams, and gears. Each of these terms will be fully defined in succeeding chapters, but it may be useful to show some examples of kinematic applications in this introductory chapter. You probably have used many of these systems without giving any thought to their kinematics.

Virtually any machine or device that moves contains one or more kinematic elements such as linkages, cams, gears, belts, chains. Your bicycle is a simple example of a kinematic system that contains a chain drive to provide torque multiplication and simple cable-operated linkages for braking. An automobile contains many more examples of kinematic devices. Its steering system, wheel suspensions, and piston-engine all contain linkages; the engine's valves are opened by cams; and the transmission is full of gears. Even the windshield wipers are linkage-driven. Figure l-la shows a spatial linkage used to control the rear wheel movement of a modem automobile over bumps.

(a) Spatial linkage rear suspension
Courtesy of Daimler Benz Co.

(b) Utility tractor with backhoe
Courtesy of John Deere Co.

(c) Linkage-driven exercise mechanism
Courtesy of ICON Health & Fitness, Inc.

FIGURE 1-1
Examples of kinematic devices in general use

Construction equipment such as tractors, cranes, and backhoes all use linkages extensively in their design. Figure 1-1b shows a small backhoe that is a linkage driven by hydraulic cylinders. Another application using linkages is thatof exercise equipment as shown in Figure I-Ie. The examples in Figure 1-1 are all of consumer goods which you may encounter in your daily travels. Many other kinematic examples occur in the realm of producer goods-machines used to make the many consumer products that we use. You are less likely to encounter these outside of a factory environment. Once you become familiar with the terms and principles of kinematics, you will no longer be able to look at any machine or product without seeing its kinematic aspects.

1.5 THE DESIGN PROCESS

Design, Invention, Creativity

These are all familiar terms but may mean different things to different people. These terms can encompass a wide range of activities from styling the newest look in clothing, to creating impressive architecture, to engineering a machine for the manufacture of facial tissues. **Engineering design,** which we are concerned with here, embodies all three of these activities as well as many others. The word **design** is derived from the Latin **designare,** which means *"to designate, or mark out."* Webster's gives several definitions, the most applicable being *"to outline, plot, or plan, as action or work ... to conceive, invent- contrive."* **Engineering design** has been defined as "... *the process ofapplying the various techniques and scientific principles for the purpose of defining a device, a process or a system in sufficient detail to permit its realization ... Design may be simple or enormously complex, easy or difficult, mathematical or nonmathematical; it may involve a trivial problem or one of great importance."* **Design** is a universal constituent of engineering practice. But the complexity of engineering subjects usually re-

TABLE 1-1
A Design Process

1	Identification of Need
2	Background Research
3	Goal Statement
4	Performance Specifications
5	Ideation and Invention
6	Analysis
7	Selection
8	Detailed Design
9	Prototyping and Testing
10	Production

Blank paper syndrome

quires that the student be served with a collection of structured, set-piece problems designed to elucidate a particular concept or concepts related to the particular topic. These textbook problems typically take the form of *"given A, B,* C, *and D, find E."* Unfortunately, real-life engineering problems are almost never so structured. Real design problems more often take the form of *"What we need is a framus to stuff this widget into that hole within the time allocated to the transfer of this other gizmo."* The new engineering graduate will search in vain among his or her textbooks for much guidance to solve such a problem. This unstructured problem statement usually leads to what is commonly called "blank paper syndrome." Engineers often find themselves staring at a blank sheet of paper pondering how to begin solving such an ill-defined problem.

Much of engineering education deals with topics of analysis, which means *to decompose, to take apart, to resolve into its constituent parts.* This is quite necessary. The engineer must know how to analyze systems of various types, mechanical, electrical, thermal, or fluid. Analysis requires a thorough understanding of both the appropriate mathematical techniques and the fundamental physics of the system's function. But, before any system can be analyzed, it must exist, and a blank sheet of paper provides little substance for analysis. Thus the first step in any engineering design exercise is that of synthesis, which means *putting together.*

The design engineer, in practice, regardless of discipline, continuously faces the challenge of *structuring the unstructured problem.* Inevitably, the problem as posed to the engineer is ill-defined and incomplete. Before any attempt can be made to *analyze the situation* he or she must first carefully define the problem, using an engineering approach, to ensure that any proposed solution will solve the right problem. Many examples exist of excellent engineering solutions which were ultimately rejected because they solved the wrong problem, i.e., a different one than the client really had.

Much research has been devoted to the definition of various "design processes" intended to provide means to structure the unstructured problem and lead to a viable solution. Some of these processes present dozens of steps, others only a few. The one presented in Table 1-1 contains 10 steps and has, in the author's experience, proven successful in over 30 years of practice in engineering design.

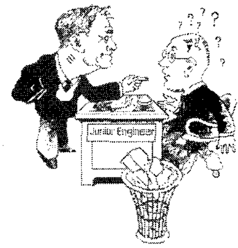
ITERATION Before discussing each of these steps in detail it is necessary to point out that this is not a process in which one proceeds from step one through ten in a linear fashion. Rather it is, by its nature, an iterative process in which progress is made haltingly, two steps forward and one step back. It is inherently *circular.* To iterate means *to repeat, to return to a previous state.* If, for example, your apparently great idea, upon analysis, turns out to violate the second law of thermodynamics, you can return to the ideation step and get a better idea! Or, if necessary, you can return to an earlier step in the process, perhaps the background research, and learn more about the problem. With the understanding that the actual execution of the process involves iteration, for simplicity, we will now discuss each step in the order listed in Table 1-1.

Identification of Need

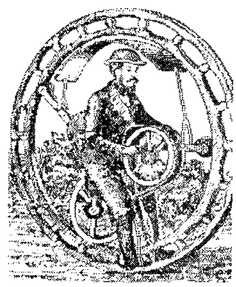
This first step is often done for you by someone, boss or client, saying "What we need is ... " Typically this statement will be brief and lacking in detail. It will fall far short of providing you with a structured problem statement. For example, the problem statement might be "We need a better lawn mower."

Background Research

This is the most important phase in the process, and is unfortunately often the most neglected. The term research, used in this context, should *not* conjure up visions of white-coated scientists mixing concoctions in test tubes. Rather this is research of a more mundane sort, gathering background information on the relevant physics, chemistry, or other aspects of the problem. Also it is desirable to find out if this, or a similar problem, has been solved before. There is no point in reinventing the wheel. If you are lucky enough to find a ready-made solution on the market, it will no doubt be more economical to purchase it than to build your own. Most likely this will not be the case, but you may learn a great deal about the problem to be solved by investigating the existing "art" associated with similar technologies and products. The patent literature and technical publications in the subject area are obvious sources of information and are accessible via the worldwide web. Clearly, if you find that the solution exists and is covered by a patent still in force, you have only a few ethical choices: buy the patentee's existing solution, design something which does not conflict with the patent, or drop the project. It is very important that sufficient energy and time be expended on this research and preparation phase of the process in order to avoid the embarrassment of concocting a great solution to the wrong problem. Most inexperienced (and some experienced) engineers give too little attention to this phase and jump too quickly into the ideation and invention stage of the process. *This must be avoided!* You must discipline yourself to *not* try to solve the problem before thoroughly preparing yourself to do so.



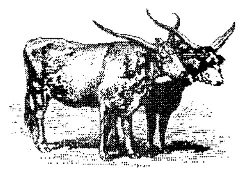
Identifying the need



Reinventing the whee

Goal Statement

Once the background of the problem area as originally stated is fully understood, you will be ready to recast that problem into a more coherent goal statement. This new problem statement should have three characteristics. It should be concise, be general, and be uncolored by any terms which predict a solution. It should be couched in terms of functional visualization, *meaning to visualize its function,* rather than any particular embodiment. For example, if the original statement of need was *"Design a Better Lawn Mower,"* after research into the myriad of ways to cut grass that have been devised over the ages, the wise designer might restate the goal as "Design a Means to Shorten Grass." The original problem statement has a built-in trap in the form of the *colored* words "lawn mower." For most people, this phrase will conjure up a vision of something with whirring blades and a noisy engine. For the ideation phase to be most successful, it is necessary to avoid such images and to state the problem generally, clearly, and concisely. As an exercise, list 10 ways to shorten grass. Most of them would not occur to you had you been asked for 10 better lawn mower designs. You should use functional visualization to avoid unnecessarily limiting your creativity!

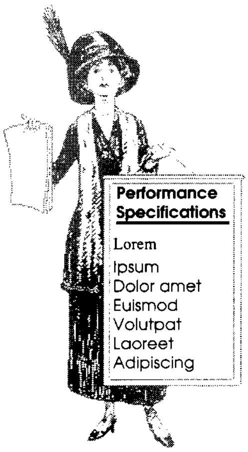


Grass shorteners

Performance Specifications'

When the background is understood, and the goal clearly stated, you are ready to formulate a set of performance specifications. These should not be design specifications. The difference is that performance specifications define what *the system must do,* while design specifications define how *it must do it.* At this stage of the design process it is unwise to attempt to specify *how* the goal is to be accomplished. That is left for the ideation phase. The purpose of the performance specifications is to carefully define and

* Orson Welles, famous author and filmmaker, onc said, "*The enemy of art is the absence of limitations*" We can paraphrase that as *The enemy of design is th absence of specifications.*



constrain the problem so that it both *can be solved* and *can be shown to have been solved* after the fact. A sample set of performance specifications for our "grass shortener" is shown in Table 1-2.

TABLE 1-2
Performance Specifications

1	Device to have self-contained power supply.
2	Device to be corrosion resistant.
3	Device to cost less than $100.00.
4	Device to emit < 80 dB sound intensity at 50 feet.
5	Device to shorten 1/4 acre of grass per hour.
6	etc . . . etc.

Note that these specifications constrain the design without overly restricting the engineer's design freedom. It would be inappropriate to require a gasoline engine for specification 1, since other possibilities exist which will provide the desired mobility. Likewise, to demand stainless steel for all components in specification 2 would be unwise, since corrosion resistance can be obtained by other, less-expensive means. In short, the performance specifications serve to define the problem in as complete and as general a manner as possible, and they serve as a contractual definition of what is to be accomplished. The finished design can be tested for compliance with the specifications.

Ideation and Invention

This step is full of both fun and frustration. This phase is potentially the most satisfying to most designers, but it is also the most difficult. A great deal of research has been done to explore the phenomenon of "creativity." It is, most agree, a common human trait. It is certainly exhibited to a very high degree by all young children. The rate and degree of development that occurs in the human from birth through the first few years of life certainly requires some innate creativity. Some have claimed that our methods of Western education tend to stifle children's natural creativity by encouraging conformity and restricting individuality. From "coloring within the lines" in kindergarten to imitating the textbook's writing patterns in later grades, individuality is suppressed in favor of a socializing conformity. This is perhaps necessary to avoid anarchy but probably does have the effect of reducing the individual's ability to think creatively. Some claim that creativity can be taught, some that it is only inherited. No hard evidence exists for either theory. It is probably true that one's lost or suppressed creativity can be rekindled. Other studies suggest that most everyone underutilizes his or her potential creative abilities. You can enhance your creativity through various techniques.

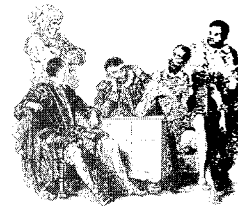
CREATIVE PROCESS Many techniques have been developed to enhance or inspire creative problem solving. In fact, just as design processes have been defined, so has the *creative process* shown in Table 1-3. This creative process can be thought of as a subset of the design process and to exist within it. The ideation and invention step can thus be broken down into these four substeps.

TABLE 1-3
The Creative Process

5a	Idea Generation
5b	Frustration
5c	Incubation
5d	Eureka!

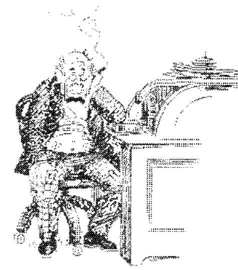
IDEA GENERATION is the most difficult of these steps. Even very creative people have difficulty in inventing "on demand." Many techniques have been suggested to improve the yield of ideas. The most important technique is that of *deferred judgment,* which means that your criticality should be temporarily suspended. Do not try to judge the quality of your ideas at this stage. That will be taken care of later, in the analysis phase. The goal here is to obtain as large a *quantity* of potential designs as possible. Even superficially ridiculous suggestions should be welcomed, as they may trigger new insights and suggest other more realistic and practical solutions.

BRAINSTORMING is a technique for which some claim great success in generating creative solutions. This technique requires a group, preferably 6 to 15 people, and attempts to circumvent the largest barrier to creativity, which is *fear of ridicule.* Most people, when in a group, will not suggest their real thoughts on a subject, for fear of being laughed at. Brainstorming's rules require that no one is allowed to make fun of or criticize anyone's suggestions, no matter how ridiculous. One participant acts as "scribe" and is duty bound to record all suggestions, no matter how apparently silly. When done properly, this technique can be fun and can sometimes result in a "feeding frenzy" of ideas which build upon each other. Large quantities of ideas can be generated in a short time. Judgment on their quality is deferred to a later time.



Brainstorming

When working alone, other techniques are necessary. Analogies and inversion are often useful. Attempt to draw analogies between the problem at hand and other physical contexts. If it is a mechanical problem, convert it by analogy to a fluid or electrical one. Inversion turns the problem inside out. For example, consider what you want moved to be stationary and vice versa. Insights often follow. Another useful aid to creativity is the use of synonyms. Define the action verb in the problem statement, and then list as many synonyms for that verb as possible. For example:



Frustration

Problem statement: Move this object from point A to point B.

The action verb is *"move." Some synonyms are push, pull, slip, slide, shove, throw, eject. jump, spill.*

By whatever means, the aim in this ideation step is to generate a large number of ideas without particular regard to quality. But, at some point, your "mental well" will go dry. You will have then reached the step in the creative process called frustration. It is time to leave the problem and do something else for a time. While your conscious mind is occupied with other concerns, your subconscious mind will still be hard at work on the problem. This is the step called incubation. Suddenly, at a quite unexpected time and place, an idea will pop into your consciousness, and it will seem to be the obvious and "right" solution to the problem ... Eureka! Most likely, later analysis will discover some flaw in this solution. If so, back up and iterate! More ideation, perhaps more research, and possibly even a redefinition of the problem may be necessary.

Eureka!

In "Unlocking Human Creativity"[S] Wallen describes three requirements for creative insight:

- *Fascination with a problem.*
- *Saturation with the facts, technical ideas, data, and the background of the problem.*
- *A period of reorganization.*

The first of these provides the motivation to solve the problem. The second is the background research step described above. The period of reorganization refers to the frustration phase when your subconscious works on the problem. Wallen[S] reports that testimony from creative people tells us that in this period of reorganization they have no conscious concern with the particular problem and that the moment of insight frequently appears in the midst of relaxation or sleep. So to enhance your creativity, saturate yourself in the problem and related background material. Then relax and let your subconscious do the hard work!

Analysis

Once you are at this stage, you have structured the problem, at least temporarily, and can now apply more sophisticated analysis techniques to examine the performance of the design in the **analysis phase** of the design process. (Some of these analysis methods will be discussed in detail in the following chapters.) Further iteration will be required as problems are discovered from the analysis. Repetition of as many earlier steps in the design process as necessary must be done to ensure the success of the design.

Selection

When the technical analysis indicates that you have some potentially viable designs, the best one available must be **selected** for **detailed design, prototyping,** and **testing.** The selection process usually involves a comparative analysis of the available design solutions. A **decision matrix** sometimes helps to identify the best solution by forcing you to consider a variety of factors in a systematic way. A decision matrix for our better grass shortener is shown in Figure 1-2. Each design occupies a row in the matrix. The columns are assigned categories in which the designs are to be judged, such as cost, ease of use, efficiency, performance, reliability, and any others you deem appropriate to the particular problem. Each category is then assigned a **weighting factor,** which measures its relative importance. For example, reliability may be a more important criterion to the user than cost, or vice versa. You as the design engineer have to exercise your judgment as to the selection and weighting of these categories. The body of the matrix is then filled with numbers which rank each design on a convenient scale, such as 1 to 10, in each of the categories. Note that this is ultimately a *subjective ranking* on your part. You must examine the designs and decide on a score for each. The scores are then multiplied by the weighting factors (which are usually chosen so as to sum to a convenient number such as 1) and the products summed for each design. The weighted scores then give a ranking of designs. Be cautious in applying these results. Remember the source and subjectivity of your scores and the weighting factors! There is a temptation to put more faith in these results than is justified. After all, they look impressive! They can even be taken out to several decimal places! (But they shouldn't be.) The real value of a decision

	Cost	*Safety*	*Performance*	*Reliability*	*RANK*
Weighting Factor	.35	.30	.15	.20	1.0
Design 1	3 / 1.05	6 / 1.80	4 / .60	9 / 1.80	5.3
Design 2	4 / 1.40	2 / .60	7 / 1.05	2 / .40	3.5
Design 3	1 / .35	9 / 2.70	4 / .60	5 / 1.00	4.7
Design 4	9 / 3.15	1 / .30	6 / .90	7 / 1.40	5.8
Design 5	7 / 2.45	4 / 1.20	2 / .30	6 / 1.20	5.2

FIGURE 1-2

A decision matrix

matrix is that it breaks the problem into more tractable pieces and forces you to think about the relative value of each design in many categories. You can then make a more informed decision as to the "best" design.

Detailed Design

This step usually includes the creation of a complete set of assembly and detail drawings or computer-aided design (CAD) part files, for *each and every part* used in the design. Each detail drawing must specify all the dimensions and the material specifications necessary to make that part. From these drawings (or CAD files) a prototype test model (or models) must be constructed for physical testing. Most likely the tests will discover more flaws, requiring further iteration.

Prototyping and Testing

MODELS Ultimately, one cannot be sure of the correctness or viability of any design until it is built and tested. This usually involves the construction of a prototype physical model. A mathematical model, while very useful, can never be as complete and accurate a representation of the actual physical system as a physical model, due to the need to make simplifying assumptions. Prototypes are often very expensive but may be the most economical way to prove a design, short of building the actual, full-scale device. Prototypes can take many forms, from working scale models to full-size, but simplified, representations of the concept. Scale models introduce their own complications in regard to proper scaling of the physical parameters. For example, volume of material varies as the cube of linear dimensions, but surface area varies as the square. Heat transfer to the environment may be proportional to surface area, while heat generation may be proportional to volume. So linear scaling of a system, either up or down, may lead to behavior different from that of the full-scale system. One must exercise caution in scaling physical models. You will find as you begin to design linkage mechanisms that a simple cardboard model of your chosen link lengths, coupled together with thumbtacks for pivots, will tell you a great deal about the quality and character of the mechanism's motions. You should get into the habit of making such simple articulated models for all your linkage designs.

TESTING of the model or prototype may range from simply actuating it and observing its function to attaching extensive instrumentation to accurately measure displacements, velocities, accelerations, forces, temperatures, and other parameters. Tests may need to be done under controlled environmental conditions such as high or low temperature or humidity. The microcomputer has made it possible to measure many phenomena more accurately and inexpensively than could be done before.

Production

Finally, with enough time, money, and perseverance, the design will be ready for production. This might consist of the manufacture of a single final version of the design, but more likely will mean making thousands or even millions of your widget. The danger, expense, and embarrassment of finding flaws in your design after making large quantities of defective devices should inspire you to use the greatest care in the earlier steps of the design process to ensure that it is properly engineered.

The design process is widely used in engineering. Engineering is usually defined in terms of what an engineer does, but engineering can also be defined in terms of *how* the engineer does what he or she does. Engineering is *as much a method, an approach, a process, a state of mind for problem solving, as it is an activity.* The engineering approach is that of thoroughness, attention to detail, and consideration of all the possibilities. While it may seem a contradiction in terms to emphasize "attention to detail" while extolling the virtues of open-minded, freewheeling, creative thinking, it is not. The two activities are not only compatible, they are symbiotic. It ultimately does no good to have creative, original ideas if you do not, or cannot, carry out the execution of those ideas and "reduce them to practice." To do this you must discipline yourself to suffer the nitty-gritty, nettlesome, tiresome details which are so necessary to the completion of any one phase of the creative design process. For example, to do a creditable job in the design of anything, you must *completely* define the problem. If you leave out some detail of the problem definition, you will end up solving the wrong problem. Likewise, you must *thoroughly* research the background information relevant to the problem. You must *exhaustively* pursue conceptual potential solutions to your problem. You must then *extensively* analyze these concepts for validity. And, finally, you must *detail* your chosen design down to the last nut and bolt to be confident it will work. If you wish to be a good designer and engineer, you must discipline yourself to do things thoroughly and in a logical, orderly manner, even while thinking great creative thoughts and iterating to a solution. Both attributes, creativity and attention to detail, are necessary for success in engineering design.

1.6 OTHER APPROACHES TO DESIGN

In recent years, an increased effort has been directed toward a better understanding of design methodology and the design process. Design methodology is the study of the process of designing. One goal of this research is to define the design process in sufficient detail to allow it to be encoded in a form amenable to execution in a computer, using "artificial intelligence" (AI).

Dixon[6] defines a design as a *state of information* which may be in any of several forms:

> ... words, graphics, electronic data, and/or others. It may be partial or complete. It ranges from a small amount of highly abstract information early in the design process to a very large amount of detailed information later in the process sufficient to perform manufacturing. It may include, but is not limited to, information about size and shape, function, materials, marketing, simulated performance, manufacturing processes, tolerances, and more. Indeed, any and all information relevant to the physical or economic life of a designed object is part of its design.

He goes on to describe several generalized states of information such as the *requirements* state which is analogous to our performance specifications. Information about the physical concept is referred to as the *conceptual* state of information and is analogous to our ideation phase. His *feature configuration* and *parametric* states of information are similar in concept to our detailed design phase. Dixon then defines a design process as:

> The series of activities by which the information about the designed object is changed from one information state to another.

Axiomatic Design

N. P. Suh[7] suggests an *axiomatic approach* to design in which there are four domains: customer domain, functional domain, physical domain, and the process domain. These represent a range from "what" to "how," i.e., from a state of defining what the customer wants through determining the functions required and the needed physical embodiment, to how a process will achieve the desired end. He defines two axioms that need to be satisfied to accomplish this:

1 Maintain the independence of the functional requirements.

2 Minimize the information content.

The first of these refers to the need to create a complete and nondependent set of performance specifications. The second indicates that the best design solution will have the lowest information content (i.e., the least complexity). Others have earlier referred to this second idea as *KISS,* which stands, somewhat crudely, for *"keep it simple, stupid."*

The implementation of both Dixon's and Suh's approaches to the design process is somewhat complicated. The interested reader is referred to the literature cited in the bibliography to this chapter for more complete information.

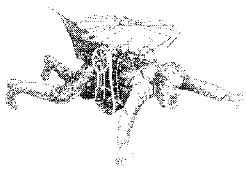
1.7 MULTIPLE SOLUTIONS

Note that by the nature of the design process, there is not anyone correct answer or solution to any design problem. Unlike the structured "engineering textbook" problems, which most students are used to, there is no right answer "in the back of the book" for any real design problem. * There are as many potential solutions as there are designers willing to attempt them. Some solutions will be better than others, but many will work. Some will not! There is no "one right answer" in design engineering, which is what makes it interesting. The only way to determine the relative merits of various potential design solutions is by thorough analysis, which usually will include physical testing of constructed prototypes. Because this is a very expensive process, it is desirable to do as much analysis on paper, or in the computer, as possible before actually building the device. Where feasible, mathematical models of the design, or parts of the design, should be created. These may take many forms, depending on the type of physical system involved. In the design of mechanisms and machines it is usually possible to write the equations for the rigid-body dynamics of the system, and solve them in "closed form" with (or without) a computer. Accounting for the elastic deformations of the members of the mechanism or machine usually requires more complicated approaches using finite difference techniques or the finite element method (FEM).

1.8 HUMAN FACTORS ENGINEERING

With few exceptions, all machines are designed to be used by humans. Even robots must be programmed by a human. Human factors engineering is the study of the human-machine interaction and is defined as *an applied science that coordinates the design of devices, systems, and physical working conditions with the capacities and requirements of the worker.* The machine designer must be aware of this subject and design devices to "fit the man" rather than expect the man to adapt to fit the machine. The term ergonom-

* A student once commented that *"Life is an odd-numbered problem."* This (slow) author had to ask for an explanation, which was: *"The answer is not in the back of the book."*



Make the machine fit the man

ics is synonymous with *human factors engineering.* We often see reference to the good or bad ergonomics of an automobile interior or a household appliance. A machine designed with poor ergonomics will be uncomfortable and tiring to use and may even be dangerous. (Have you programmed your VCR lately, or set its clock?)

There is a wealth of human factors data available in the literature. Some references are noted in the bibliography. The type of information which might be needed for a machine design problem ranges from dimensions of the human body and their distribution among the population by age and gender, to the ability of the human body to withstand accelerations in various directions, to typical strengths and force generating ability in various positions. Obviously, if you are designing a device that will be controlled by a human (a grass shortener, perhaps), you need to know how much force the user can exert with hands held in various positions, what the user's reach is, and how much noise the ears can stand without damage. If your device will carry the user on it, you need data on the limits of acceleration which the body can tolerate. Data on all these topics exist. Much of it was developed by the government which regularly tests the ability of military personnel to withstand extreme environmental conditions. Part of the background research of any machine design problem should include some investigation of human factors.

1.9 THE ENGINEERING REPORT

Communication of your ideas and results is a very important aspect of engineering. Many engineering students picture themselves in professional practice spending most of their time doing calculations of a nature similar to those they have done as students. Fortunately, this is seldom the case, as it would be very boring. Actually, engineers spend the largest percentage of their time communicating with others, either orally or in writing. Engineers write proposals and technical reports, give presentations, and interact with support personnel and managers. When your design is done, it is usually necessary to present the results to your client, peers, or employer. The usual form of presentation is a formal engineering report. Thus, it is very important for the engineering student to develop his or her communication skills. *You may be the cleverest person in the world, but no one will know that if you cannot communicate your ideas clearly and concisely.* In fact, if you cannot explain what you have done, you probably don't understand it yourself. To give you some experience in this important skill, the design project assignments in later chapters are intended to be written up in formal engineering reports. Information on the writing of engineering reports can be found in the suggested readings in the bibliography at the end of this chapter.

1.10 UNITS

There are several systems of units used in engineering. The most common in the United States are the U.S. **foot-pound-second** (fps) system, the U.S. **inch-pound-second** (ips) system, and the System **International** (SI). All systems are created from the choice of three of the quantities in the general expression of Newton's second law

$$F = \frac{ml}{t^2} \tag{1.1a}$$

where F is force, m is mass, l is length, and t is time. The units for any three of these variables can be chosen and the other is then derived in terms of the chosen units. The three chosen units are called *base units*, and the remaining one is then a *derived unit.*

Most of the confusion that surrounds the conversion of computations between either one of the U.S. systems and the SI system is due to the fact that the SI system uses a different set of base units than the U.S. systems. Both U.S. systems choose ***force***, *length,* **and** ***time*** as the base units. Mass is then a derived unit in the U.S. systems, and they are referred to as *gravitational systems* because the value of mass is dependent on the local gravitational constant. The SI system chooses ***mass***, *length,* and *time* as the base units and force is the derived unit. SI is then referred to as an *absolute system* since the mass is a base unit whose value is not dependent on local gravity.

The **U.S. foot-pound-second (fps)** system requires that all lengths be measured in **feet** (ft), forces in pounds (lb), and time in seconds (sec). Mass is then derived from Newton's law as

$$m = \frac{Ft^2}{l} \tag{1.1b}$$

and the units are:

Pounds seconds squared per **foot** (lb-sec^2/ft) = **slugs**

The **U.S. inch-pound-second (ips)** system requires that all lengths be measured in **inches** (in), forces in pounds (lb), and time in seconds (sec). Mass is still derived from Newton's law, equation 1.1b, but the units are now:

Pounds seconds squared per **inch** (lb-sec^2/in) = **blobs**

This mass unit is not slugs! It is worth twelve slugs or one blob.*

Weight is defined as the force exerted on an object by gravity. Probably the most common units error that students make is to mix up these two unit systems (**fps** and **ips**) when converting weight units (which are pounds force) to mass units. Note that the gravitational acceleration constant (g) on earth at sea level is approximately 32.2 **feet** per second squared which is equivalent to 386 **inches** per second squared. The relationship between mass and weight is:

Mass = weight / gravitational acceleration

$$m = \frac{W}{g} \tag{1.2}$$

It should be obvious that, if you measure all your lengths in **inches** and then use g = 32.2 **feet**/sec^2 to compute mass, you will have an error of a *factor of 12* in your results. This is a serious error, large enough to crash the airplane you designed. Even worse off is the student who neglects to convert weight to mass *at all* in his calculations. He will have an error of either 32.2 or 386 in his results. This is enough to sink the ship!

To even further add to the student's confusion about units is the common use of the unit of **pounds mass** (lb$_m$). This unit is often used in fluid dynamics and thermodynamics and comes about through the use of a slightly different form of Newton's equation:

* It is unfortunate that the mass unit in the ips system has never officially been given a name such as the term *slug* used for mass in the fps system. The author boldly suggests (with tongue only slightly in cheek) that this unit of mass in the **ips** system be called a *blob* (bl) to distinguish it more clearly from the *slug* (sl), and to help the student avoid some of the common units errors listed above.

Twelve slugs = one **blob.**

Blob does not sound any sillier than slug, is easy to remember, implies mass, and has a convenient abbreviation (bl) which is an anagram for the abbreviation for pound (Ib). Besides, if you have ever seen a garden slug, you know it looks just like a *"little blob."*

$$F = \frac{ma}{g_c} \tag{1.3}$$

where m = mass in *Ibm'* a = acceleration and g_c = the gravitational constant.

The value of the mass of an object measured in pounds mass *(lbm)* is *numerically equal* to its weight in pounds force (Ib/). However the student *must remember to divide* the value of m in *Ibm* by g_c when substituting into this form of Newton's equation. Thus the *Ibm* will be divided either by 32.2 or by 386 when calculating the dynamic force. The result will be the same as when the mass is expressed in either slugs or blobs in the $F = ma$ form of the equation. Remember that in round numbers at sea level on earth:

I Ibm = llbf I slug = 32.2 Ibf I blob = 386 Ibf

The SI system requires that lengths be measured in meters (m), mass in kilograms (kg), and time in seconds (sec). This is sometimes also referred to as the mks system. Force is derived from Newton's law, equation 1.1b and the units are:

kilogram-meters per second^2 (kg-m/sec^2) = newtons

Thus in the SI system there are distinct names for mass and force which helps alleviate confusion. When converting between SI and U.S. systems, be alert to the fact that mass converts from kilograms (kg) to either slugs (sl) or blobs (bl), and force converts from newtons (N) to pounds (Ib). The gravitational constant *(g)* in the SI system is approximately 9.81 m/sec^2.

The principal system of units used in this textbook will be the U.S. ips system. Most machine design in the United States is still done in this system. Table 1-4 shows some of the variables used in this text and their units. The inside front cover contains a table of conversion factors between the U.S, and SI systems.

The student is cautioned to always check the units in any equation written for a problem solution, whether in school or in professional practice after graduation. If properly written, an equation should cancel all units across the equal sign. If it does not, then you can be *absolutely sure it is incorrect.* Unfortunately, a unit balance in an equation does not guarantee that it is correct, as many other errors are possible. Always double-check your results. You might save a life.

1.11 WHAT'S TO COME

In this text we will explore the topic of machine design in respect to the synthesis of mechanisms in order to accomplish desired motions or tasks, and also the analysis of these mechanisms in order to determine their rigid-body dynamic behavior. On the premise that we cannot analyze anything until it has been synthesized into existence, we will first explore the topic of synthesis of mechanisms. Then we will investigate the analysis of those and other mechanisms for their kinematic behavior. Finally, in Part II we will deal with the dynamic analysis of the forces and torques generated by these moving machines. These topics cover the essence of the early stages of a design project. Once the kinematics and kinetics of a design have been determined, most of the conceptual design will have been accomplished. What then remains is detailed design-sizing the parts against failure. The topic of *detailed design* is discussed in other texts such as reference [8].

Table 1-4 Variables and Units
Base Units in Boldface – Abbreviations in ( )

Variable	Symbol	ips unit	fps unit	SI unit
Force	F	**pounds (lb)**	**pounds (lb)**	newtons (N)
Length	l	**inches (in)**	**feet (ft)**	**meters (m)**
Time	t	**seconds (sec)**	**seconds (sec)**	**seconds (sec)**
Mass	m	lb–sec^2 / in (bl)	lb–sec^2 / ft (sl)	**kilograms (kg)**
Weight	W	pounds (lb)	pounds (lb)	newtons (N)
Velocity	v	in / sec	ft / sec	m / sec
Acceleration	a	in / sec^2	ft / sec^2	m / sec^2
Jerk	j	in / sec^3	ft / sec^3	m / sec^3
Angle	θ	degrees (deg)	degrees (deg)	degrees (deg)
Angle	θ	radians (rad)	radians (rad)	radians (rad)
Angular velocity	ω	rad / sec	rad / sec	rad / sec
Angular acceleration	α	rad / sec^2	rad / sec^2	rad / sec^2
Angular jerk	φ	rad / sec^3	rad / sec^3	rad / sec^3
Torque	T	lb–in	lb–ft	N–m
Mass moment of inertia	I	lb–in–sec^2	lb–ft–sec^2	N–m–sec^2
Energy	E	in–lb	ft–lb	joules
Power	P	in–lb / sec	ft–lb / sec	watts
Volume	V	in^3	ft^3	m^3
Weight density	γ	lb / in^3	lb / ft^3	N / m^3
Mass density	ρ	bl / in^3	sl / ft^3	kg / m^3

1.12 REFERENCES

1 **Rosenauer, N., and A. H. Willis.** (1967). *Kinematics of Mechanisms.* Dover Publications: New York, p. 275 ff.

2 **de Jonge, A. E. R.** (1942). "What Is Wrong with 'Kinematics' and 'Mechanisms'?" *Mechanical Engineering*, **64**(April), pp. 273-278.

3 **Artobolevsky, I. I.** (1975). *Mechanisms in Modern Engineering Design.* N. Weinstein, translator. Vols. 1 - 5. MIR Publishers: Moscow.

4 **Erdman, A. E.**, ed. (1993). *Modern Kinematics: Developments in the Last Forty Years.* Wiley Series in Design Engineering, John Wiley & Sons: New York.

5 **Wallen, R. W.** (1957). "Unlocking Human Creativity." *Proc. of Fourth Conference on Mechanisms*, Purdue University, pp. 2-8.

6 **Dixon, J. R.** (1995). "Knowledge Based Systems for Design." *Journal of Mechanical Design*, **117b**(2), p. 11.

7 Suh, N. P. (1995). "Axiomatic Design of Mechanical Systems." *Journal of Mechanical Design,* 117b(2), p. 2.

8 Norton, R. L. (1996). *Machine Design: An Integrated Approach.* Prentice-Hall: Upper Saddle River, NJ.

For additional information on the history of kinematics, the following are recommended:

Artobolevsky, I. I. (1976). "Past Present and Future of the Theory of Machines and Mechanisms." *Mechanism and Machine Theory,* 11, pp. 353-361.

Brown, H. T. (1869). *Five Hundred and Seven Mechanical Movements.* Brown, Coombs & Co.: New York, republished by USM Corporation, Beverly, MA., 1970.

de Jonge, A. E. R. (1942). "What Is Wrong with 'Kinematics' and 'Mechanisms'?" *Mechanical Engineering,* 64(April), pp. 273-278.

de Jonge, A. E. R. (\ 943). "A Brief Account of Modem Kinematics." *Transactions of the ASME,* pp. 663-683.

Erdman, A. E., ed. (1993). *Modern Kinematics: Developments in the Last Forty Years.* Wiley Series in Design Engineering, John Wiley & Sons: New York.

Ferguson, E. S. (1962). "Kinematics of Mechanisms from the Time of Watt." *United States National Museum Bulletin,* 228(27), pp. 185-230.

Freudenstein, F. (1959). "Trends in the Kinematics of Mechanisms." *Applied Mechanics Reviews,* 12(9), September, pp. 587-590.

Hartenberg, R. S., and J. Denavit. (1964). *Kinematic Synthesis of Linkages.* McGraw-Hill: New York, pp. 1-27.

Nolle, H. (1974). "Linkage Coupler Curve Synthesis: A Historical Review - II. Developments after 1875." *Mechanism and Machine Theory,* 9, pp. 325 - 348.

Nolle, H. (1974). "Linkage Coupler Curve Synthesis: A Historical Review -I. Developments up to 1875." *Mechanism and Machine Theory,* 9, pp. 147-168.

Nolle, H. (\ 975). "Linkage Coupler Curve Synthesis: A Historical Review - III. Spatial Synthesis and Optimization." *Mechanism and Machine Theory,* 10, pp. 41-55.

Reuleaux, F. (1963). *The Kinematics of Machinery,* A. B. W. Kennedy, translator. Dover Publications: New York, pp. 29-55.

Strandh, S. (1979). *A History of the Machine.* A&W Publishers: New York.

For additional information on creativity and the design process, the following are recommended:

Alger, J. R. M., and C. V. Hays. (1964). *Creative Synthesis in Design.* Prentice-Hall: Upper Saddle River, NJ.

Allen, M. S. (1962). *Morphological Creativity.* Prentice-Hall: Upper Saddle River, NJ.

Altschuller, G. (1984). *Creativity as an Exact Science.* Gordon and Breach: New York.

Buhl, H. R. (\960). *Creative Engineering Design.* Iowa State University Press: Ames, IA.

Dixon. J. R., and C. Poli. (1995). *Engineering Design and Design for Manufacturing-A Structured Approach.* Field Stone Publishers: Conway, MA.

Fey, V., et aI. (1994). "Application of the Theory ofInventive Problem Solving to Design and Manufacturing Systems." *CIRP Annals,* 43(1), pp. 107-110.

Gordon, W. J. J. (1962). *Synectics.* Harper & Row: New York.

Haefele, W. J. (1962). *Creativity and Innovation.* Van Nostrand Reinhold: New York.

Harrisberger, L. (1982). *Engineersmanship.* Brooks/Cole: Monterey, CA.

Osborn, A. F. (1963). *Applied Imagination.* Scribners: New York.

Pleuthner, W. (1956). "Brainstorming." *Machine Design,* January 12, 1956.

Soh, N. P. (1990). *The Principles of Design.* Oxford University Press: New York.

Taylor, C. W. (1964). *Widening Horizons in Creativity.* John Wiley & Sons: New York.

Von Fange, E. K. (1959). *Professional Creativity.* Prentice-Hall: Upper Saddle River, NJ.

For additional information on Human Factors, the following are recommended:

Bailey, R. W. (1982). *Human Performance Engineering: A Guidefor System Designers.* Prentice-Hall: Upper Saddle River, NJ.

Burgess, W. R. (1986). *Designing for Humans: The Human Factor in Engineering.* Petrocelli Books.

Clark, T. S., and E. N. Corlett. (1984). *The Ergonomics of Workspaces and Machines.* Taylor and Francis.

Huchinson, R. D. (1981). *New Horizons for Human Factors in Design.* McGraw-Hill: New York.

McCormick, D. J. (1964). *Human Factors Engineering.* McGraw-Hill: New York.

Osborne, D. J. (1987). *Ergonomics at Work.* John Wiley & Sons: New York.

Pheasant, S. (1986). *Bodyspace: Anthropometry, Ergonomics & Design.* Taylor and Francis.

Salvendy, G. (1987). *Handbook of Human Factors.* John Wiley & Sons: New York.

Sanders, M. S. (1987). *Human Factors in Engineering and Design.* McGraw-Hill: New York.

Woodson, W. E. (1981). *Human Factors Design Handbook.* McGraw-Hill: New York.

For additional information on writing engineering reports, the following are recommended:

Barrass, R. (1978). *Scientists Must Write.* John Wiley & Sons: New York.

Crouch, W. G., and R. L. Zetler. (1964). *A Guide to Technical Writing.* The Ronald Press: New York.

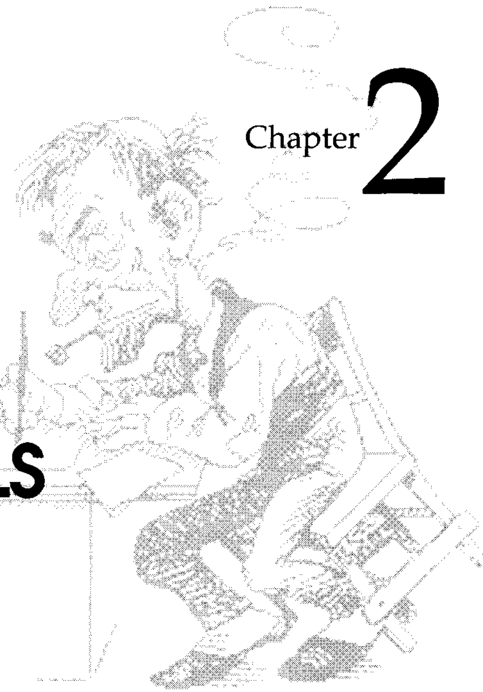
Davis, D. S. (1963). *Elements of Engineering Reports.* Chemical Publishing Co.: New York.

Gray, D. E. (1963). *So You Have to Write a Technical Report.* Information Resources Press: Washington, D.C.

Michaelson, H. B. (1982). *How to Write and Publish Engineering Papers and Reports.* ISI Press: Philadelphia, PA.

Nelson, J. R. (1952). *Writing the Technical Report.* McGraw-Hill: New York.

Chapter 2



KINEMATICS FUNDAMENTALS

Chance favors the prepared mind
PASTEUR

2.0 INTRODUCTION

This chapter will present definitions of a number of terms and concepts fundamental to the synthesis and analysis of mechanisms. It will also present some very simple but powerful analysis tools which are useful in the synthesis of mechanisms.

2.1 DEGREESOF FREEDOM (*DOF)*

Any mechanical system can be classified according to the number of **degrees of freedom** *(DOF)* which it possesses. The system's *DOF* is equal to *the number of independent parameters (measurements) which are needed to uniquely define its position in space at any instant of time.* Note that *DOF* is defined with respect to a selected frame of reference. Figure 2-1 shows a pencil lying on a flat piece of paper with an x, y coordinate system added. If we constrain this pencil to always remain in the plane of the paper, three parameters *(DOF)* are required to completely define the position of the pencil on the paper, two linear coordinates (x, y) to define the position of anyone point on the pencil and one angular coordinate (8) to define the angle of the pencil with respect to the axes. The minimum number of measurements needed to define its position are shown in the figure as x, y, and 8. This system of the pencil in a plane then has **three** *DOF.* Note that the particular parameters chosen to define its position are not unique. Any alternate set of three parameters could be used. There is an infinity of sets of parameters possible, but in this case there must be three parameters per set, **such as two lengths and an angie,** to define the system's position because *a rigid body in plane motion has three DOF.*

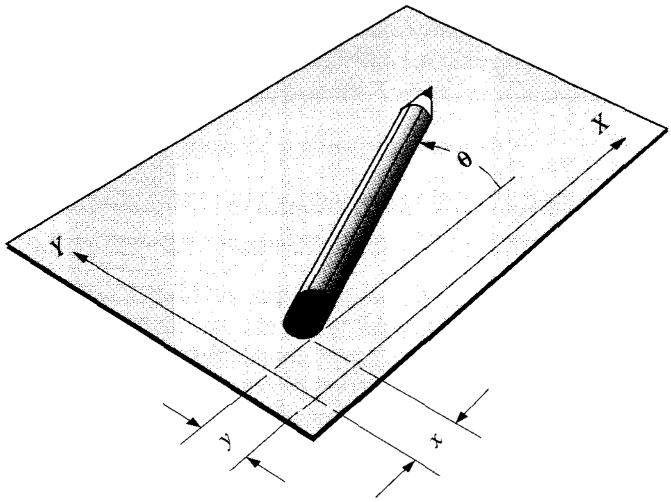


FIGURE 2-1

A rigid body in a plane has three *DOF*

Now allow the pencil to exist in a three-dimensional world. Hold it above your desktop and move it about. You now will need six parameters to define its six *DOF.* One possible set of parameters which could be used are three lengths, *(x, y, z),* plus three angles (a, ◁▷, p). *Any rigid body in three-space has six degrees of freedom.* Try to identify these six *DOF* by moving your pencil or pen with respect to your desktop.

The pencil in these examples represents a rigid body, or link, which for purposes of kinematic analysis we will assume to be incapable of deformation. This is merely a convenient fiction to allow us to more easily define the gross motions of the body. We can later superpose any deformations due to external or inertial loads onto our kinematic motions to obtain a more complete and accurate picture of the body's behavior. But remember, we are typically facing a *blank sheet of paper* at the beginning stage of the design process. We cannot determine deformations of a body until we define its size, shape, material properties, and loadings. Thus, at this stage we will assume, for purposes of initial kinematic synthesis and analysis, that *our kinematic bodies are rigid and massless.*

2.2 TYPESOF MOTION

A rigid body free to move within a reference frame will, in the general case, have complex motion, which is a simultaneous combination of rotation and translation. In three-dimensional space, there may be rotation about any axis (any skew axis or one of the three principal axes) and also simultaneous translation which can be resolved into components along three axes. In a plane, or two-dimensional space, complex motion becomes a combination of simultaneous rotation about one axis (perpendicular to the plane) and also translation resolved into components along two axes in the plane. For simplicity, we will limit our present discussions to the case of planar (2-0) kinematic systems. We will define these terms as follows for our purposes, in planar motion:

Pure rotation
the body possesses one point (center of rotation) which has no motion with respect to the "stationary" frame of reference. All other points on the body describe arcs about that center. A reference line drawn on the body through the center changes only its angular orientation.

Pure translation
all points on the body describe parallel (curvilinear or rectilinear) paths. A reference line drm\"n on the body changes its linear position but does not change its angular orientation.

Complex motion
a simultaneous combination of rotation and translation. Any reference line drawn on the body will change both its linear position and its angular orientation. Points on the body will travel nonparallel paths, and there will be, at every instant, a center of rota· tion, which will continuously change location.

Translation and **rotation** represent independent motions of the body. Each can exist without the other. If we define a 2-D coordinate system as shown in Figure 2-1, the x and y terms represent the translation components of motion, and the e term represents the rotation component.

2.3 LINKS, JOINTS, AND KINEMATIC CHAINS

We will begin our exploration of the kinematics of mechanisms with an investigation of the subject of **linkage design.** Linkages are the basic building blocks of all mechanisms. We will show in later chapters that all common forms of mechanisms (cams, gears, belts, chains) are in fact variations on a common theme of linkages. Linkages are made up of links and joints.

A **link,** as shown in Figure 2-2, is an (assumed) rigid body which possesses at least two **nodes** which are *points for attachment to other links.*

Binary link - *one with two nodes.*

Ternary link - *one with three nodes.*

Quaternary link - *one with four nodes.*

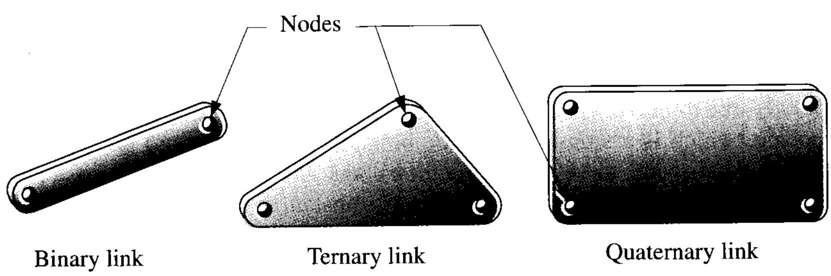


FIGURE 2-2
Links of different order

A **joint** is *a connection between two or more links (at their nodes), which allows some motion, or potential motion, between the connected links.* **Joints** (also called **kinematic pairs)** can be classified in several ways:

1 By the type of contact between the elements, line, point, or surface.

2 By the number of degrees of freedom allowed at the joint.

3 By the type of physical closure of the joint: either **force or form** closed.

4 By the number of links joined (order of the joint).

Reuleaux [1] coined the term **lower pair** to describe joints with surface contact (as with a pin surrounded by a hole) and the term **higher pair** to describe joints with point or line contact. However, if there is any clearance between pin and hole (as there must be for motion), so-called surface contact in the pin joint actually becomes line contact, as the pin contacts only one "side" of the hole. Likewise, at a microscopic level, a block sliding on a flat surface actually has contact only at discrete points, which are the tops of the surfaces' asperities. The main practical advantage of lower pairs over higher pairs is their better ability to trap lubricant between their enveloping surfaces. This is especially true for the rotating pin joint. The lubricant is more easily squeezed out of a higher pair, nonenveloping joint. As a result, the pin joint is preferred for low wear and long life, even over its lower pair cousin, the prismatic or slider joint.

Figure 2-3a shows the six possible lower pairs, their degrees of freedom, and their one-letter symbols. The revolute (R) and the prismatic (P) pairs are the only lower pairs usable in a planar mechanism. The screw (H), cylindric (C), spherical, and flat (F) lower pairs are all combinations of the revolute and/or prismatic pairs and are used in spatial (3-D) mechanisms. The Rand P pairs are the basic building blocks of all other pairs which are combinations of those two as shown in Table 2-1.

A more useful means to classify joints (pairs) is by the number of degrees of freedom that they allow between the two elements joined. Figure 2-3 also shows examples of both one- and two-freedom joints commonly found in planar mechanisms. Figure 2-3b shows two forms of a planar, **one-freedom** joint (or pair), namely, a rotating pin joint (R) and a translating slider joint (P). These are also referred to as **full joints** (i.e., full = 1 *DOF)* and are **lower pairs.** The pin joint allows one rotational *DOF,* and the slider joint allows one translational *DOF* between the joined links. These are both contained within (and each is a limiting case of) another common, one-freedom joint, the screw and nut (Figure 2-3a). Motion of either the nut or the screw with respect to the other results in helical motion. If the helix angle is made zero, the nut rotates without advancing and it becomes the pin joint. If the helix angle is made 90 degrees, the nut will translate along the axis of the screw, and it becomes the slider joint.

Figure 2-3c shows examples of two-freedom joints (h1gherpairs) which simultaneously allow two independent, relative motions, namely translation and rotation, between the joined links. Paradoxically, this **two-freedom joint** is sometimes referred to as a **"half joint,"** with its two freedoms placed in the denominator. The **half joint** is also called a **roll-slide joint** because it allows both rolling and sliding. A spherical, or ball-and-socket joint (Figure 2-3a), is an example of a three-freedom joint, which allows three independent angular motions between the two links joined. This *ball joint* would typically be used in a three-dimensional mechanism, one example being the ball joints in an automotive suspension system.

TABLE 2-1
The Six Lower Pairs

Name (Symbol)	DOF	Contains
Revolute (R)	1	R
Prismatic (P)	1	P
Helical (H)	1	RP
Cylindric (C)	2	RP
Spherical (S)	3	RRR
Planar (F)	3	RPP

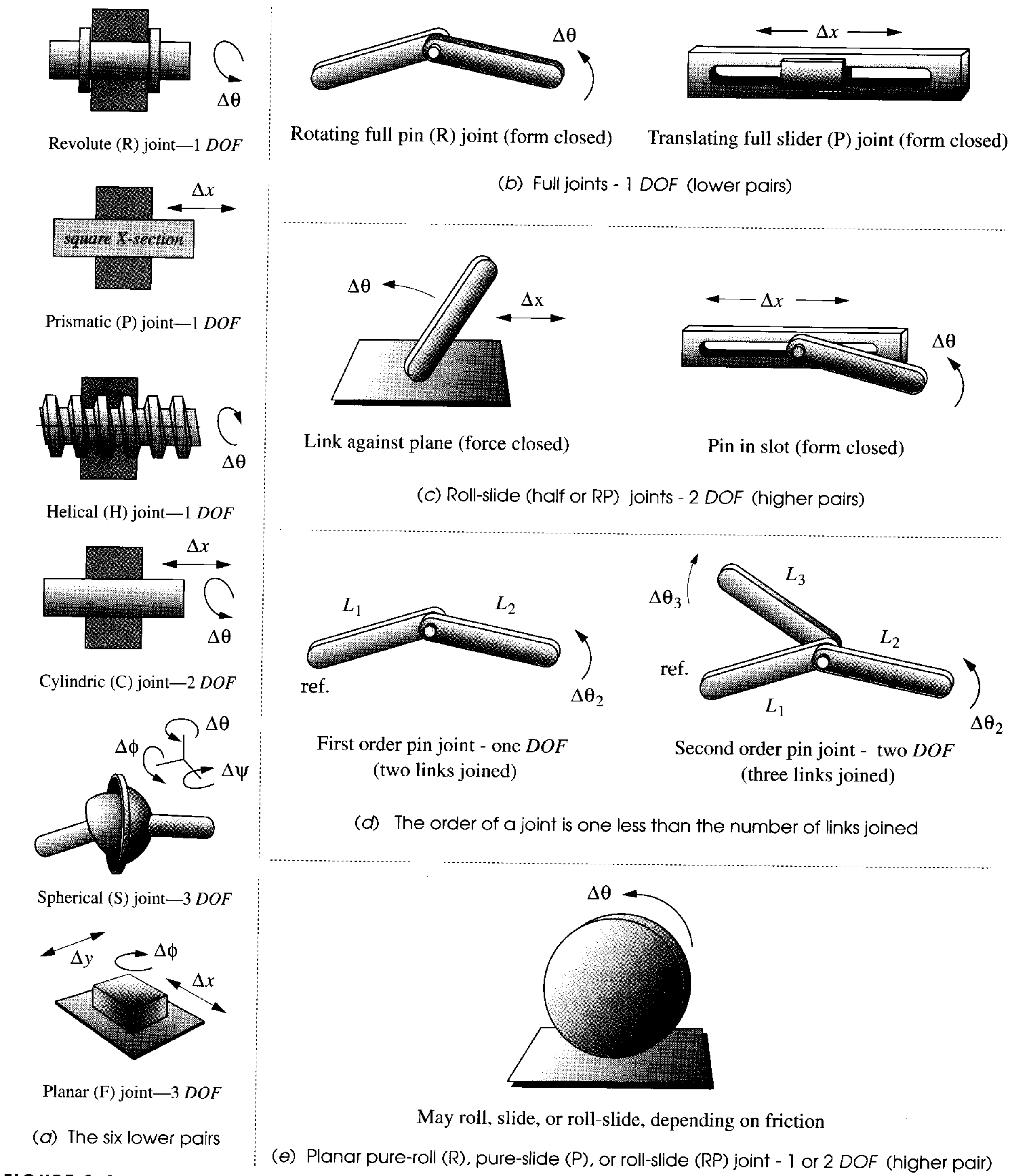


FIGURE 2-3
Joints (pairs) of various types

A joint with more than one freedom may also be a higher pair as shown in Figure 2-3c. Full joints (lower pairs) and half joints (higher pairs) are both used in planar (2-D), ~ in spatial (3-D) mechanisms. Note that if you do not allow the two links in Hgore 2-3c connected by a roll-slide joint to slide, perhaps by providing a high friction coefficient between them, you can "lock out" the translating (At) freedom and make it behave as a full joint. This is then called a pure rolling joint and has rotational freedom (AD) only. A cornmon example of this type of joint is your automobile tire rolling against die road, as shown in Figure 2-3e. In normal use there is pure rolling and no sliding at Ibis joint, unless, of course, you encounter an icy road or become too enthusiastic about accelerating or cornering. If you lock your brakes on ice, this joint converts to a pure sliding one like the slider block in Figure 2-3b. Friction determines the actual number of freedoms at this kind of joint. It can be pure roll, pure slide, or roll-slide.

To visualize the degree of freedom of a joint in a mechanism, it is helpful to "mentally disconnect" the two links which create the joint from the rest of the mechanism. You can then more easily see how many freedoms the two joined links have with respect to one another.

Figure 2-3c also shows examples of both form-closed and force-closed joints. A form-closed joint is kept together or *closed by its geometry.* A pin in a hole or a slider in a two-sided slot are form closed. In contrast, a force-closed joint, such as a pin in a half-bearing or a slider on a surface, *requires some external force to keep it together or closed.* This force could be supplied by gravity, a spring, or any external means. There can be substantial differences in the behavior of a mechanism due to the choice of force or form closure, as we shall see. The choice should be carefully considered. In linkages, form closure is usually preferred, and it is easy to accomplish. But for cam-follower systems, force closure is often preferred. This topic will be explored further in later chapters.

Figure 2-3d shows examples of joints of various orders, where order is defined as *the number of links joined minus one.* It takes two links to make a single joint; thus the simplest joint combination of two links has order one. As additional links are placed on the same joint, the order is increased on a one for one basis. Joint order has significance in the proper determination of overall degree of freedom for the assembly. We gave definitions for a mechanism and a machine in Chapter 1. With the kinematic elements of links and joints now defined, we can define those devices more carefully based on Reuleaux's classifications of the kinematic chain, mechanism, and machine. [1]

A kinematic chain is defined as:

An assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied input motion.

A mechanism is defined as:

A kinematic chain in which at least one link has been "grounded," or attached, to the frame of reference (which itself may be in motion).

A machine is defined as:

A combination of resistant bodies arranged to compel the mechanical forces of nature to do work accompanied by determinate motions.

By Reuleaux's definition [1] a machine is *a collection of mechanisms arranged to transmit forces and do work.* He viewed all energy or force transmitting devices as machines which utilize mechanisms as their building blocks to provide the necessary motion constraints.

We will now define a crank as *a link which makes a complete revolution and* is *pivoted to ground,* a rocker as *a link which has oscillatory (back andforth) rotation and* is *pivoted to ground,* and a coupler (or connecting rod) *which has complex motion and* is *not pivoted to ground.* Ground is defined as *any link or links that are fixed* (nonmoving) with respect to the reference frame. Note that the reference frame may in fact itself be in motion.

2.4 DETERMINING DEGREE OF FREEDOM

The concept of degree offreedom *(DOF)* is fundamental to both the synthesis and analysis of mechanisms. We need to be able to quickly determine the *DOF* of any collection of links and joints which may be suggested as a solution to a problem. Degree of freedom (also called the mobility *M)* of a system can be defined as:

Degree of Freedom

the number of inputs which need to be provided in order to create a predictable output;

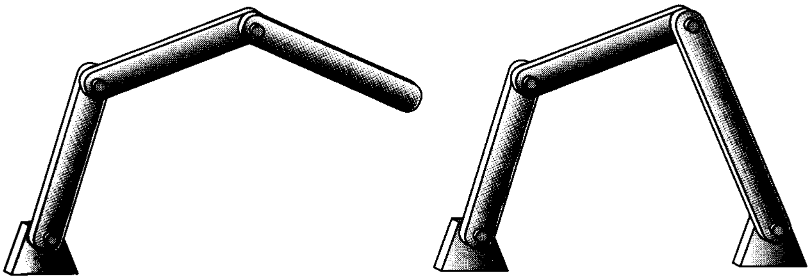
also:

the number of independent coordinates required to define its position.

At the outset of the design process, some general definition of the desired output motion is usually available. The number of inputs needed to obtain that output mayor may not be specified. Cost is the principal constraint here. Each required input will need some type of actuator, either a human operator or a "slave" in the fonn of a motor, solenoid, air cylinder, or other energy conversion device. (These devices are discussed in Section 2.15.) These multiple input devices will have to have their actions coordinated by a "controller," which must have some intelligence. This control is now often provided by a computer but can also be mechanically programmed into the mechanism design. There is no requirement that a mechanism have only one *DOF,* although that is often desirable for simplicity. Some machines have many *DOF.* For example, picture the number of control levers or actuating cylinders on a bulldozer or crane. See Figure I-lb (p.7).

Kinematic chains or mechanisms may be either open or closed. Figure 2-4 shows both open and closed mechanisms. A closed mechanism will have no open attachment points or nodes and may have one or more degrees of freedom. An open mechanism of more than one link will always have more than one degree of freedom, thus requiring as many actuators (motors) as it has *DOF.* A common example of an open mechanism is an industrial robot. *An open kinematic chain of two binary links and one joint* is called a dyad. The sets of links shown in Figure 2-3a and b are dyads.

Reuleaux limited his definitions to closed kinematic chains and to mechanisms having only one *DOF,* which he called *constrained.* [1] The somewhat broader definitions above are perhaps better suited to current-day applications. A *multi-DOF* mechanism, such as a robot, will be constrained in its motions as long as the necessary number of inputs are supplied to control all its *DOF.*



(*a*) Open mechanism chain (*b*) Closed mechanism chain

FIGURE 2-4
Mechanism chains

Degree of Freedom in Planar Mechanisms

To determine the overall *DOF* of any mechanism, we must account for the number of links and joints, and for the interactions among them. The *DOF* of any assembly of links can be predicted from an investigation of the Gruebler condition. [2] Any link in a plane bas 3 *DOF*. Therefore, a system of *L* unconnected links in the same plane will have *3L DOF,* as shown in Figure 2-5a where the two unconnected links have a total of six *DOF.* When these links are connected by a full joint in Figure 2-5b, ¥l and *¥2* are combined as *¥,* and Lixl and *Lix2* are combined as *Lix.* This removes two *DOF,* leaving four *DOF.* In Figure 2-5c the half joint removes only one *DOF* from the system (because a half joint has two *DOF),* leaving the system of two links connected by a half joint with a total of five *DOF.* In addition, when any link is grounded or attached to the reference frame, all three of its *DOF* will be removed. This reasoning leads to Gruebler's equation:

$$M=3L-2J-3G \tag{2.1a}$$

where: *M* = *degree offreedom or mobility*
L = *number of links*
J = *number of joints*
G = *number of grounded links*

Note that in any real mechanism, even if more than one link of the kinematic chain is grounded, the net effect will be to create one larger, higher-order ground link, as there can be only one ground plane. Thus G is always one, and Gruebler's equation becomes:

$$M = 3(L-1) - 2J \tag{2.1b}$$

The value of *J* in equations 2.1a and 2.1b must reflect the value of all joints in the mechanism. That is, half joints count as 1/2 because they only remove one *DOF*. It is less confusing if we use **Kutzbach's** modification of Gruebler's equation in this form:

$$M = 3(L-1) - 2J_1 - J_2 \tag{2.1c}$$

where: *M* = *degree offreedom or mobility*
L = *number of links*
Jl = *number of* 1*DOF (full) joints*
J2 = *number of* 2 *DOF (half) joints*

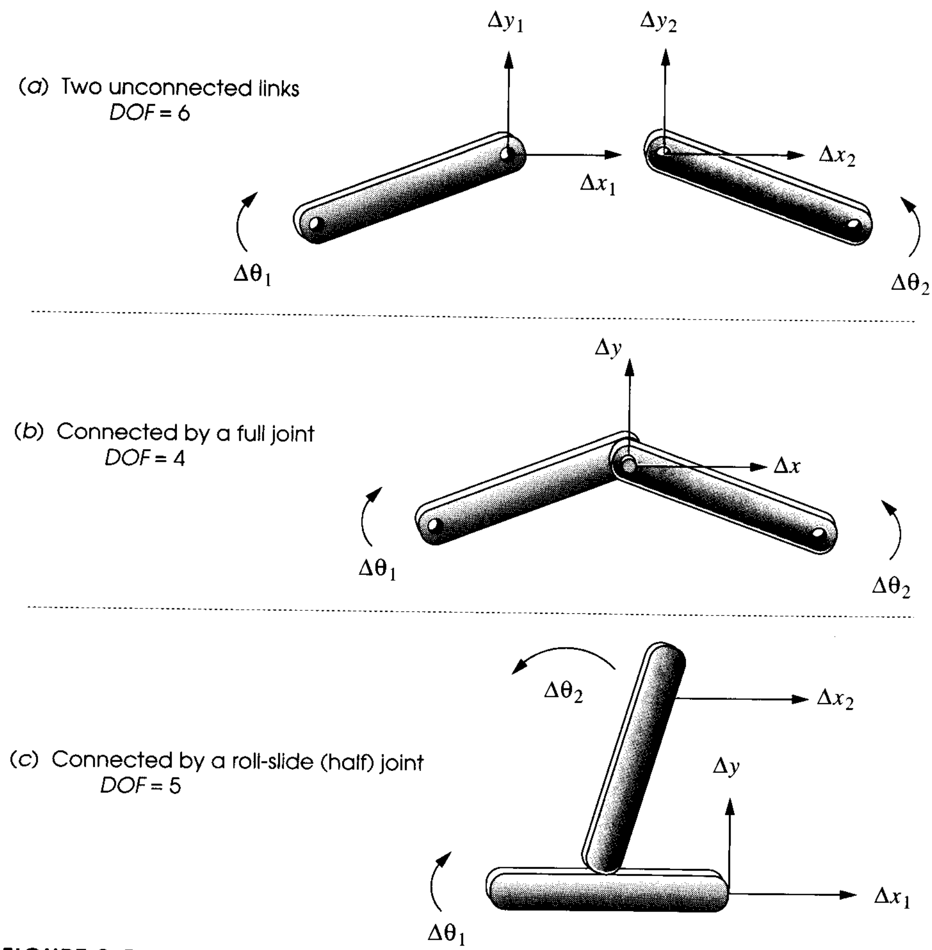
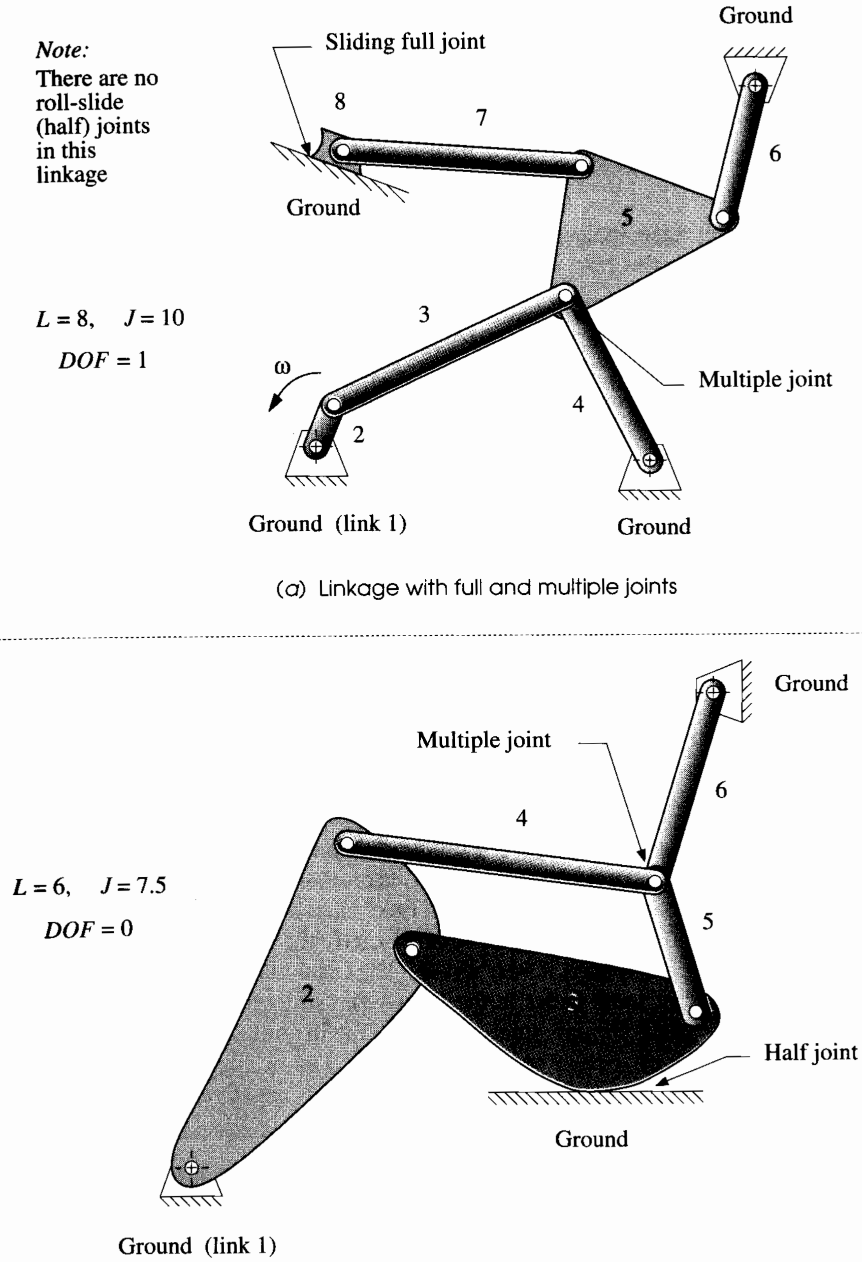


FIGURE 2-5

Joints remove degrees of freedom

The value of *J1* and lz in these equations must still be carefully determined to account for all full, half, and multiple joints in any linkage. Multiple joints count as one less than the number oflinks joined at that joint and add to the "full" (11) category. The *DOF* of any proposed mechanism can be quickly ascertained from this expression before investing any time in more detailed design. It is interesting to note that this equation has no information in it about link sizes or shapes, only their quantity. Figure 2-6a shows a mechanism with one *DOF* and only full joints in it.

Figure 2-6b shows a structure with zero *DOF* and which contains both half and multiple joints. Note the schematic notation used to show the ground link. The ground link need not be drawn in outline as long as all the grounded joints are identified. Note also the joints labeled **"multiple"** and **"half'** in Figure 2-6a and b. As an exercise, compute the *DOF* of these examples with **Kutzbach's** equation.



(a) Linkage with full and multiple joints

(b) Linkage with full, half, and multiple joints

FIGURE 2-6

Linkages containing joints of various types

Degree of Freedom in Spatial Mechanisms

The approach used to determine the mobility of a planar mechanism can be easily extended to three dimensions. Each unconnected link in three-space has 6 *DOF,* and any one of the six lower pairs can be used to connect them, as can higher pairs with more freedom. A one-freedom joint removes 5 *DOF,* a two-freedom joint removes 4 *DOF,* etc. Grounding a link removes 6 *DOF.* This leads to the Kutzbach mobility equation for spatiallinkages:

$$M = 6(L-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5 \tag{2.2}$$

where the subscript refers to the number of freedoms of the joint. We will limit our study to 2-D mechanisms in this text.

2.5 MECHANISMS AND STRUCTURES

The degree of freedom of an assembly of links completely predicts its character. There are only three possibilities. *If the DOF is positive, it will be a mechanism,* and the links will have relative motion. *If the DOF is exactly zero, then it will be a structure,* and no motion is possible. *If the DOF is negative, then it is a preloaded structure,* which means that no motion is possible and some stresses may also be present at the time of assembly. Figure 2-7 shows examples of these three cases. One link is grounded in each case.

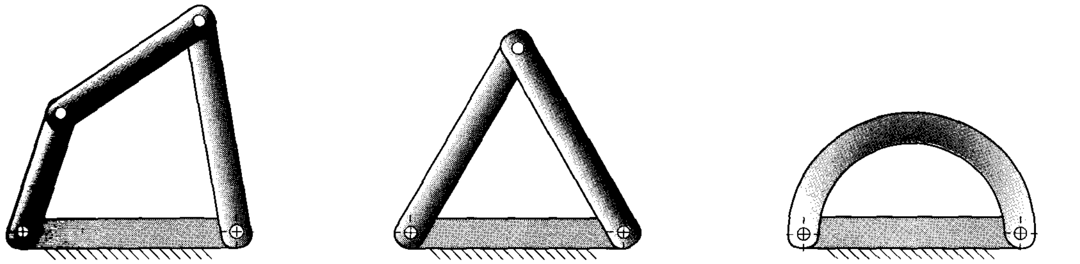
Figure 2-7a shows four links joined by four full joints which, from the Gruebler equation, gives one *DOF.* It will move, and only one input is needed to give predictable results.

Figure 2-7b shows three links joined by three full joints. It has zero *DOF* and is thus a structure. Note that if the link lengths will allow connection, * all three pins can be inserted into their respective pairs of link holes (nodes) without straining the structure, as a position can always be found to allow assembly.

Figure 2-7c shows two links joined by two full joints. It has a *DOF* of minus one, making it a **preloaded** structure. In order to insert the two pins without straining the links, the center distances of the holes in both links must be exactly the same. Practically speaking, it is impossible to make two parts exactly the same. There will always be some manufacturing error, even if very small. Thus you may have to force the second pin into place, creating some stress in the links. The structure will then be preloaded. You have probably met a similar situation in a course in applied mechanics in the form of an indeterminate beam, one in which there were too many supports or constraints for the equations available. An indeterminate beam also has negative *DOF,* while a *simply supported* beam has zero *DOF.*

Both structures and preloaded structures are commonly encountered in engineering. In fact the true structure of zero *DOF* is rare in engineering practice. Most buildings, bridges, and machine frames are preloaded structures, due to the use of welded and riveted joints rather than pin joints. Even simple structures like the chair you are sitting in are often preloaded. Since our concern here is with mechanisms, we will concentrate on devices with positive *DOF* only.

* If the sum of the lengths of any two links is less than the length of the third link, then their interconnection is impossible.



(*a*) Mechanism—*DOF* = + 1 (*b*) Structure—*DOF* = 0 (*c*) Preloaded structure—*DOF* = −1

FIGURE 2-7

Mechanisms, structures and preloaded structures

2.6 NUMBER SYNTHESIS

The term number synthesis has been coined to mean *the determination of the number and order of links and joints necessary to produce motion of a particular DOF.* Order in this context refers to the number of nodes perlink, i.e., binary, ternary, quaternary, etc. The value of number synthesis is to allow the exhaustive determination of all possible combinations of links which will yield any chosen *DOF.* This then equips the designer with a definitive catalog of potential linkages to solve a variety of motion control problems.

As an example we will now derive all the possible link combinations for one *DOF,* including sets of up to eight links, and link orders up to and including hexagonal links. For simplicity we will assume that the links will be connected with only full rotating joints. We can later introduce half joints, multiple joints, and sliding joints through linkage transformation. First let's look at some interesting attributes of linkages as defined by the above assumption regarding full joints.

Hypothesis: If all joints are full joints, an odd number of *DOF requires* an even number of links and vice versa.

Proof: Given: All even integers can be denoted by *2m* or by *2n,* and all odd integers can be denoted by *2m* - I or by *2n* - 1, where *n* and *m* are any positive integers. The number of joints must be a positive integer.

Let: L = number of links, J = number of joints, and $M = DOF = 2m$ (i.e., all even numbers)

Then: rewriting Gruebler's equation (Equation 2.1b) to solve for *J,*

$$J = \frac{3}{2}(L-1) - \frac{M}{2} \qquad \text{(a)}$$

Try: Substituting $M = 2m$, and $L = 2n$ (i.e., both any even numbers):

$$J = 3n - m - \frac{3}{2} \qquad \text{(b)}$$