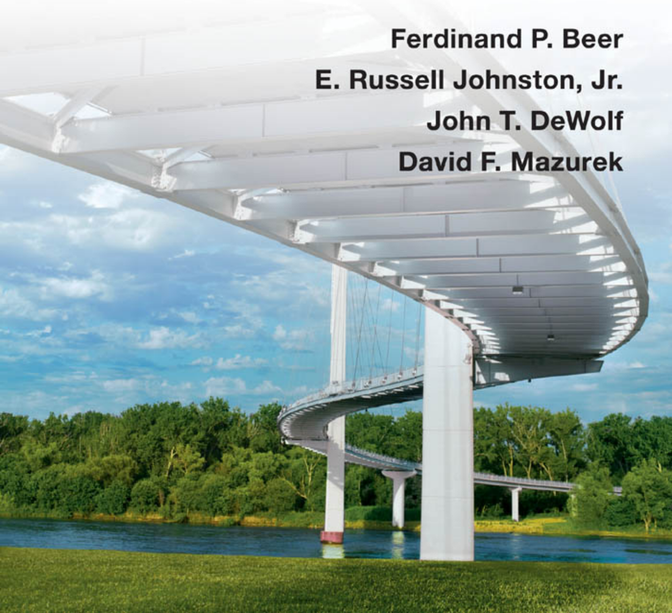
SIXTH EDITION

MECHANICS of MATERIALS

Ferdinand P. Beer
E. Russell Johnston, Jr.
John T. DeWolf
David F. Mazurek



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SI Prefixes

Multiplication Factor	Prefix†	Symbol
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera	T
$1\ 000\ 000\ 000 = 10^{9}$	giga	G
$1\ 000\ 000 = 10^{6}$	mega	M
$1\ 000 = 10^{3}$	kilo	k
$100 = 10^{2}$	hecto‡	h
$10 = 10^{1}$	deka‡	da
$0.1 = 10^{-1}$	deci‡	d
$0.01 = 10^{-2}$	centi‡	c
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	μ
$0.000\ 000\ 001 = 10^{-9}$	nano	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico	p
$0.000\ 000\ 000\ 000\ 001 = 10^{-15}$	femto	f
$0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$	atto	a

† The first syllable of every prefix is accented so that the prefix will retain its identity. Thus, the preferred pronunciation of kilometer places the accent on the first syllable, not the second.

‡ The use of these prefixes should be avoided, except for the measurement of areas and volumes and for the nontechnical use of centimeter, as for body and clothing measurements.

Principal SI Units Used in Mechanics

Quantity	Unit	Symbol	Formula
Acceleration	Meter per second squared	⋯	m/s^2
Angle	Radian	rad	†
Angular acceleration	Radian per second squared	⋯	rad/s^2
Angular velocity	Radian per second	⋯	rad/s
Area	Square meter	⋯	m^2
Density	Kilogram per cubic meter	⋯	kg/m^3
Energy	Joule	J	$N \cdot m$
Force	Newton	N	$kg \cdot m/s^2$
Frequency	Hertz	Hz	s^{-1}
Impulse	Newton-second	⋯	$kg \cdot m/s$
Length	Meter	m	‡
Mass	Kilogram	kg	‡
Moment of a force	Newton-meter	⋯	$N \cdot m$
Power	Watt	W	J/s
Pressure	Pascal	Pa	N/m^2
Stress	Pascal	Pa	N/m^2
Time	Second	s	‡
Velocity	Meter per second	⋯	m/s
Volume, solids	Cubic meter	⋯	m^3
Liquids	Liter	L	$10^{-3}\ m^3$
Work	Joule	J	$N \cdot m$

† Supplementary unit (1 revolution $= 2\pi$ rad $= 360°$).

‡ Base unit.

U.S. Customary Units and Their SI Equivalents

Quantity	U.S. Customary Units	SI Equivalent
Acceleration	ft/s^2	$0.3048 m/s^2$
	$in./s^2$	$0.0254 m/s^2$
Area	ft^2	$0.0929 m^2$
	in^2	$645.2 mm^2$
Energy	ft · lb	1.356 J
Force	kip	4.448 kN
	lb	4.448 N
	oz	0.2780 N
Impulse	lb · s	4.448 N · s
Length	ft	0.3048 m
	in.	25.40 mm
	mi	1.609 km
Mass	oz mass	28.35 g
	lb mass	0.4536 kg
	slug	14.59 kg
	ton	907.2 kg
Moment of a force	lb · ft	1.356 N · m
	lb · in.	0.1130 N · m
Moment of inertia		
Of an area	in^4	$0.4162 \times 10^6 mm^4$
Of a mass	$lb \cdot ft \cdot s^2$	$1.356 kg \cdot m^2$
Power	ft · lb/s	1.356 W
	hp	745.7 W
Pressure or stress	lb/ft^2	47.88 Pa
	lb/in^2 (psi)	6.895 kPa
Velocity	ft/s	0.3048 m/s
	in./s	0.0254 m/s
	mi/h (mph)	0.4470 m/s
	mi/h (mph)	1.609 km/h
Volume, solids	ft^3	$0.02832 m^3$
	in^3	$16.39 cm^3$
Liquids	gal	3.785 L
	qt	0.9464 L
Work	ft · lb	1.356 J

MECHANICS OF MATERIALS

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Sixth Edition

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The photos on the front and back cover show the Bob Kerrey Pedestrian Bridge, which spans the Missouri River between Omaha, Nebraska, and Council Bluffs, lowa. This S-curved structure utilizes a cable-stayed design, and is the longest pedestrian bridge to connect two states.

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About the Authors

As publishers of the books written by Ferd Beer and Russ Johnston, we are often asked how did they happen to write the books together, with one of them at Lehigh and the other at the University of Connecticut.

The answer to this question is simple. Russ Johnston's first teaching appointment was in the Department of Civil Engineering and Mechanics at Lehigh University. There he met Ferd Beer, who had joined that department two years earlier and was in charge of the courses in mechanics. Born in France and educated in France and Switzerland (he held an M.S. degree from the Sorbonne and an Sc.D. degree in the field of theoretical mechanics from the University of Geneva), Ferd had come to the United States after serving in the French army during the early part of World War II and had taught for four years at Williams College in the Williams-MIT joint arts and engineering program. Born in Philadelphia, Russ had obtained a B.S. degree in civil engineering from the University of Delaware and an Sc.D. degree in the field of structural engineering from MIT.

Ferd was delighted to discover that the young man who had been hired chiefly to teach graduate structural engineering courses was not only willing but eager to help him reorganize the mechanics courses. Both believed that these courses should be taught from a few basic principles and that the various concepts involved would be best understood and remembered by the students if they were presented to them in a graphic way. Together they wrote lecture notes in statics and dynamics, to which they later added problems they felt would appeal to future engineers, and soon they produced the manuscript of the first edition of *Mechanics for Engineers*. The second edition of *Mechanics for Engineers* and the first edition of *Vector Mechanics for Engineers* found Russ Johnston at Worcester Polytechnic Institute and the next editions at the University of Connecticut. In the meantime, both Ferd and Russ had assumed administrative responsibilities in their departments, and both were involved in research, consulting, and supervising graduate students—Ferd in the area of stochastic processes and random vibrations, and Russ in the area of elastic stability and structural analysis and design. However, their interest in improving the teaching of the basic mechanics courses had not subsided, and they both taught sections of these courses as they kept revising their texts and began writing together the manuscript of the first edition of *Mechanics of Materials*.

Ferd and Russ's contributions to engineering education earned them a number of honors and awards. They were presented with the Western Electric Fund Award for excellence in the instruction of engineering students by their respective regional sections of the American Society for Engineering Education, and they both received the Distinguished Educator Award from the Mechanics Division of the same society. In 1991 Russ received the Outstanding Civil Engineer Award from the Connecticut Section of the American Society of Civil Engineers, and in 1995 Ferd was awarded an honorary Doctor of Engineering degree by Lehigh University.

John T. DeWolf, Professor of Civil Engineering at the University of Connecticut, joined the Beer and Johnston team as an author on the second edition of *Mechanics of Materials.* John holds a B.S. degree in civil engineering from the University of Hawaii and M.E. and Ph.D. degrees in structural engineering from Cornell University. His research interests are in the area of elastic stability, bridge monitoring, and structural analysis and design. He is a registered Professional Engineering and a member of the Connecticut Board of Professional Engineers. He was selected as the University of Connecticut Teaching Fellow in 2006.

David F. Mazurek, Professor of Civil Engineering at the United States Coast Guard Academy, joined the team in the fourth edition. David holds a B.S. degree in ocean engineering and an M.S. degree in civil engineering from the Florida Institute of Technology, and a Ph.D. degree in civil engineering from the University of Connecticut. He is a registered Professional Engineer. He has served on the American Railway Engineering & Maintenance of Way Association's Committee 15—Steel Structures for the past seventeen years. Professional interests include bridge engineering, structural forensics, and blast-resistant design.

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Preface

OBJECTIVES

The main objective of a basic mechanics course should be to develop in the engineering student the ability to analyze a given problem in a simple and logical manner and to apply to its solution a few fundamental and well-understood principles. This text is designed for the first course in mechanics of materials—or strength of materials—offered to engineering students in the sophomore or junior year. The authors hope that it will help instructors achieve this goal in that particular course in the same way that their other texts may have helped them in statics and dynamics.

GENERAL APPROACH

In this text the study of the mechanics of materials is based on the understanding of a few basic concepts and on the use of simplified models. This approach makes it possible to develop all the necessary formulas in a rational and logical manner, and to clearly indicate the conditions under which they can be safely applied to the analysis and design of actual engineering structures and machine components.

Free-body Diagrams Are Used Extensively. Throughout the text free-body diagrams are used to determine external or internal forces. The use of "picture equations" will also help the students understand the superposition of loadings and the resulting stresses and deformations.

Design Concepts Are Discussed Throughout the Text Whenever Appropriate. A discussion of the application of the factor of safety to design can be found in Chap. 1, where the concepts of both allowable stress design and load and resistance factor design are presented.

A Careful Balance Between SI and U.S. Customary Units Is Consistently Maintained. Because it is essential that students be able to handle effectively both SI metric units and U.S. customary units, half the examples, sample problems, and problems to be assigned have been stated in SI units and half in U.S. customary units. Since a large number of problems are available, instructors can assign problems using each system of units in whatever proportion they find most desirable for their class.

Optional Sections Offer Advanced or Specialty Topics. Topics such as residual stresses, torsion of noncircular and thin-walled members, bending of curved beams, shearing stresses in non-symmetrical members, and failure criteria, have been included in optional sections for use in courses of varying emphases. To preserve the integrity of the subject, these topics are presented in the proper sequence, wherever they logically belong. Thus, even when not covered in the course, they are highly visible and can be easily referred to by the students if needed in a later course or in engineering practice. For convenience all optional sections have been indicated by asterisks.

CHAPTER ORGANIZATION

It is expected that students using this text will have completed a course in statics. However, Chap. 1 is designed to provide them with an opportunity to review the concepts learned in that course, while shear and bending-moment diagrams are covered in detail in Secs. 5.2 and 5.3. The properties of moments and centroids of areas are described in Appendix A; this material can be used to reinforce the discussion of the determination of normal and shearing stresses in beams (Chaps. 4, 5, and 6).

The first four chapters of the text are devoted to the analysis of the stresses and of the corresponding deformations in various structural members, considering successively axial loading, torsion, and pure bending. Each analysis is based on a few basic concepts, namely, the conditions of equilibrium of the forces exerted on the member, the relations existing between stress and strain in the material, and the conditions imposed by the supports and loading of the member. The study of each type of loading is complemented by a large number of examples, sample problems, and problems to be assigned, all designed to strengthen the students' understanding of the subject.

The concept of stress at a point is introduced in Chap. 1, where it is shown that an axial load can produce shearing stresses as well as normal stresses, depending upon the section considered. The fact that stresses depend upon the orientation of the surface on which they are computed is emphasized again in Chaps. 3 and 4 in the cases of torsion and pure bending. However, the discussion of computational techniques—such as Mohr's circle—used for the transformation of stress at a point is delayed until Chap. 7, after students have had the opportunity to solve problems involving a combination of the basic loadings and have discovered for themselves the need for such techniques.

The discussion in Chap. 2 of the relation between stress and strain in various materials includes fiber-reinforced composite materials. Also, the study of beams under transverse loads is covered in two separate chapters. Chapter 5 is devoted to the determination of the normal stresses in a beam and to the design of beams based on the allowable normal stress in the material used (Sec. 5.4). The chapter begins with a discussion of the shear and bending-moment diagrams (Secs. 5.2 and 5.3) and includes an optional section on the use of singularity functions for the determination of the shear and bending moment in a beam (Sec. 5.5). The chapter ends with an optional section on nonprismatic beams (Sec. 5.6).

Chapter 6 is devoted to the determination of shearing stresses in beams and thin-walled members under transverse loadings. The formula for the shear flow, $q = VQ/I$, is derived in the traditional way. More advanced aspects of the design of beams, such as the determination of the principal stresses at the junction of the flange and web of a W-beam, are in Chap. 8, an optional chapter that may be covered after the transformations of stresses have been discussed in Chap. 7. The design of transmission shafts is in that chapter for the same reason, as well as the determination of stresses under combined loadings that can now include the determination of the principal stresses, principal planes, and maximum shearing stress at a given point.

Statically indeterminate problems are first discussed in Chap. 2 and considered throughout the text for the various loading conditions encountered. Thus, students are presented at an early stage with a method of solution that combines the analysis of deformations with the conventional analysis of forces used in statics. In this way, they will have become thoroughly familiar with this fundamental method by the end of the course. In addition, this approach helps the students realize that stresses themselves are statically indeterminate and can be computed only by considering the corresponding distribution of strains.

The concept of plastic deformation is introduced in Chap. 2, where it is applied to the analysis of members under axial loading. Problems involving the plastic deformation of circular shafts and of prismatic beams are also considered in optional sections of Chaps. 3, 4, and 6. While some of this material can be omitted at the choice of the instructor, its inclusion in the body of the text will help students realize the limitations of the assumption of a linear stress-strain relation and serve to caution them against the inappropriate use of the elastic torsion and flexure formulas.

The determination of the deflection of beams is discussed in Chap. 9. The first part of the chapter is devoted to the integration method and to the method of superposition, with an optional section (Sec. 9.6) based on the use of singularity functions. (This section should be used only if Sec. 5.5 was covered earlier.) The second part of Chap. 9 is optional. It presents the moment-area method in two lessons.

Chapter 10 is devoted to columns and contains material on the design of steel, aluminum, and wood columns. Chapter 11 covers energy methods, including Castigliano's theorem.

PEDAGOGICAL FEATURES

Each chapter begins with an introductory section setting the purpose and goals of the chapter and describing in simple terms the material to be covered and its application to the solution of engineering problems.

Chapter Lessons. The body of the text has been divided into units, each consisting of one or several theory sections followed by sample problems and a large number of problems to be assigned.

Each unit corresponds to a well-defined topic and generally can be covered in one lesson.

Examples and Sample Problems. The theory sections include many examples designed to illustrate the material being presented and facilitate its understanding. The sample problems are intended to show some of the applications of the theory to the solution of engineering problems. Since they have been set up in much the same form that students will use in solving the assigned problems, the sample problems serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions.

Homework Problem Sets. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and help the students understand the basic principles used in mechanics of materials. The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty. Problems requiring special attention have been indicated by asterisks. Answers to problems are given at the end of the book, except for those with a number set in italics.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in the chapter. Notes in the margin have been included to help the students organize their review work, and cross references provided to help them find the portions of material requiring their special attention.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

Computer Problems. Computers make it possible for engineering students to solve a great number of challenging problems. A group of six or more problems designed to be solved with a computer can be found at the end of each chapter. These problems can be solved using any computer language that provides a basis for analytical calculations. Developing the algorithm required to solve a given problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply the skills acquired in their computer programming course to the solution of a meaningful engineering problem. These problems can be solved using any computer language that provide a basis for analytical calculations.

Fundamentals of Engineering Examination. Engineers who seek to be licensed as *Professional Engineers* must take two exams. The first exam, the *Fundamentals of Engineering Examination*, includes subject material from *Mechanics of Materials.* Appendix E lists the topics in *Mechanics of Materials* that are covered in this exam along with problems that can be solved to review this material.

SUPPLEMENTAL RESOURCES

Instructor's Solutions Manual. The Instructor's and Solutions Manual that accompanies the sixth edition continues the tradition of exceptional accuracy and keeping solutions contained to a single page for easier reference. The manual also features tables designed to assist instructors in creating a schedule of assignments for their courses. The various topics covered in the text are listed in Table I, and a suggested number of periods to be spent on each topic is indicated. Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. Sample lesson schedules are also found within the manual.

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ADDITIONAL ONLINE RESOURCES

Mechanics of Materials 6e also features a companion website (www.mhhe.com/beerjohnston) for instructors. Included on the website are lecture PowerPoints, an image library, and animations. Via the website, instructors can also request access to C.O.S.M.O.S., a complete online solutions manual organization system that allows instructors to create custom homework, quizzes, and tests using end-of-chapter problems from the text. For access to this material, contact your sales representative for a user name and password.

Hands-On Mechanics. Hands-On Mechanics is a website designed for instructors who are interested in incorporating three-dimensional, hands-on teaching aids into their lectures. Developed through a partnership between McGraw-Hill and the Department of Civil and Mechanical Engineering at the United States Military Academy at West Point, this website not only provides detailed instructions for how to build 3-D teaching tools using materials found in any lab or local hardware store but also provides a community where educators can share ideas, trade best practices, and submit their own demonstrations for posting on the site. Visit www.handsonmechanics.com to see how you can put this to use in your classroom.

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The authors thank the many companies that provided photographs for this edition. We also wish to recognize the determined efforts and patience of our photo researcher Sabina Dowell.

Our special thanks go to Professor Dean Updike, of the Department of Mechanical Engineering and Mechanics, Lehigh University for his patience and cooperation as he checked the solutions and answers of all the problems in this edition.

We also gratefully acknowledge the help, comments and suggestions offered by the many reviewers and users of previous editions of *Mechanics of Materials.*

John T. DeWolf
David F. Mazurek

List of Symbols

a	Constant; distance
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Forces; reactions
$A, B, C, \ldots$	Points
$A, \mathcal{A}$	Area
b	Distance; width
c	Constant; distance; radius
C	Centroid
$C_1, C_2, \ldots$	Constants of integration
C_P	Column stability factor
d	Distance; diameter; depth
D	Diameter
e	Distance; eccentricity; dilatation
E	Modulus of elasticity
f	Frequency; function
$\mathbf{F}$	Force
$F.S.$	Factor of safety
G	Modulus of rigidity; shear modulus
h	Distance; height
$\mathbf{H}$	Force
H, J, K	Points
$I, I_x, \ldots$	Moment of inertia
$I_{xy}, \ldots$	Product of inertia
J	Polar moment of inertia
k	Spring constant; shape factor; bulk modulus; constant
K	Stress concentration factor; torsional spring constant
l	Length; span
L	Length; span
L_e	Effective length
m	Mass
$\mathbf{M}$	Couple
$M, M_x, \ldots$	Bending moment
M_D	Bending moment, dead load (LRFD)
M_L	Bending moment, live load (LRFD)
M_U	Bending moment, ultimate load (LRFD)
n	Number; ratio of moduli of elasticity; normal direction
p	Pressure
$\mathbf{P}$	Force; concentrated load
P_D	Dead load (LRFD)
P_L	Live load (LRFD)
P_U	Ultimate load (LRFD)
q	Shearing force per unit length; shear flow
$\mathbf{Q}$	Force
Q	First moment of area

r	Radius; radius of gyration
$\mathbf{R}$	Force; reaction
R	Radius; modulus of rupture
s	Length
S	Elastic section modulus
t	Thickness; distance; tangential deviation
$\mathbf{T}$	Torque
T	Temperature
u, v	Rectangular coordinates
u	Strain-energy density
U	Strain energy; work
$\mathbf{v}$	Velocity
$\mathbf{V}$	Shearing force
V	Volume; shear
w	Width; distance; load per unit length
$\mathbf{W}, W$	Weight, load
x, y, z	Rectangular coordinates; distance; displacements; deflections
$\bar{x}, \bar{y}, \bar{z}$	Coordinates of centroid
Z	Plastic section modulus
α, β, γ	Angles
α	Coefficient of thermal expansion; influence coefficient
γ	Shearing strain; specific weight
γ_D	Load factor, dead load (LRFD)
γ_L	Load factor, live load (LRFD)
δ	Deformation; displacement
ϵ	Normal strain
θ	Angle; slope
λ	Direction cosine
ν	Poisson's ratio
ρ	Radius of curvature; distance; density
σ	Normal stress
τ	Shearing stress
ϕ	Angle; angle of twist; resistance factor
ω	Angular velocity

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MECHANICS OF MATERIALS

This chapter is devoted to the study of the stresses occurring in many of the elements contained in these excavators, such as two-force members, axles, bolts, and pins.

CHAPTER 1

Introduction—Concept of Stress

Chapter 1 Introduction—Concept of Stress

1.1 INTRODUCTION

The main objective of the study of the mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load-bearing structures.

Both the analysis and the design of a given structure involve the determination of *stresses* and *deformations*. This first chapter is devoted to the concept of *stress*.

Section 1.2 is devoted to a short review of the basic methods of statics and to their application to the determination of the forces in the members of a simple structure consisting of pin-connected members. Section 1.3 will introduce you to the concept of *stress* in a member of a structure, and you will be shown how that stress can be determined from the *force* in the member. After a short discussion of engineering analysis and design (Sec. 1.4), you will consider successively the *normal stresses* in a member under axial loading (Sec. 1.5), the *shearing stresses* caused by the application of equal and opposite transverse forces (Sec. 1.6), and the *bearing stresses* created by bolts and pins in the members they connect (Sec. 1.7). These various concepts will be applied in Sec. 1.8 to the determination of the stresses in the members of the simple structure considered earlier in Sec. 1.2.

The first part of the chapter ends with a description of the method you should use in the solution of an assigned problem (Sec. 1.9) and with a discussion of the numerical accuracy appropriate in engineering calculations (Sec. 1.10).

In Sec. 1.11, where a two-force member under axial loading is considered again, it will be observed that the stresses on an *oblique* plane include both *normal* and *shearing* stresses, while in Sec. 1.12 you will note that *six components* are required to describe the state of stress at a point in a body under the most general loading conditions.

Finally, Sec. 1.13 will be devoted to the determination from test specimens of the *ultimate strength* of a given material and to the use of a *factor of safety* in the computation of the *allowable load* for a structural component made of that material.

1.2 A SHORT REVIEW OF THE METHODS OF STATICS

In this section you will review the basic methods of statics while determining the forces in the members of a simple structure.

Consider the structure shown in Fig. 1.1, which was designed to support a 30-kN load. It consists of a boom *AB* with a 30 × 50-mm rectangular cross section and of a rod *BC* with a 20-mm-diameter circular cross section. The boom and the rod are connected by a pin at *B* and are supported by pins and brackets at *A* and *C*, respectively. Our first step should be to draw a *free-body diagram* of the structure by detaching it from its supports at *A* and *C*, and showing the reactions that these supports exert on the structure (Fig. 1.2). Note that the sketch of the structure has been simplified by omitting all unnecessary details. Many of you may have recognized at this point that *AB* and *BC* are *two-force members*. For those of you who have not, we will pursue our analysis, ignoring that fact and assuming that the directions of the reactions at *A* and *C* are unknown. Each of these

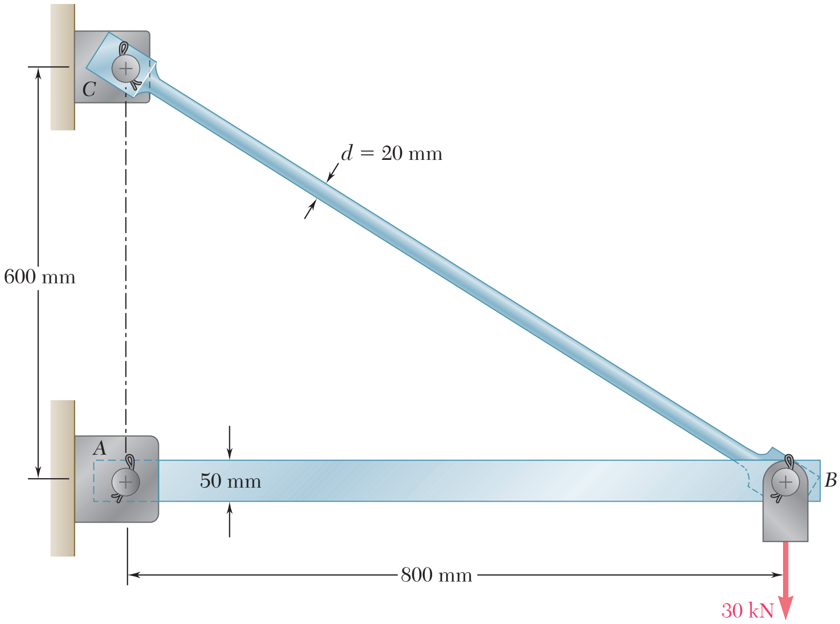


Fig. 1.1 Boom used to support a 30-kN load.

reactions, therefore, will be represented by two components, $\mathbf{A}_x$ and $\mathbf{A}_y$ at A, and $\mathbf{C}_x$ and $\mathbf{C}_y$ at C. We write the following three equilibrium equations:

$$+\circlearrowleft \Sigma M_C = 0: \qquad A_x(0.6 \text{ m}) - (30 \text{ kN})(0.8 \text{ m}) = 0$$

$$A_x = +40 \text{ kN} \tag{1.1}$$

$$\xrightarrow{+} \Sigma F_x = 0: \qquad A_x + C_x = 0$$

$$C_x = -A_x \qquad C_x = -40 \text{ kN} \tag{1.2}$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + C_y - 30 \text{ kN} = 0$$

$$A_y + C_y = +30 \text{ kN} \tag{1.3}$$

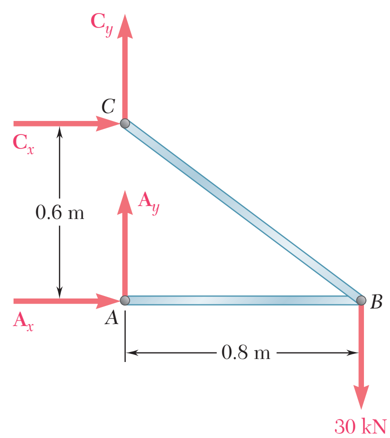


Fig. 1.2

We have found two of the four unknowns, but cannot determine the other two from these equations, and no additional independent equation can be obtained from the free-body diagram of the structure. We must now dismember the structure. Considering the free-body diagram of the boom AB (Fig. 1.3), we write the following equilibrium equation:

$$+\circlearrowleft \Sigma M_B = 0: \qquad -A_y(0.8 \text{ m}) = 0 \qquad A_y = 0 \tag{1.4}$$

Substituting for A_y from (1.4) into (1.3), we obtain $C_y = +30$ kN. Expressing the results obtained for the reactions at A and C in vector form, we have

$$\mathbf{A} = 40 \text{ kN} \rightarrow \qquad \mathbf{C}_x = 40 \text{ kN} \leftarrow, \mathbf{C}_y = 30 \text{ kN} \uparrow$$

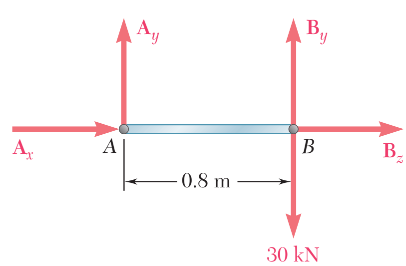


Fig. 1.3

We note that the reaction at A is directed along the axis of the boom AB and causes compression in that member. Observing that the components C_x and C_y of the reaction at C are, respectively, proportional to the horizontal and vertical components of the distance from B to C, we conclude that the reaction at C is equal to 50 kN, is directed along the axis of the rod BC, and causes tension in that member.

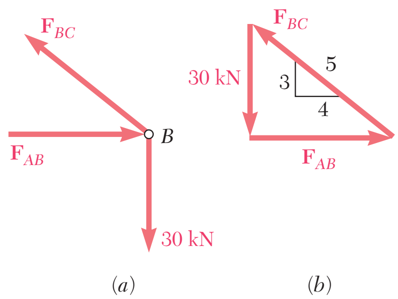


Fig. 1.4

These results could have been anticipated by recognizing that AB and BC are two-force members, i.e., members that are subjected to forces at only two points, these points being A and B for member AB, and B and C for member BC. Indeed, for a two-force member the lines of action of the resultants of the forces acting at each of the two points are equal and opposite and pass through both points. Using this property, we could have obtained a simpler solution by considering the free-body diagram of pin B. The forces on pin B are the forces $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$ exerted, respectively, by members AB and BC, and the 30-kN load (Fig. 1.4*a*). We can express that pin B is in equilibrium by drawing the corresponding force triangle (Fig. 1.4*b*).

Since the force $\mathbf{F}_{BC}$ is directed along member BC, its slope is the same as that of BC, namely, 3/4. We can, therefore, write the proportion

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30 \text{ kN}}{3}$$

from which we obtain

$$F_{AB} = 40 \text{ kN} \qquad F_{BC} = 50 \text{ kN}$$

The forces $\mathbf{F}'_{AB}$ and $\mathbf{F}'_{BC}$ exerted by pin B, respectively, on boom AB and rod BC are equal and opposite to $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$ (Fig. 1.5).

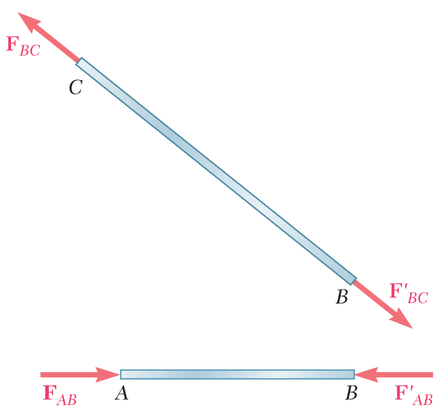


Fig. 1.5

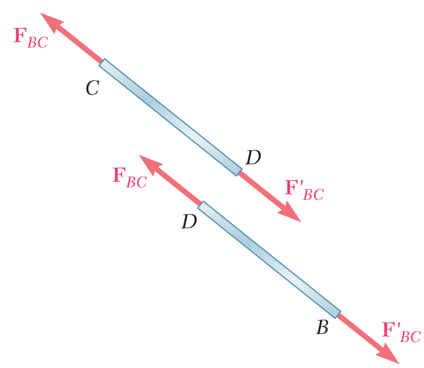


Fig. 1.6

Knowing the forces at the ends of each of the members, we can now determine the internal forces in these members. Passing a section at some arbitrary point D of rod BC, we obtain two portions BD and CD (Fig. 1.6). Since 50-kN forces must be applied at D to both portions of the rod to keep them in equilibrium, we conclude that an internal force of 50 kN is produced in rod BC when a 30-kN load is applied at B. We further check from the directions of the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ in Fig. 1.6 that the rod is in tension. A similar procedure would enable us to determine that the internal force in boom AB is 40 kN and that the boom is in compression.

1.3 STRESSES IN THE MEMBERS OF A STRUCTURE

While the results obtained in the preceding section represent a first and necessary step in the analysis of the given structure, they do not tell us whether the given load can be safely supported. Whether rod BC, for example, will break or not under this loading depends not only upon the value found for the internal force F_{BC}, but also upon the cross-sectional area of the rod and the material of which the rod is made. Indeed, the internal force F_{BC} actually represents the resultant of elementary forces distributed over the entire area A of the cross section (Fig. 1.7) and the average intensity of these distributed forces is equal to the force per unit area, F_{BC}/A, in the section. Whether or not the rod will break under the given loading clearly depends upon the ability of the material to withstand the corresponding value F_{BC}/A of the intensity of the distributed internal forces. It thus depends upon the force F_{BC}, the cross-sectional area A, and the material of the rod.

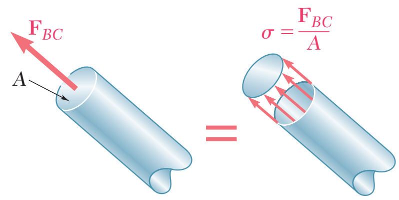


Fig. 1.7

The force per unit area, or intensity of the forces distributed over a given section, is called the *stress* on that section and is denoted by the Greek letter σ (sigma). The stress in a member of cross-sectional area A subjected to an axial load $\mathbf{P}$ (Fig. 1.8) is therefore obtained by dividing the magnitude P of the load by the area A:

$$\sigma = \frac{P}{A} \tag{1.5}$$

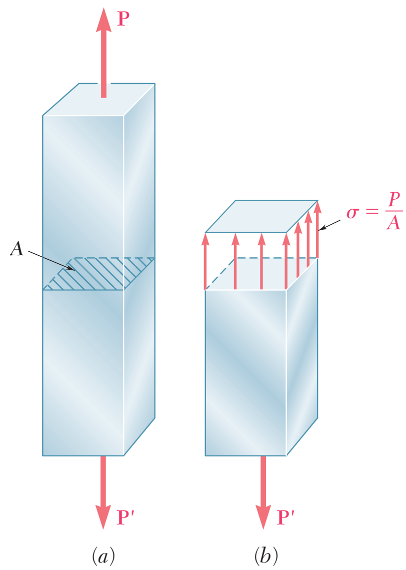


Fig. 1.8 Member with an axial load.

A positive sign will be used to indicate a tensile stress (member in tension) and a negative sign to indicate a compressive stress (member in compression).

Since SI metric units are used in this discussion, with P expressed in newtons (N) and A in square meters (m^2), the stress σ will be expressed in N/m^2. This unit is called a *pascal* (Pa). However, one finds that the pascal is an exceedingly small quantity and that, in practice, multiples of this unit must be used, namely, the kilopascal (kPa), the megapascal (MPa), and the gigapascal (GPa). We have

$$1 \text{ kPa} = 10^3 \text{ Pa} = 10^3 \text{ N/m}^2$$

$$1 \text{ MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2$$

$$1 \text{ GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2$$

When U.S. customary units are used, the force P is usually expressed in pounds (lb) or kilopounds (kip), and the cross-sectional area A in square inches (in^2). The stress σ will then be expressed in pounds per square inch (psi) or kilopounds per square inch (ksi).†

†The principal SI and U.S. customary units used in mechanics are listed in tables inside the front cover of this book. From the table on the right-hand side, we note that 1 psi is approximately equal to 7 kPa, and 1 ksi approximately equal to 7 MPa.

1.4 ANALYSIS AND DESIGN

Considering again the structure of Fig. 1.1, let us assume that rod *BC* is made of a steel with a maximum allowable stress $\sigma_{\text{all}} = 165$ MPa. Can rod *BC* safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was found earlier to be 50 kN. Recalling that the diameter of the rod is 20 mm, we use Eq. (1.5) to determine the stress created in the rod by the given loading. We have

$$P = F_{BC} = +50 \text{ kN} = +50 \times 10^3 \text{ N}$$

$$A = \pi r^2 = \pi\left(\frac{20 \text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3} \text{ m})^2 = 314 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = +159 \times 10^6 \text{ Pa} = +159 \text{ MPa}$$

Since the value obtained for σ is smaller than the value σ_{all} of the allowable stress in the steel used, we conclude that rod *BC* can safely support the load to which it will be subjected. To be complete, our analysis of the given structure should also include the determination of the compressive stress in boom *AB*, as well as an investigation of the stresses produced in the pins and their bearings. This will be discussed later in this chapter. We should also determine whether the deformations produced by the given loading are acceptable. The study of deformations under axial loads will be the subject of Chap. 2. An additional consideration required for members in compression involves the *stability* of the member, i.e., its ability to support a given load without experiencing a sudden change in configuration. This will be discussed in Chap. 10.

The engineer's role is not limited to the analysis of existing structures and machines subjected to given loading conditions. Of even greater importance to the engineer is the *design* of new structures and machines, that is, the selection of appropriate components to perform a given task. As an example of design, let us return to the structure of Fig. 1.1, and assume that aluminum with an allowable stress $\sigma_{\text{all}} = 100$ MPa is to be used. Since the force in rod *BC* will still be $P = F_{BC} = 50$ kN under the given loading, we must have, from Eq. (1.5),

$$\sigma_{\text{all}} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{\text{all}}} = \frac{50 \times 10^3 \text{ N}}{100 \times 10^6 \text{ Pa}} = 500 \times 10^{-6} \text{ m}^2$$

and, since $A = \pi r^2$,

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6} \text{ m}^2}{\pi}} = 12.62 \times 10^{-3} \text{ m} = 12.62 \text{ mm}$$

$$d = 2r = 25.2 \text{ mm}$$

We conclude that an aluminum rod 26 mm or more in diameter will be adequate.

1.5 AXIAL LOADING; NORMAL STRESS

As we have already indicated, rod BC of the example considered in the preceding section is a two-force member and, therefore, the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ acting on its ends B and C (Fig. 1.5) are directed along the axis of the rod. We say that the rod is under *axial loading*. An actual example of structural members under axial loading is provided by the members of the bridge truss shown in Photo 1.1.

Photo 1.1 This bridge truss consists of two-force members that may be in tension or in compression.

Returning to rod BC of Fig. 1.5, we recall that the section we passed through the rod to determine the internal force in the rod and the corresponding stress was perpendicular to the axis of the rod; the internal force was therefore normal to the plane of the section (Fig. 1.7) and the corresponding stress is described as a *normal stress*. Thus, formula (1.5) gives us the *normal stress in a member under axial loading*:

$$\sigma = \frac{P}{A} \tag{1.5}$$

We should also note that, in formula (1.5), σ is obtained by dividing the magnitude P of the resultant of the internal forces distributed over the cross section by the area A of the cross section; it represents, therefore, the *average value* of the stress over the cross section, rather than the stress at a specific point of the cross section.

To define the stress at a given point Q of the cross section, we should consider a small area ΔA (Fig. 1.9). Dividing the magnitude of $\Delta\mathbf{F}$ by ΔA, we obtain the average value of the stress over ΔA. Letting ΔA approach zero, we obtain the stress at point Q:

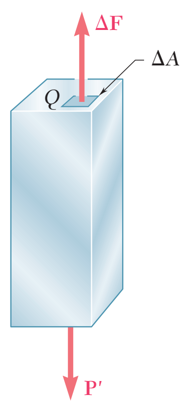


Fig. 1.9

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.6}$$

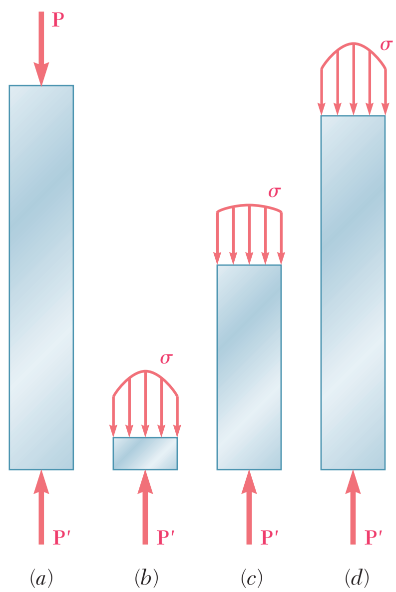


Fig. 1.10 Stress distributions at different sections along axially loaded member.

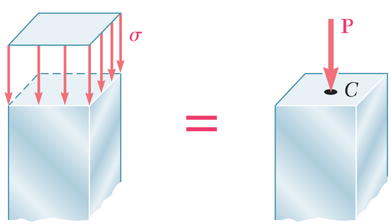


Fig. 1.11

In general, the value obtained for the stress σ at a given point Q of the section is different from the value of the average stress given by formula (1.5), and σ is found to vary across the section. In a slender rod subjected to equal and opposite concentrated loads **P** and **P**′ (Fig. 1.10*a*), this variation is small in a section away from the points of application of the concentrated loads (Fig. 1.10*c*), but it is quite noticeable in the neighborhood of these points (Fig. 1.10*b* and *d*).

It follows from Eq. (1.6) that the magnitude of the resultant of the distributed internal forces is

$$\int dF = \int_A \sigma\, dA$$

But the conditions of equilibrium of each of the portions of rod shown in Fig. 1.10 require that this magnitude be equal to the magnitude P of the concentrated loads. We have, therefore,

$$P = \int dF = \int_A \sigma\, dA \tag{1.7}$$

which means that the volume under each of the stress surfaces in Fig. 1.10 must be equal to the magnitude P of the loads. This, however, is the only information that we can derive from our knowledge of statics, regarding the distribution of normal stresses in the various sections of the rod. The actual distribution of stresses in any given section is *statically indeterminate*. To learn more about this distribution, it is necessary to consider the deformations resulting from the particular mode of application of the loads at the ends of the rod. This will be discussed further in Chap. 2.

In practice, it will be assumed that the distribution of normal stresses in an axially loaded member is uniform, except in the immediate vicinity of the points of application of the loads. The value σ of the stress is then equal to σ_{ave} and can be obtained from formula (1.5). However, we should realize that, when we assume a uniform distribution of stresses in the section, i.e., when we assume that the internal forces are uniformly distributed across the section, it follows from elementary statics† that the resultant **P** of the internal forces must be applied at the centroid C of the section (Fig. 1.11). This means that *a uniform distribution of stress is possible only if the line of action of the concentrated loads* **P** *and* **P**′ *passes through the centroid of the section considered* (Fig. 1.12). This type of loading is called *centric loading* and will be assumed to take place in all straight two-force members found in trusses and pin-connected structures, such as the one considered in Fig. 1.1. However, if a two-force member is loaded axially, but *eccentrically* as shown in Fig. 1.13*a*, we find from the conditions of equilibrium of the portion of member shown in Fig. 1.13*b* that the internal forces in a given section must be

†See Ferdinand P. Beer and E. Russell Johnston, Jr., *Mechanics for Engineers*, 5th ed., McGraw-Hill, New York, 2008, or *Vector Mechanics for Engineers*, 9th ed., McGraw-Hill, New York, 2010, Secs. 5.2 and 5.3.

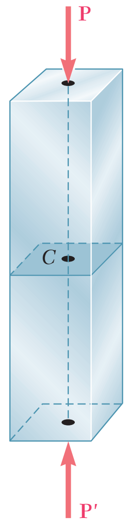


Fig. 1.12

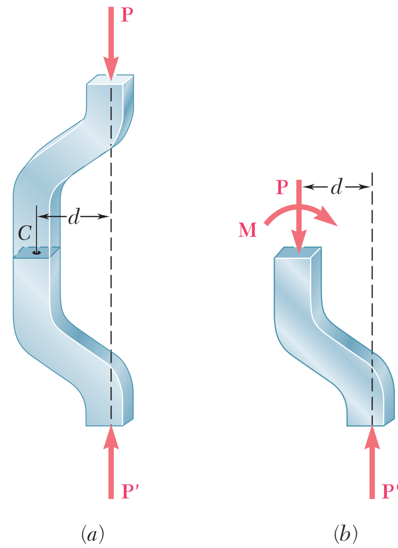


Fig. 1.13 Eccentric axial loading.

equivalent to a force **P** applied at the centroid of the section and a couple **M** of moment $M = Pd$. The distribution of forces—and, thus, the corresponding distribution of stresses—*cannot be uniform*. Nor can the distribution of stresses be symmetric as shown in Fig. 1.10. This point will be discussed in detail in Chap. 4.

1.6 SHEARING STRESS

The internal forces and the corresponding stresses discussed in Secs. 1.2 and 1.3 were normal to the section considered. A very different type of stress is obtained when transverse forces **P** and **P′** are applied to a member AB (Fig. 1.14). Passing a section at C between the points of application of the two forces (Fig. 1.15*a*), we obtain the diagram of portion AC shown in Fig. 1.15*b*. We conclude that internal forces must exist in the plane of the section, and that their resultant is equal to **P.** These elementary internal forces are called *shearing forces*, and the magnitude P of their resultant is the *shear* in the section. Dividing the shear P by the area A of the cross section, we

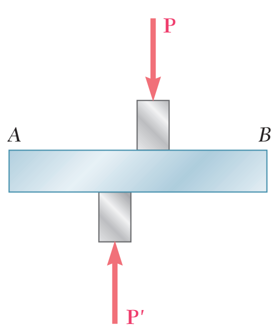


Fig. 1.14 Member with transverse loads.

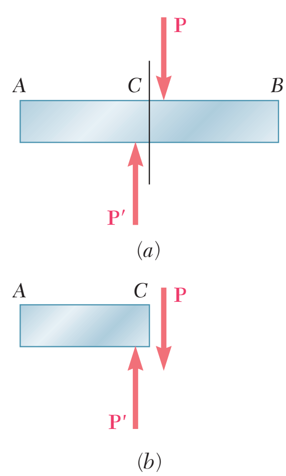


Fig. 1.15

obtain the *average shearing stress* in the section. Denoting the shearing stress by the Greek letter τ (tau), we write

$$\tau_{\text{ave}} = \frac{P}{A} \tag{1.8}$$

It should be emphasized that the value obtained is an average value of the shearing stress over the entire section. Contrary to what we said earlier for normal stresses, the distribution of shearing stresses across the section *cannot* be assumed uniform. As you will see in Chap. 6, the actual value τ of the shearing stress varies from zero at the surface of the member to a maximum value $\tau_{\max}$ that may be much larger than the average value τ_{ave}.

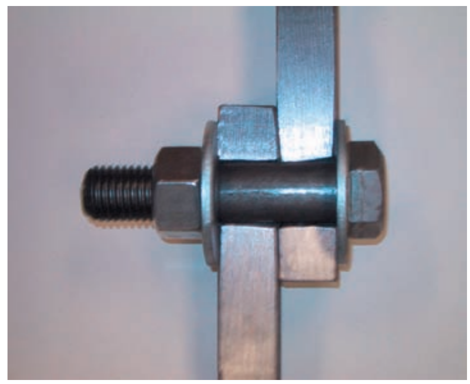


Photo 1.2 Cutaway view of a connection with a bolt in shear.

Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components (Photo 1.2). Consider the two plates A and B, which are connected by a bolt CD (Fig. 1.16). If the plates are subjected to tension forces of magnitude F, stresses will develop in the section of bolt corresponding to the plane EE'. Drawing the diagrams of the bolt and of the portion located above the plane EE' (Fig. 1.17), we conclude that the shear P in the section is equal to F. The average shearing stress in the section is obtained, according to formula (1.8), by dividing the shear $P = F$ by the area A of the cross section:

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A} \tag{1.9}$$

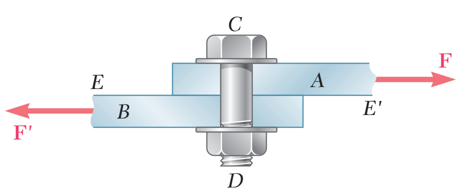


Fig. 1.16 Bolt subject to single shear.

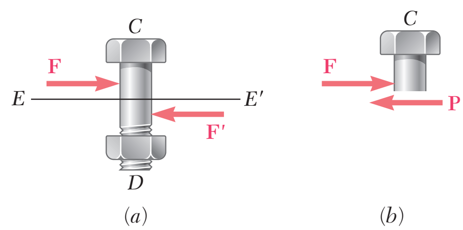


Fig. 1.17

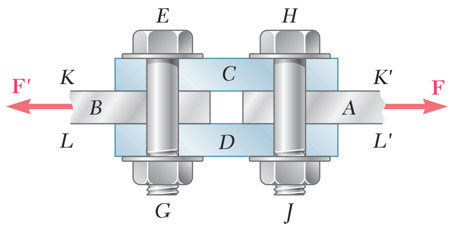


Fig. 1.18 Bolts subject to double shear.

The bolt we have just considered is said to be in *single shear*. Different loading situations may arise, however. For example, if splice plates C and D are used to connect plates A and B (Fig. 1.18), shear will take place in bolt HJ in each of the two planes KK' and LL' (and similarly in bolt EG). The bolts are said to be in *double shear*. To determine the average shearing stress in each plane, we draw free-body diagrams of bolt HJ and of the portion of bolt located between the two planes (Fig. 1.19). Observing that the shear P in each of the sections is $P = F/2$, we conclude that the average shearing stress is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \tag{1.10}$$

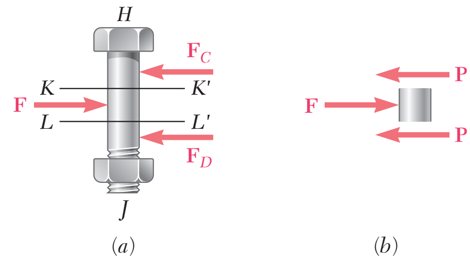


Fig. 1.19

1.7 BEARING STRESS IN CONNECTIONS

Bolts, pins, and rivets create stresses in the members they connect, along the *bearing surface,* or surface of contact. For example, consider again the two plates A and B connected by a bolt CD that we have discussed in the preceding section (Fig. 1.16). The bolt exerts on plate A a force **P** equal and opposite to the force **F** exerted by the plate on the bolt (Fig. 1.20). The force **P** represents the resultant of elementary forces distributed on the inside surface of a half-cylinder of diameter d and of length t equal to the thickness of the plate. Since the distribution of these forces—and of the corresponding stresses—is quite complicated, one uses in practice an average nominal value σ_b of the stress, called the *bearing stress,* obtained by dividing the load P by the area of the rectangle representing the projection of the bolt on the plate section (Fig. 1.21). Since this area is equal to td, where t is the plate thickness and d the diameter of the bolt, we have

$$\boxed{\sigma_b = \frac{P}{A} = \frac{P}{td}} \tag{1.11}$$

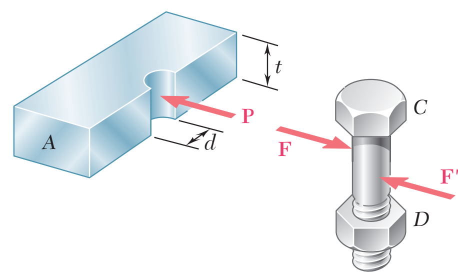


Fig. 1.20

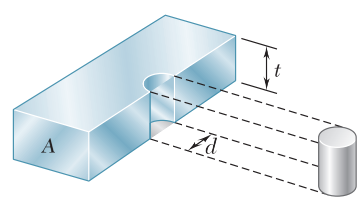


Fig. 1.21

1.8 APPLICATION TO THE ANALYSIS AND DESIGN OF SIMPLE STRUCTURES

We are now in a position to determine the stresses in the members and connections of various simple two-dimensional structures and, thus, to design such structures.

As an example, let us return to the structure of Fig. 1.1 that we have already considered in Sec. 1.2 and let us specify the supports and connections at A, B, and C. As shown in Fig. 1.22, the 20-mm-diameter rod BC has flat ends of 20×40-mm rectangular cross section, while boom AB has a 30×50-mm rectangular cross section and is fitted with a clevis at end B. Both members are connected at B by a pin from which the 30-kN load is suspended by means of a U-shaped bracket. Boom AB is supported at A by a pin fitted into a double bracket, while rod BC is connected at C to a single bracket. All pins are 25 mm in diameter.

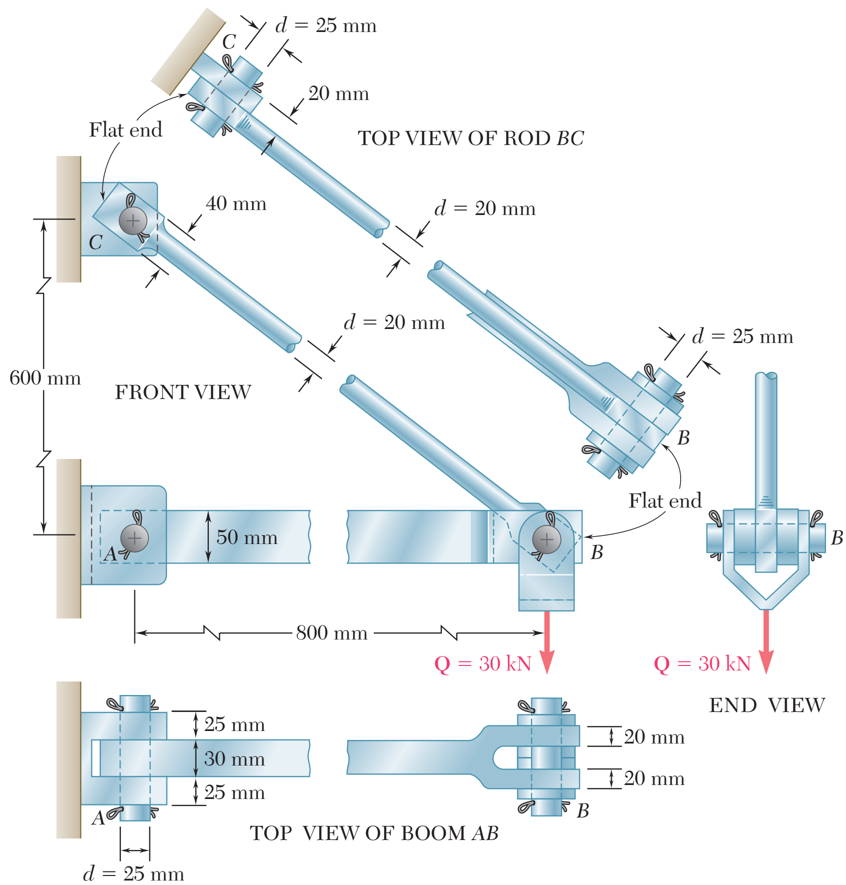


Fig. 1.22

a. Determination of the Normal Stress in Boom *AB* and Rod *BC*. As we found in Secs. 1.2 and 1.4, the force in rod BC is $F_{BC} = 50$ kN (tension) and the area of its circular cross section is $A = 314 \times 10^{-6}\ \text{m}^2$; the corresponding average normal stress is $\sigma_{BC} = +159$ MPa. However, the flat parts of the rod are also under tension and at the narrowest section, where a hole is located, we have

$$A = (20 \text{ mm})(40 \text{ mm} - 25 \text{ mm}) = 300 \times 10^{-6}\ \text{m}^2$$

The corresponding average value of the stress, therefore, is

$$(\sigma_{BC})_{\text{end}} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{300 \times 10^{-6} \text{ m}^2} = 167 \text{ MPa}$$

Note that this is an *average value*; close to the hole, the stress will actually reach a much larger value, as you will see in Sec. 2.18. It is clear that, under an increasing load, the rod will fail near one of the holes rather than in its cylindrical portion; its design, therefore, could be improved by increasing the width or the thickness of the flat ends of the rod.

Turning now our attention to boom AB, we recall from Sec. 1.2 that the force in the boom is $F_{AB} = 40$ kN (compression). Since the area of the boom's rectangular cross section is $A = 30 \text{ mm} \times 50 \text{ mm} = 1.5 \times 10^{-3} \text{ m}^2$, the average value of the normal stress in the main part of the rod, between pins A and B, is

$$\sigma_{AB} = -\frac{40 \times 10^3 \text{ N}}{1.5 \times 10^{-3} \text{ m}^2} = -26.7 \times 10^6 \text{ Pa} = -26.7 \text{ MPa}$$

Note that the sections of minimum area at A and B are not under stress, since the boom is in compression, and, therefore, *pushes* on the pins (instead of *pulling* on the pins as rod BC does).

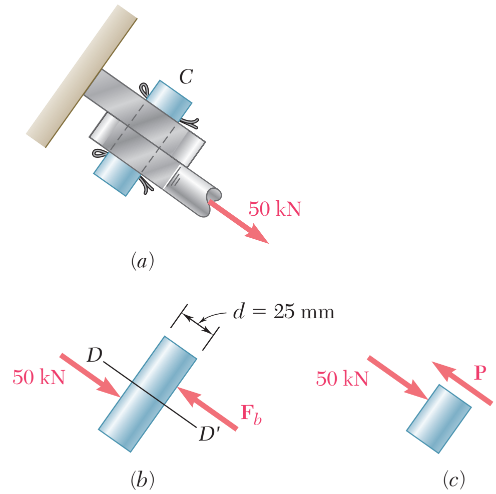


Fig. 1.23

b. Determination of the Shearing Stress in Various Connections. To determine the shearing stress in a connection such as a bolt, pin, or rivet, we first clearly show the forces exerted by the various members it connects. Thus, in the case of pin C of our example (Fig. 1.23*a*), we draw Fig. 1.23*b*, showing the 50-kN force exerted by member BC on the pin, and the equal and opposite force exerted by the bracket. Drawing now the diagram of the portion of the pin located below the plane DD' where shearing stresses occur (Fig. 1.23*c*), we conclude that the shear in that plane is $P = 50$ kN. Since the cross-sectional area of the pin is

$$A = \pi r^2 = \pi\left(\frac{25 \text{ mm}}{2}\right)^2 = \pi(12.5 \times 10^{-3} \text{ m})^2 = 491 \times 10^{-6} \text{ m}^2$$

we find that the average value of the shearing stress in the pin at C is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{491 \times 10^{-6} \text{ m}^2} = 102 \text{ MPa}$$

Considering now the pin at A (Fig. 1.24), we note that it is in double shear. Drawing the free-body diagrams of the pin and of the portion of pin located between the planes DD' and EE' where shearing stresses occur, we conclude that $P = 20$ kN and that

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{20 \text{ kN}}{491 \times 10^{-6} \text{ m}^2} = 40.7 \text{ MPa}$$

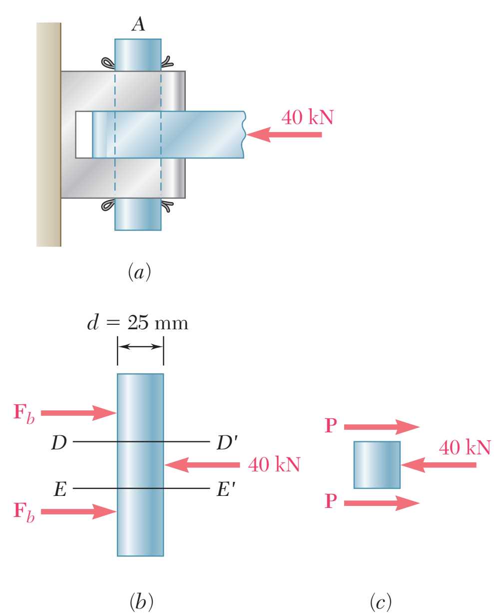


Fig. 1.24

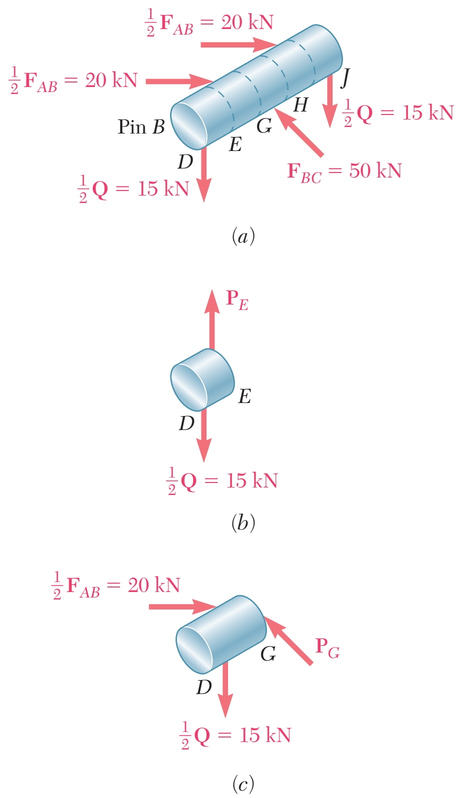


Fig. 1.25

Considering the pin at B (Fig. 1.25a), we note that the pin may be divided into five portions which are acted upon by forces exerted by the boom, rod, and bracket. Considering successively the portions DE (Fig. 1.25b) and DG (Fig. 1.25c), we conclude that the shear in section E is $P_E = 15$ kN, while the shear in section G is $P_G = 25$ kN. Since the loading of the pin is symmetric, we conclude that the maximum value of the shear in pin B is $P_G = 25$ kN, and that the largest shearing stresses occur in sections G and H, where

$$\tau_{\text{ave}} = \frac{P_G}{A} = \frac{25 \text{ kN}}{491 \times 10^{-6} \text{ m}^2} = 50.9 \text{ MPa}$$

c. Determination of the Bearing Stresses. To determine the nominal bearing stress at A in member AB, we use formula (1.11) of Sec. 1.7. From Fig. 1.22, we have $t = 30$ mm and $d = 25$ mm. Recalling that $P = F_{AB} = 40$ kN, we have

$$\sigma_b = \frac{P}{td} = \frac{40 \text{ kN}}{(30 \text{ mm})(25 \text{ mm})} = 53.3 \text{ MPa}$$

To obtain the bearing stress in the bracket at A, we use $t = 2(25 \text{ mm}) = 50$ mm and $d = 25$ mm:

$$\sigma_b = \frac{P}{td} = \frac{40 \text{ kN}}{(50 \text{ mm})(25 \text{ mm})} = 32.0 \text{ MPa}$$

The bearing stresses at B in member AB, at B and C in member BC, and in the bracket at C are found in a similar way.

1.9 METHOD OF PROBLEM SOLUTION

You should approach a problem in mechanics of materials as you would approach an actual engineering situation. By drawing on your own experience and intuition, you will find it easier to understand and formulate the problem. Once the problem has been clearly stated, however, there is no place in its solution for your particular fancy. Your solution must be based on the fundamental principles of statics and on the principles you will learn in this course. Every step you take must be justified on that basis, leaving no room for your "intuition." After an answer has been obtained, it should be checked. Here again, you may call upon your common sense and personal experience. If not completely satisfied with the result obtained, you should carefully check your formulation of the problem, the validity of the methods used in its solution, and the accuracy of your computations.

The *statement* of the problem should be clear and precise. It should contain the given data and indicate what information is required. A simplified drawing showing all essential quantities involved should be included. The solution of most of the problems you will encounter will necessitate that you first determine the *reactions at supports* and *internal forces and couples*. This will require the drawing of one or several *free-body diagrams,* as was done in Sec. 1.2, from which you will write *equilibrium equations*. These equations can be solved for the unknown forces, from which the required *stresses* and *deformations* will be computed.

After the answer has been obtained, it should be *carefully checked*. Mistakes in *reasoning* can often be detected by carrying the units through your computations and checking the units obtained for the answer. For example, in the design of the rod discussed in Sec. 1.4, we found, after carrying the units through our computations, that the required diameter of the rod was expressed in millimeters, which is the correct unit for a dimension; if another unit had been found, we would have known that some mistake had been made.

Errors in *computation* will usually be found by substituting the numerical values obtained into an equation which has not yet been used and verifying that the equation is satisfied. The importance of correct computations in engineering cannot be overemphasized.

1.10 NUMERICAL ACCURACY

The accuracy of the solution of a problem depends upon two items: (1) the accuracy of the given data and (2) the accuracy of the computations performed.

The solution cannot be more accurate than the less accurate of these two items. For example, if the loading of a beam is known to be 75,000 lb with a possible error of 100 lb either way, the relative error which measures the degree of accuracy of the data is

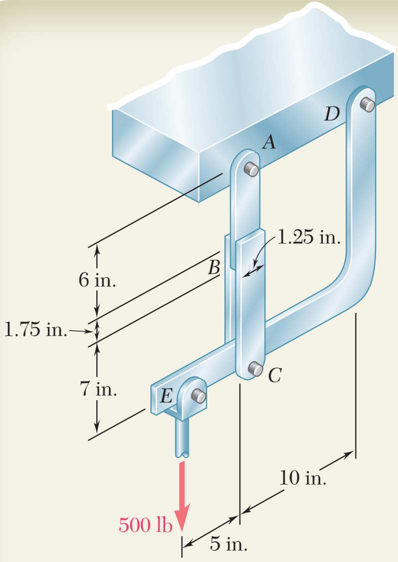
$$\frac{100 \text{ lb}}{75{,}000 \text{ lb}} = 0.0013 = 0.13\%$$

In computing the reaction at one of the beam supports, it would then be meaningless to record it as 14,322 lb. The accuracy of the solution cannot be greater than 0.13%, no matter how accurate the computations are, and the possible error in the answer may be as large as $(0.13/100)(14{,}322 \text{ lb}) \approx 20$ lb. The answer should be properly recorded as $14{,}320 \pm 20$ lb.

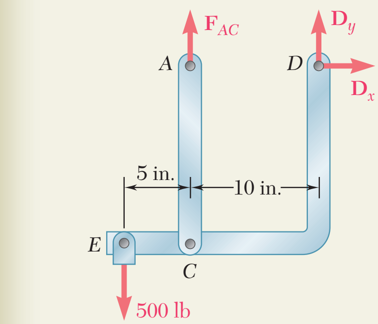
In engineering problems, the data are seldom known with an accuracy greater than 0.2%. It is therefore seldom justified to write the answers to such problems with an accuracy greater than 0.2%. A practical rule is to use 4 figures to record numbers beginning with a “1” and 3 figures in all other cases. Unless otherwise indicated, the data given in a problem should be assumed known with a comparable degree of accuracy. A force of 40 lb, for example, should be read 40.0 lb, and a force of 15 lb should be read 15.00 lb.

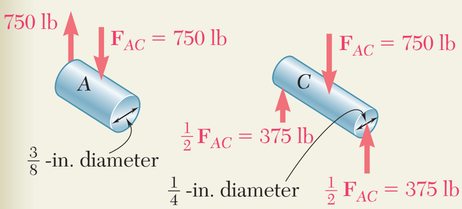
Pocket calculators and computers are widely used by practicing engineers and engineering students. The speed and accuracy of these devices facilitate the numerical computations in the solution of many problems. However, students should not record more significant figures than can be justified merely because they are easily obtained. As noted above, an accuracy greater than 0.2% is seldom necessary or meaningful in the solution of practical engineering problems.

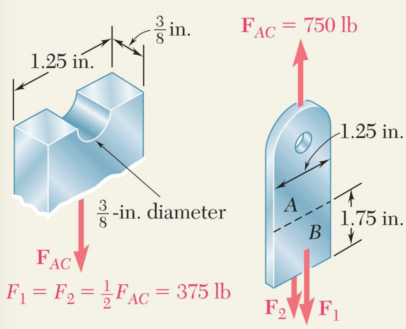
SAMPLE PROBLEM 1.1

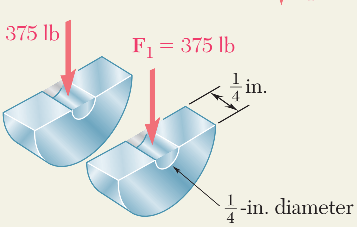


In the hanger shown, the upper portion of link ABC is $\frac{3}{8}$ in. thick and the lower portions are each $\frac{1}{4}$ in. thick. Epoxy resin is used to bond the upper and lower portions together at B. The pin at A is of $\frac{3}{8}$-in. diameter while a $\frac{1}{4}$-in.-diameter pin is used at C. Determine (*a*) the shearing stress in pin A, (*b*) the shearing stress in pin C, (*c*) the largest normal stress in link ABC, (*d*) the average shearing stress on the bonded surfaces at B, (*e*) the bearing stress in the link at C.









SOLUTION

Free Body: Entire Hanger. Since the link ABC is a two-force member, the reaction at A is vertical; the reaction at D is represented by its components $\mathbf{D}_x$ and $\mathbf{D}_y$. We write

$$+\circlearrowleft \Sigma M_D = 0: \qquad (500 \text{ lb})(15 \text{ in.}) - F_{AC}(10 \text{ in.}) = 0$$

$$F_{AC} = +750 \text{ lb} \qquad F_{AC} = 750 \text{ lb} \qquad \textit{tension}$$

a. Shearing Stress in Pin *A*. Since this $\frac{3}{8}$-in.-diameter pin is in single shear, we write

$$\tau_A = \frac{F_{AC}}{A} = \frac{750 \text{ lb}}{\frac{1}{4}\pi(0.375 \text{ in.})^2} \qquad \tau_A = 6790 \text{ psi} \blacktriangleleft$$

b. Shearing Stress in Pin *C*. Since this $\frac{1}{4}$-in.-diameter pin is in double shear, we write

$$\tau_C = \frac{\frac{1}{2}F_{AC}}{A} = \frac{375 \text{ lb}}{\frac{1}{4}\pi(0.25 \text{ in.})^2} \qquad \tau_C = 7640 \text{ psi} \blacktriangleleft$$

c. Largest Normal Stress in Link *ABC*. The largest stress is found where the area is smallest; this occurs at the cross section at A where the $\frac{3}{8}$-in. hole is located. We have

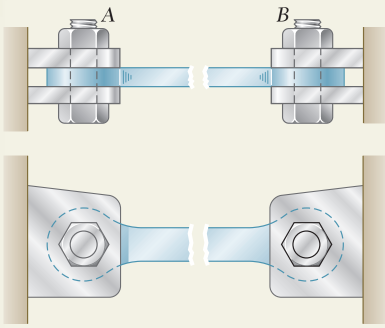
$$\sigma_A = \frac{F_{AC}}{A_{\text{net}}} = \frac{750 \text{ lb}}{(\frac{3}{8} \text{ in.})(1.25 \text{ in.} - 0.375 \text{ in.})} = \frac{750 \text{ lb}}{0.328 \text{ in}^2} \qquad \sigma_A = 2290 \text{ psi} \blacktriangleleft$$

d. Average Shearing Stress at *B*. We note that bonding exists on both sides of the upper portion of the link and that the shear force on each side is $F_1 = (750 \text{ lb})/2 = 375$ lb. The average shearing stress on each surface is thus

$$\tau_B = \frac{F_1}{A} = \frac{375 \text{ lb}}{(1.25 \text{ in.})(1.75 \text{ in.})} \qquad \tau_B = 171.4 \text{ psi} \blacktriangleleft$$

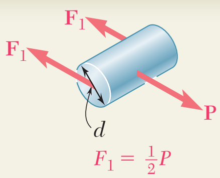
e. Bearing Stress in Link at *C*. For each portion of the link, $F_1 = 375$ lb and the nominal bearing area is $(0.25 \text{ in.})(0.25 \text{ in.}) = 0.0625 \text{ in}^2$.

$$\sigma_b = \frac{F_1}{A} = \frac{375 \text{ lb}}{0.0625 \text{ in}^2} \qquad \sigma_b = 6000 \text{ psi} \blacktriangleleft$$



SAMPLE PROBLEM 1.2

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are: $\sigma = 175$ MPa, $\tau = 100$ MPa, $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (*a*) the diameter d of the bolt, (*b*) the dimension b at each end of the bar, (*c*) the dimension h of the bar.

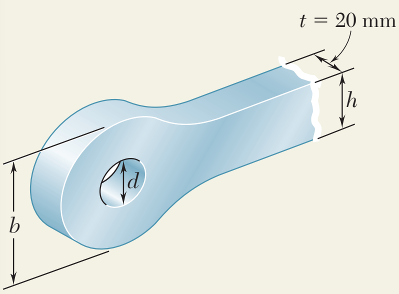


SOLUTION

a. Diameter of the Bolt. Since the bolt is in double shear, $F_1 = \frac{1}{2}P = 60$ kN.

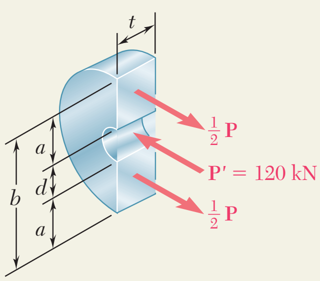
$$\tau = \frac{F_1}{A} = \frac{60\text{ kN}}{\frac{1}{4}\pi\, d^2} \qquad 100\text{ MPa} = \frac{60\text{ kN}}{\frac{1}{4}\pi\, d^2} \qquad d = 27.6\text{ mm}$$

We will use $\quad d = 28\text{ mm}$ ◀



At this point we check the bearing stress between the 20-mm-thick plate and the 28-mm-diameter bolt.

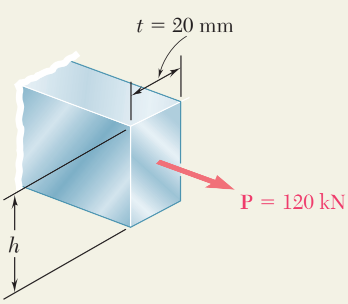
$$\tau_b = \frac{P}{td} = \frac{120\text{ kN}}{(0.020\text{ m})(0.028\text{ m})} = 214\text{ MPa} < 350\text{ MPa} \qquad \text{OK}$$



b. Dimension *b* at Each End of the Bar. We consider one of the end portions of the bar. Recalling that the thickness of the steel plate is $t = 20$ mm and that the average tensile stress must not exceed 175 MPa, we write

$$\sigma = \frac{\frac{1}{2}P}{ta} \qquad 175\text{ MPa} = \frac{60\text{ kN}}{(0.02\text{ m})a} \qquad a = 17.14\text{ mm}$$

$$b = d + 2a = 28\text{ mm} + 2(17.14\text{ mm}) \qquad b = 62.3\text{ mm} \;◀$$



c. Dimension *h* of the Bar. Recalling that the thickness of the steel plate is $t = 20$ mm, we have

$$\sigma = \frac{P}{th} \qquad 175\text{ MPa} = \frac{120\text{ kN}}{(0.020\text{ m})h} \qquad h = 34.3\text{ mm}$$

We will use $\quad h = 35\text{ mm}$ ◀

PROBLEMS

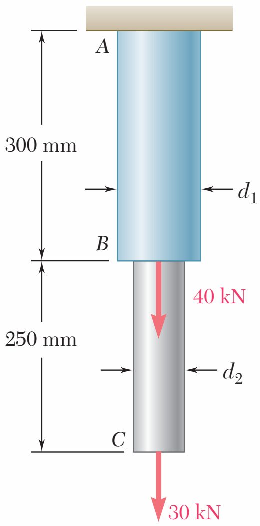


Fig. P1.1 and P1.2

1.1 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod *AB* and 150 MPa in rod *BC*, determine the smallest allowable values of d_1 and d_2.

1.2 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* has the same magnitude as the compressive stress in rod *BC*.

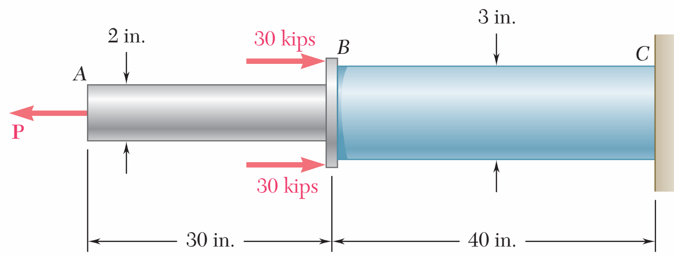


Fig. P1.3

1.4 In Prob. 1.3, knowing that $P = 40$ kips, determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

1.5 Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

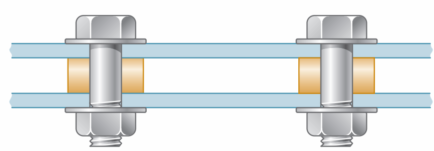


Fig. *P1.5*

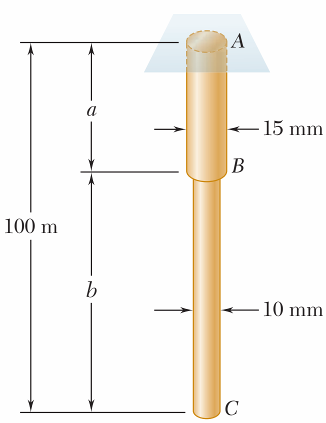


Fig. *P1.6*

1.6 Two brass rods *AB* and *BC*, each of uniform diameter, will be brazed together at *B* to form a nonuniform rod of total length 100 m which will be suspended from a support at *A* as shown. Knowing that the density of brass is 8470 kg/m^3, determine (*a*) the length of rod *AB* for which the maximum normal stress in *ABC* is minimum, (*b*) the corresponding value of the maximum normal stress.

1.7 Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

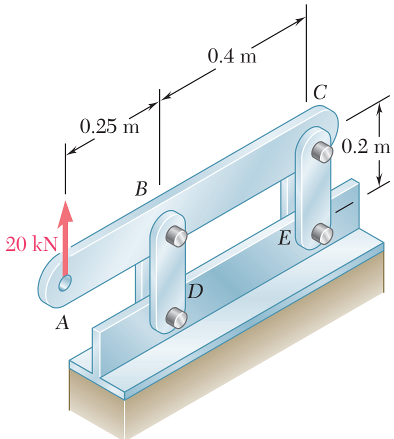


Fig. P1.7

1.8 Knowing that link *DE* is $\frac{1}{8}$ in. thick and 1 in. wide, determine the normal stress in the central portion of that link when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

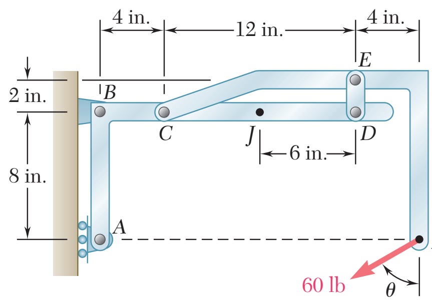


Fig. P1.8

1.9 Link *AC* has a uniform rectangular cross section $\frac{1}{16}$ in. thick and $\frac{1}{4}$ in. wide. Determine the normal stress in the central portion of the link.

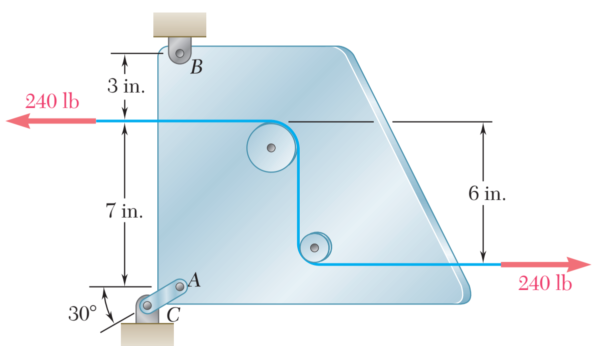


Fig. P1.9

1.10 Three forces, each of magnitude $P = 4$ kN, are applied to the mechanism shown. Determine the cross-sectional area of the uniform portion of rod *BE* for which the normal stress in that portion is $+100$ MPa.

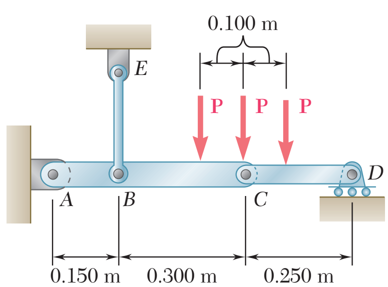


Fig. P1.10

1.11 The frame shown consists of *four* wooden members, *ABC*, *DEF*, *BE*, and *CF*. Knowing that each member has a 2 × 4-in. rectangular cross section and that each pin has a $\frac{1}{2}$-in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.

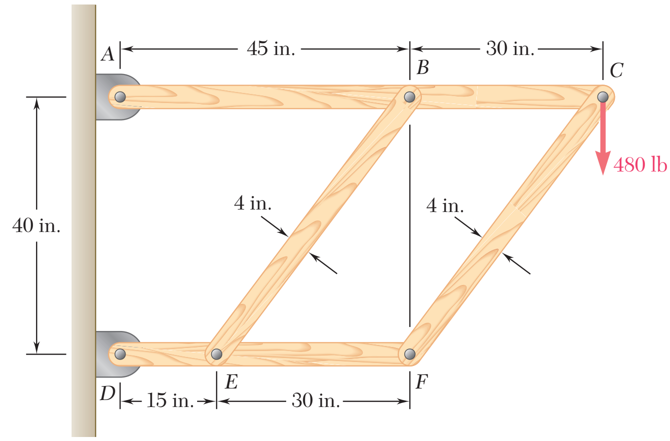


Fig. *P1.11*

1.12 For the Pratt bridge truss and loading shown, determine the average normal stress in member *BE*, knowing that the cross-sectional area of that member is 5.87 in^2.

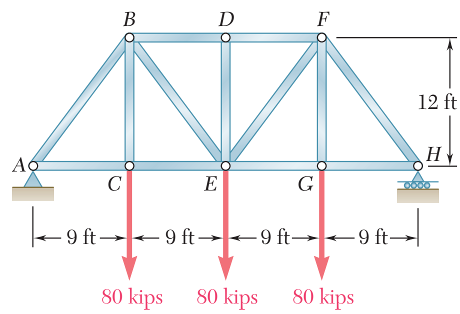


Fig. *P1.12*

1.13 An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

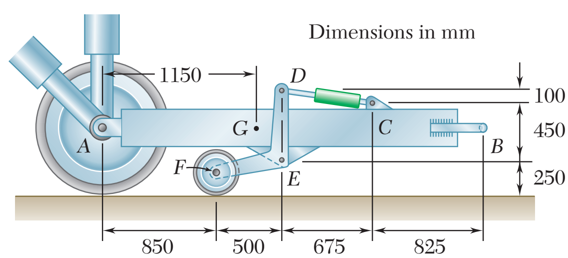


Fig. P1.13

1.14 A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.

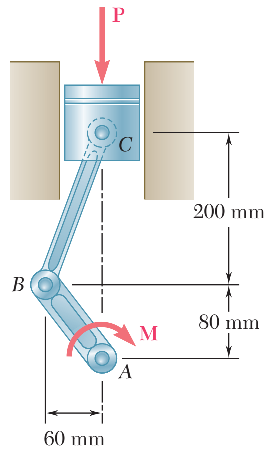


Fig. P1.14

1.15 When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

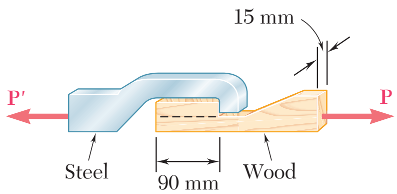


Fig. P1.15

1.16 The wooden members *A* and *B* are to be joined by plywood splice plates that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be $\frac{1}{4}$ in., determine the smallest allowable length *L* if the average shearing stress in the glue is not to exceed 120 psi.

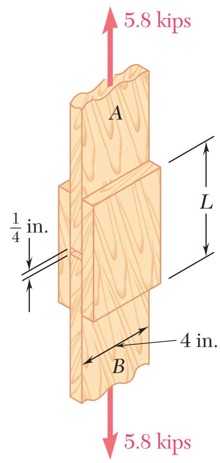


Fig. P1.16

1.17 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load **P** that can be applied to the rod.

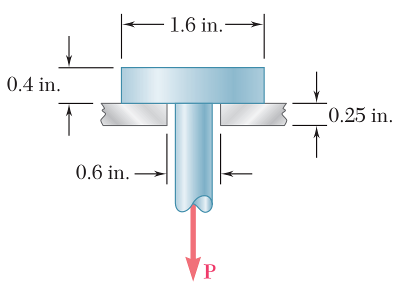


Fig. *P1.17*

1.18 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length *d* of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.

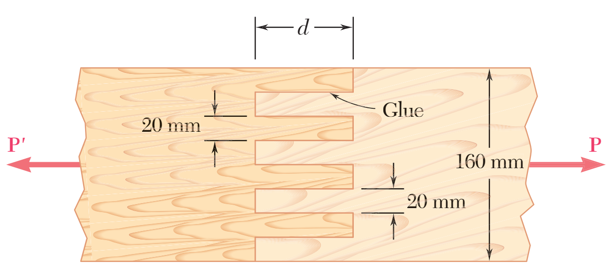


Fig. P1.18

1.19 The load **P** applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

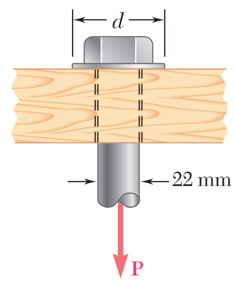


Fig. P1.19

1.20 The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

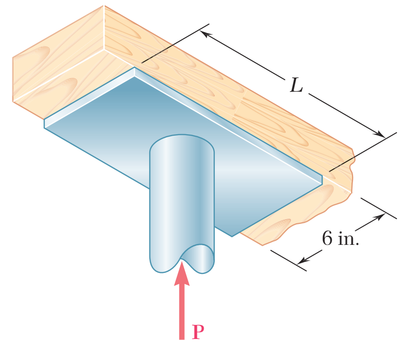


Fig. *P1.20*

1.21 An axial load **P** is supported by a short W8 × 40 column of cross-sectional area $A = 11.7\ \text{in}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side a of the plate that will provide the most economical and safe design.

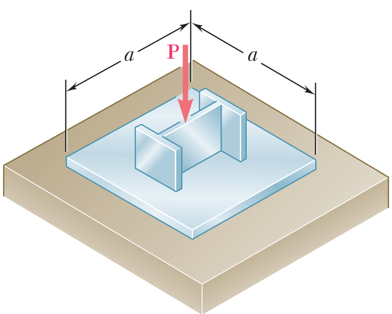


Fig. P1.21

1.22 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

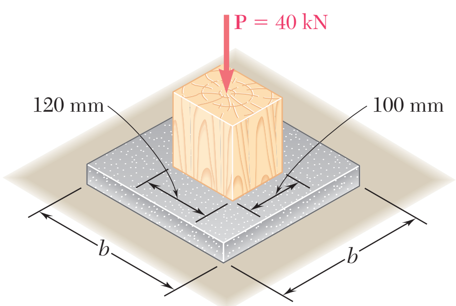


Fig. P1.22

1.23 A $\frac{5}{8}$-in.-diameter steel rod AB is fitted to a round hole near end C of the wooden member CD. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance b for which the average shearing stress is 100 psi on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

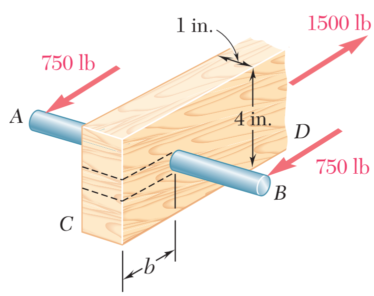


Fig. P1.23

1.24 Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at *B* if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member *AB* at *B*, (*c*) the corresponding average bearing stress in each of the support brackets at *B*.

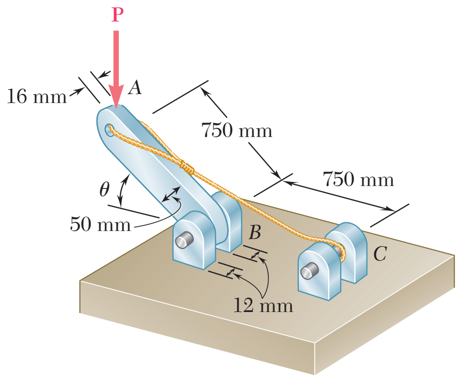


Fig. P1.24 and P1.25

1.25 Determine the largest load **P** that can be applied at *A* when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at *B* must not exceed 120 MPa and that the average bearing stress in member *AB* and in the bracket at *B* must not exceed 90 MPa.

1.26 Link *AB*, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (*a*) the diameter *d* of the pins, (*b*) the average bearing stress in the link.

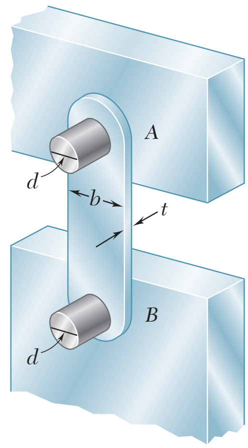


Fig. P1.26

1.27 For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10×50-mm uniform rectangular cross section.

1.28 The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is $\frac{5}{8}$ in. thick and is connected to the vertical rod by a $\frac{3}{8}$-in.-diameter bolt. Determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.

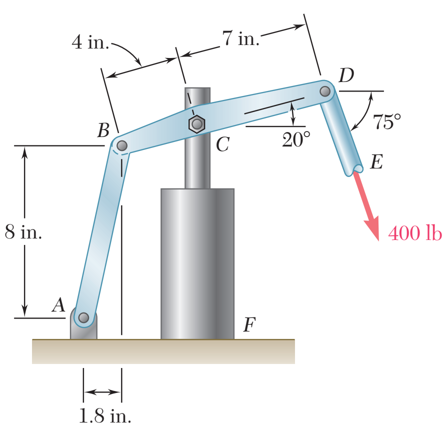


Fig. P1.28

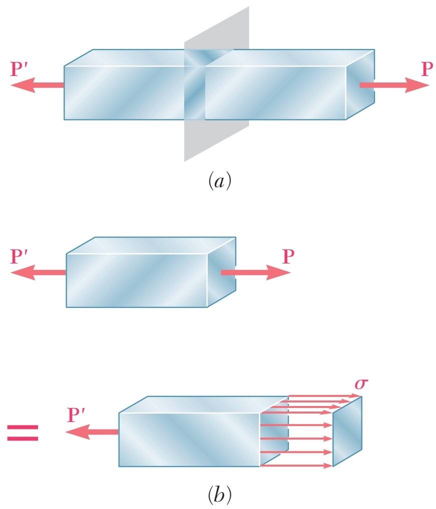


Fig. 1.26 Axial forces.

1.11 STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING

In the preceding sections, axial forces exerted on a two-force member (Fig. 1.26*a*) were found to cause normal stresses in that member (Fig. 1.26*b*), while transverse forces exerted on bolts and pins (Fig. 1.27*a*) were found to cause shearing stresses in those connections (Fig. 1.27*b*). The reason such a relation was observed between axial forces and normal stresses on one hand, and transverse forces and shearing stresses on the other, was because stresses were being determined only on planes perpendicular to the axis of the member or connection. As you will see in this section, axial forces cause both normal and shearing stresses on planes which are not perpendicular to the axis of the member. Similarly, transverse forces exerted on a bolt or a pin cause both normal and shearing stresses on planes which are not perpendicular to the axis of the bolt or pin.

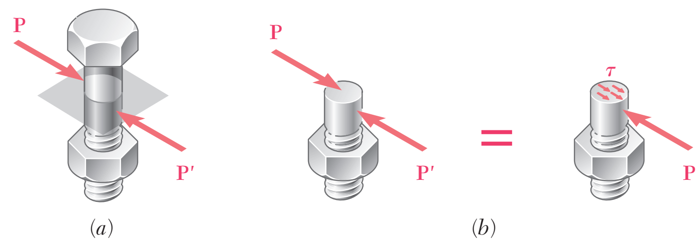


Fig. 1.27 Transverse forces.

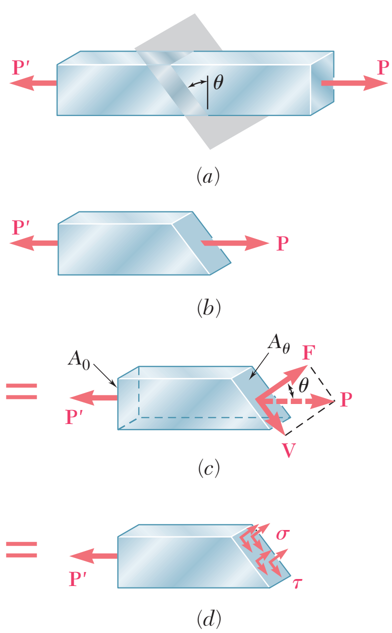


Fig. 1.28

Consider the two-force member of Fig. 1.26, which is subjected to axial forces **P** and **P**′. If we pass a section forming an angle θ with a normal plane (Fig. 1.28*a*) and draw the free-body diagram of the portion of member located to the left of that section (Fig. 1.28*b*), we find from the equilibrium conditions of the free body that the distributed forces acting on the section must be equivalent to the force **P.**

Resolving **P** into components **F** and **V,** respectively normal and tangential to the section (Fig. 1.28*c*), we have

$$F = P\cos\theta \qquad V = P\sin\theta \tag{1.12}$$

The force **F** represents the resultant of normal forces distributed over the section, and the force **V** the resultant of shearing forces (Fig. 1.28*d*). The average values of the corresponding normal and shearing stresses are obtained by dividing, respectively, F and V by the area A_θ of the section:

$$\sigma = \frac{F}{A_\theta} \qquad \tau = \frac{V}{A_\theta} \tag{1.13}$$

Substituting for F and V from (1.12) into (1.13), and observing from Fig. 1.28*c* that $A_0 = A_\theta \cos\theta$, or $A_\theta = A_0/\cos\theta$, where A_0 denotes