SI EDITION

EIGHTH EDITION

MECHANICS OF MATERIALS

JAMES M. GERE
BARRY J. GOODNO

CONVERSIONS BETWEEN U.S. CUSTOMARY UNITS AND SI UNITS

U.S. Customary unit		Times conversion factor		Equals SI unit	
		Accurate	Practical		
Acceleration (linear)					
foot per second squared	ft/s^2	0.3048*	0.305	meter per second squared	m/s^2
inch per second squared	in./s^2	0.0254*	0.0254	meter per second squared	m/s^2
Area					
circular mil	cmil	0.0005067	0.0005	square millimeter	mm^2
square foot	ft^2	0.09290304*	0.0929	square meter	m^2
square inch	in.^{2}	645.16*	645	square millimeter	mm^2
Density (mass)					
slug per cubic foot	slug/ft^3	515.379	515	kilogram per cubic meter	kg/m^3
Density (weight)					
pound per cubic foot	lb/ft^3	157.087	157	newton per cubic meter	N/m^3
pound per cubic inch	lb/in.^{3}	271.447	271	kilonewton per cubic meter	kN/m^3
Energy; work					
foot-pound	ft-lb	1.35582	1.36	joule (N·m)	J
inch-pound	in.-lb	0.112985	0.113	joule	J
kilowatt-hour	kWh	3.6*	3.6	megajoule	MJ
British thermal unit	Btu	1055.06	1055	joule	J
Force					
pound	lb	4.44822	4.45	newton (kg·m/s^2)	N
kip (1000 pounds)	k	4.44822	4.45	kilonewton	kN
Force per unit length					
pound per foot	lb/ft	14.5939	14.6	newton per meter	N/m
pound per inch	lb/in.	175.127	175	newton per meter	N/m
kip per foot	k/ft	14.5939	14.6	kilonewton per meter	kN/m
kip per inch	k/in.	175.127	175	kilonewton per meter	kN/m
Length					
foot	ft	0.3048*	0.305	meter	m
inch	in.	25.4*	25.4	millimeter	mm
mile	mi	1.609344*	1.61	kilometer	km
Mass					
slug	lb-s^2/ft	14.5939	14.6	kilogram	kg
Moment of a force; torque					
pound-foot	lb-ft	1.35582	1.36	newton meter	N·m
pound-inch	lb-in.	0.112985	0.113	newton meter	N·m
kip-foot	k-ft	1.35582	1.36	kilonewton meter	kN·m
kip-inch	k-in.	0.112985	0.113	kilonewton meter	kN·m

SELECTED PHYSICAL PROPERTIES

Property	SI	USCS
Water (fresh)		
weight density	9.81 kN/m^3	62.4 lb/ft^3
mass density	1000 kg/m^3	1.94 slugs/ft^3
Sea water		
weight density	10.0 kN/m^3	63.8 lb/ft^3
mass density	1020 kg/m^3	1.98 slugs/ft^3
Aluminum (structural alloys)		
weight density	28 kN/m^3	175 lb/ft^3
mass density	2800 kg/m^3	5.4 slugs/ft^3
Steel		
weight density	77.0 kN/m^3	490 lb/ft^3
mass density	7850 kg/m^3	15.2 slugs/ft^3
Reinforced concrete		
weight density	24 kN/m^3	150 lb/ft^3
mass density	2400 kg/m^3	4.7 slugs/ft^3
Atmospheric pressure (sea level)		
Recommended value	101 kPa	14.7 psi
Standard international value	101.325 kPa	14.6959 psi
Acceleration of gravity (sea level, approx. 45° latitude)		
Recommended value	9.81 m/s^2	32.2 ft/s^2
Standard international value	9.80665 m/s^2	32.1740 ft/s^2

SI PREFIXES

Prefix	Symbol	Multiplication factor		
tera	T	10^{12}	=	1 000 000 000 000
giga	G	10^{9}	=	1 000 000 000
mega	M	10^{6}	=	1 000 000
kilo	k	10^{3}	=	1 000
hecto	h	10^{2}	=	100
deka	da	10^{1}	=	10
deci	d	10^{-1}	=	0.1
centi	c	10^{-2}	=	0.01
milli	m	10^{-3}	=	0.001
micro	μ	10^{-6}	=	0.000 001
nano	n	10^{-9}	=	0.000 000 001
pico	p	10^{-12}	=	0.000 000 000 001

Note: The use of the prefixes hecto, deka, deci, and centi is not recommended in SI.

CONVERSIONS BETWEEN U.S. CUSTOMARY UNITS AND SI UNITS (Continued)

U.S. Customary unit		Times conversion factor		Equals SI unit	
		Accurate	Practical		
Moment of inertia (area)					
inch to fourth power	$in.^4$	416,231	416,000	millimeter to fourth power	mm^4
inch to fourth power	$in.^4$	0.416231×10^{-6}	0.416×10^{-6}	meter to fourth power	m^4
Moment of inertia (mass)					
slug foot squared	$slug\text{-}ft^2$	1.35582	1.36	kilogram meter squared	$kg{\cdot}m^2$
Power					
foot-pound per second	ft-lb/s	1.35582	1.36	watt (J/s or N·m/s)	W
foot-pound per minute	ft-lb/min	0.0225970	0.0226	watt	W
horsepower (550 ft-lb/s)	hp	745.701	746	watt	W
Pressure; stress					
pound per square foot	psf	47.8803	47.9	pascal (N/m^2)	Pa
pound per square inch	psi	6894.76	6890	pascal	Pa
kip per square foot	ksf	47.8803	47.9	kilopascal	kPa
kip per square inch	ksi	6.89476	6.89	megapascal	MPa
Section modulus					
inch to third power	$in.^3$	16,387.1	16,400	millimeter to third power	mm^3
inch to third power	$in.^3$	16.3871×10^{-6}	16.4×10^{-6}	meter to third power	m^3
Velocity (linear)					
foot per second	ft/s	0.3048*	0.305	meter per second	m/s
inch per second	in./s	0.0254*	0.0254	meter per second	m/s
mile per hour	mph	0.44704*	0.447	meter per second	m/s
mile per hour	mph	1.609344*	1.61	kilometer per hour	km/h
Volume					
cubic foot	ft^3	0.0283168	0.0283	cubic meter	m^3
cubic inch	$in.^3$	16.3871×10^{-6}	16.4×10^{-6}	cubic meter	m^3
cubic inch	$in.^3$	16.3871	16.4	cubic centimeter (cc)	cm^3
gallon (231 $in.^3$)	gal.	3.78541	3.79	liter	L
gallon (231 $in.^3$)	gal.	0.00378541	0.00379	cubic meter	m^3

*An asterisk denotes an *exact* conversion factor

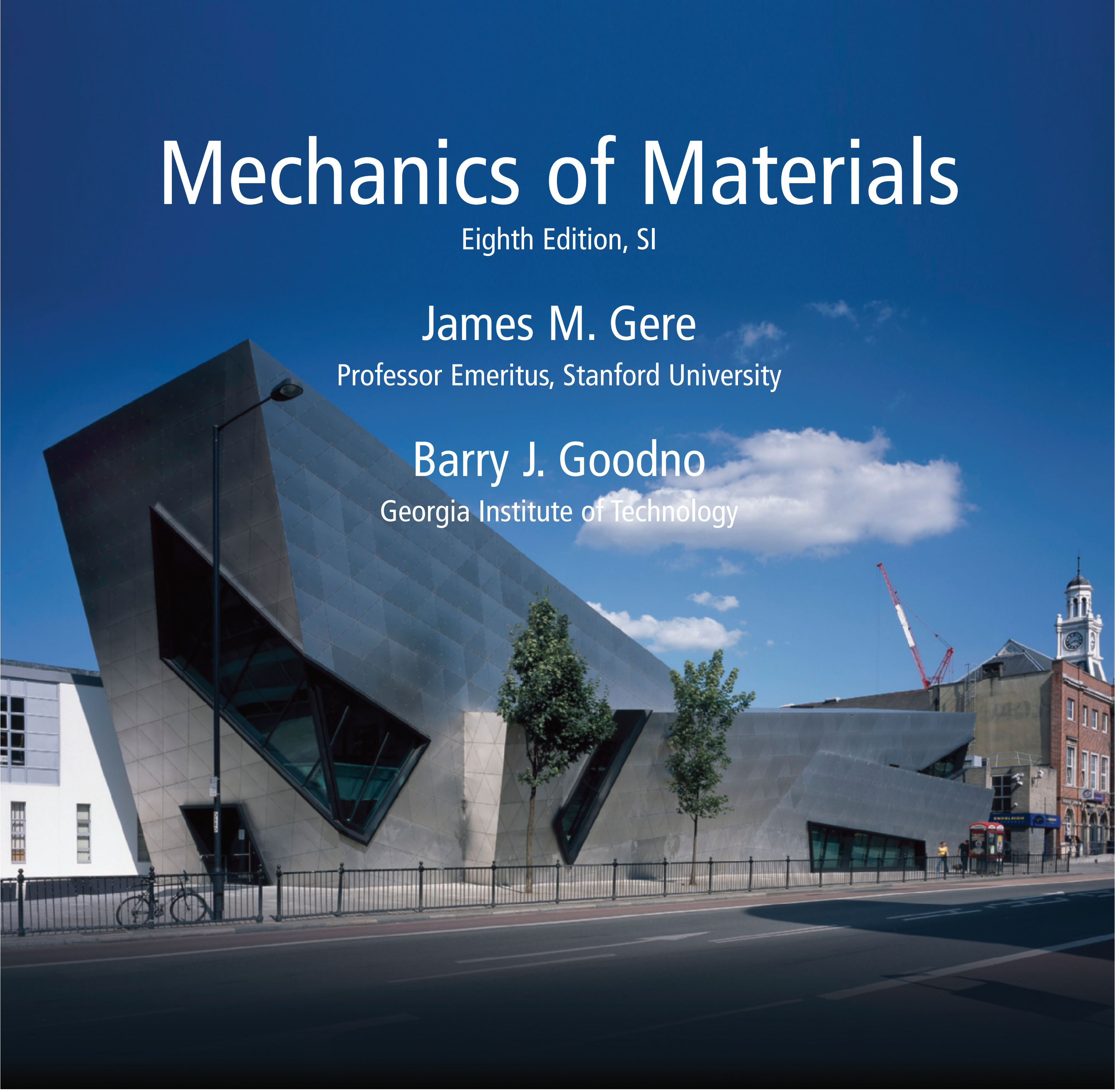
***Note:* To convert from SI units to USCS units, *divide* by the conversion factor**

Temperature Conversion Formulas

$$T(°C) = \frac{5}{9}[T(°F) - 32] = T(K) - 273.15$$

$$T(K) = \frac{5}{9}[T(°F) - 32] + 273.15 = T(°C) + 273.15$$

$$T(°F) = \frac{9}{5}T(°C) + 32 = \frac{9}{5}T(K) - 459.67$$



Mechanics of Materials

Eighth Edition, SI

James M. Gere

Professor Emeritus, Stanford University

Barry J. Goodno

Georgia Institute of Technology

CENGAGE Learning®

Australia • Brazil • Japan • Korea • Mexico • Singapore • Spain • United Kingdom • United States

CENGAGE
Learning

Mechanics of Materials, Eighth Edition, SI

James M. Gere and Barry J. Goodno

Publisher, Global Engineering: Christopher M. Shortt

Senior Acquisitions Editor: Randall Adams

Senior Developmental Editor: Hilda Gowans

Editorial Assistant: Tanya Altieri

Team Assistant: Carly Rizzo

Marketing Manager: Lauren Betsos

Media Editor: Chris Valentine

Manager, Content and Media Production: Patricia M. Boies

Senior Content Project Manager: Jennifer Ziegler

Production Service: RPK Editorial Services, Inc.

Copyeditor: Shelly Gerger-Knechtl

Proofreader: Martha McMaster

Indexer: Paul Mailhot

Compositor: Integra

Senior Art Director: Michelle Kunkler

Internal Designer: Juli Cook/Plan-It-Publishing

Cover Designer: Andrew Adams/4065042 Canada Inc.

Cover Image: © Arcaid/Corbis

Rights Acquisitions Specialist: Deanna Ettinger

Text and Image Permissions Researcher: Kristiina Paul

Manufacturing Planner: Doug Wilke

Library of Congress Control Number: 2012930214

ISBN-13: 978-1-111-57774-2

ISBN-10: 1-111-57774-9

Cengage Learning
200 First Stamford Place, Suite 400
Stamford, CT 06902
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at: **international.cengage.com/region**.

Cengage Learning products are represented in Canada by Nelson Education Ltd.

For your course and learning solutions, visit **www.cengage.com/engineering**.

Purchase any of our products at your local college store or at our preferred online store **www.cengagebrain.com**.

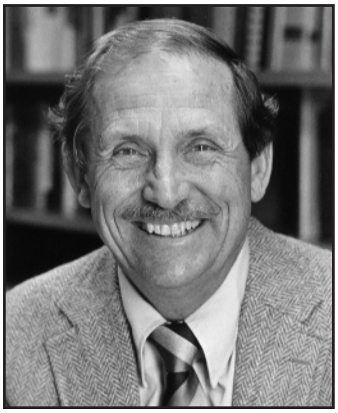
Printed in Canada
1 2 3 4 5 6 7 16 15 14 13 12

CONTENTS

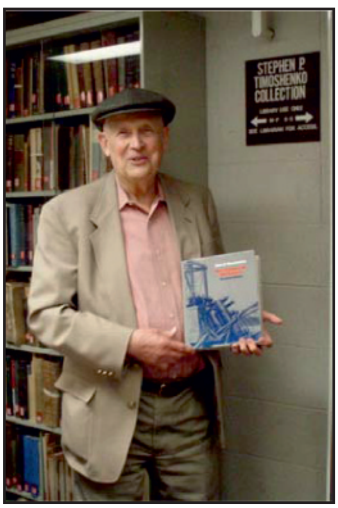
*Specialized and/or advanced topics

JAMES MONROE GERE

1925–2008



(Ed Souza/Stanford News Service)



Jim Gere in the Timoshenko Library at Stanford holding a copy of the 2nd edition of this text (photo courtesy of Richard Weingardt Consultants, Inc.)

James Monroe Gere, Professor Emeritus of Civil Engineering at Stanford University, died in Portola Valley, CA, on January 30, 2008. Jim Gere was born on June 14, 1925, in Syracuse, NY. He joined the U.S. Army Air Corps at age 17 in 1942, serving in England, France and Germany. After the war, he earned undergraduate and master's degrees in Civil Engineering from the Rensselaer Polytechnic Institute in 1949 and 1951, respectively. He worked as an instructor and later as a Research Associate for Rensselaer between 1949 and 1952. He was awarded one of the first NSF Fellowships, and chose to study at Stanford. He received his Ph.D. in 1954 and was offered a faculty position in Civil Engineering, beginning a 34-year career of engaging his students in challenging topics in mechanics, and structural and earthquake engineering. He served as Department Chair and Associate Dean of Engineering and in 1974 co-founded the John A. Blume Earthquake Engineering Center at Stanford. In 1980, Jim Gere also became the founding head of the Stanford Committee on Earthquake Preparedness, which urged campus members to brace and strengthen office equipment, furniture, and other items that could pose a life safety hazard in the event of an earthquake. That same year, he was invited as one of the first foreigners to study the earthquake-devastated city of Tangshan, China. Jim retired from Stanford in 1988 but continued to be a most valuable member of the Stanford community as he gave freely of his time to advise students and to guide them on various field trips to the California earthquake country.

Jim Gere was known for his outgoing manner, his cheerful personality and wonderful smile, his athleticism, and his skill as an educator in Civil Engineering. He authored nine textbooks on various engineering subjects starting in 1972 with *Mechanics of Materials*, a text that was inspired by his teacher and mentor Stephan P. Timoshenko. His other well-known textbooks, used in engineering courses around the world, include: *Theory of Elastic Stability*, co-authored with S. Timoshenko; *Matrix Analysis of Framed Structures* and *Matrix Algebra for Engineers*, both co-authored with W. Weaver; *Moment Distribution*; *Earthquake Tables: Structural and Construction Design Manual*, co-authored with H. Krawinkler; and *Terra Non Firma: Understanding and Preparing for Earthquakes*, co-authored with H. Shah.

Respected and admired by students, faculty, and staff at Stanford University, Professor Gere always felt that the opportunity to work with and be of service to young people both inside and outside the classroom was one of his great joys. He hiked frequently and regularly visited Yosemite and the Grand Canyon national parks. He made over 20 ascents of Half Dome in Yosemite as well as "John Muir hikes" of up to 50 miles in a day. In 1986 he hiked to the base camp of Mount Everest, saving the life of a companion on the trip. James was an active runner and completed the Boston Marathon at age 48, in a time of 3:13.

James Gere will be long remembered by all who knew him as a considerate and loving man whose upbeat good humor made aspects of daily life or work easier to bear. His last project (in progress and now being continued by his daughter Susan of Palo Alto) was a book based on the written memoirs of his great-grandfather, a Colonel (122d NY) in the Civil War.

PREFACE TO THE SI EDITION

Mechanics of materials is a basic engineering subject that, along with statics, must be understood by anyone concerned with the strength and physical performance of structures, whether those structures are man-made or natural. At the college level, statics is usually taught during the sophomore or junior year and is a prerequisite for the follow-on course in mechanics of materials. Both courses are required for most students majoring in mechanical, structural, civil, biomedical, petroleum, nuclear, aeronautical, and aerospace engineering. Furthermore, many students from such diverse fields as materials science, industrial engineering, architecture, and agricultural engineering also find it useful to study mechanics of materials.

A First Course in Mechanics of Materials

In many university engineering programs today, both statics and mechanics of materials are now taught in large sections comprised of students from the variety of engineering disciplines listed above. Instructors for the various parallel sections must cover the same material, and all of the major topics must be presented so that students are well prepared for the more advanced courses required by their specific degree programs. An essential prerequisite for success in a first course in mechanics of materials is a strong foundation in statics, which includes not only understanding of fundamental concepts but also proficiency in applying the laws of statical equilibrium to solution of both two and three dimensional problems. This eighth edition begins with a new section on review of statics in which the laws of equilibrium and boundary (or support) conditions are reviewed, as well as types of applied forces and internal stress resultants, all based upon and derived from a properly drawn free body diagram. Numerous examples and end of chapter problems are included to help the student review the analysis of plane and space trusses, shafts in torsion, beams and plane and space frames and to reinforce basic concepts learned in the prerequisite course.

Many instructors like to present the basic theory of say, beam bending, and then use real world examples to motivate student interest in the subject of beam flexure, beam design, etc. In many cases, structures on campus offer easy access to beams, frames, and bolted connections which can be dissected in lecture, or on homework problems, to find reactions at supports, forces and moments in members and stresses in connections. In addition, study of causes of failures in structures and components also offers the opportunity for students to begin the process of learning from actual designs and even past engineering mistakes. A number of the new example problems and also the new or revised end-of-chapter problems in this eighth edition are based upon actual components or structures and are accompanied by photographs so that the student can see the real world problem alongside the simplified mechanics model and free body diagrams to be used in its analysis.

An increasing number of universities are using rich media lecture (and/or classroom) capture software in their large undergraduate courses in mathematics, physics, and engineering and the many new photos and enhanced graphics in the eighth edition are designed to support this enhanced lecture mode.

NEW TO THE EIGHTH EDITION OF *MECHANICS OF MATERIALS*, SI EDITION

The main topics covered in this book are the analysis and design of structural members subjected to tension, compression, torsion, and bending, including the fundamental concepts mentioned above. Other important topics are the transformations of stress and strain, combined loadings and combined stress, deflections of beams, and stability of columns. Some additional specialized topics include the following: stress concentrations, dynamic and impact loadings, non-prismatic members, shear centers, bending of beams of two materials (or composite beams), bending of unsymmetric beams, maximum stresses in beams, energy based approaches for computing deflections of beams, and statically indeterminate beams. Review material on centroids and moments of inertia is presented in Chapter 12.

As an aid to the student reader, each chapter begins with a *Chapter Overview* which highlights the major topics to be covered in that chapter, and closes with a *Chapter Summary & Review* in which the key points as well as major mathematical formulas presented in the chapter are listed for quick review (in preparation for examinations on the material). Each chapter also opens with a photograph of a component or structure which illustrates the key concepts to be discussed in that chapter.

Some of the notable features of this eighth edition, which have been added as new or updated material to meet the needs of a modern course in mechanics of materials, are as follows:

- **Statics review**—A new section entitled *Statics Review* has been added to Chapter 1. New Section 1.2 includes four example problems which illustrate calculation of support reactions and internal stress resultants for truss, beam, circular shaft and plane frame structures. Twenty six end-of-chapter problems on statics provide the student with two and three dimensional structures to be used as practice, review or homework assignment problems of varying difficulty.
- **Expanded** *Chapter Overview* and also *Chapter Summary & Review sections*– The *Chapter Overview* and *Chapter Summary* sections have been expanded and now include key equations presented in that chapter. These summary sections will serve as a convenient review for the student of key topics and equations presented in each chapter.
- **Increased emphasis** on equilibrium, constitutive, and strain-displacement/compatibility equations in problem solutions–Example problem and end-of-chapter problem solutions have been updated to emphasize an orderly process of explicitly writing out the equilibrium, constitutive and strain-displacement/compatibility equations before attempting a solution.
- New/**expanded topic coverage—**The following topics have been added or have received expanded coverage: stress concentrations in axially loads bars (Sec. 2.10); torsion of noncircular shafts (Sec. 3.10); stress concentrations in bending (Sec. 5.13); and transformed section analysis for composite beams (Sec. 6.3).
- **New example and end-of-chapter problems**—Forty-eight new example problems have been added to the eighth edition. In addition, close to 800 of the end-of-chapter problems are new or revised, out of a total of almost 1200 problems.

- **Review problems**—A total of one hundred and nineteen **review problems** have been added at the ends of chapters 1 to 11. The student must select from 4 available answers (A, B, C or D), only one of which is the correct answer. The correct answer choices are listed in the Answers section at the back of this text, and the detailed solution for each problem is available on the student website. Solution of these problems will provide the student with a quick check on his or her mastery of the subject matter presented in that chapter.

EXAMPLES

Examples are presented throughout the book to illustrate the theoretical concepts and show how those concepts may be used in practical situations. In some cases, new photographs have been added showing actual engineering structures or components to reinforce the tie between theory and application. In both lecture and text examples, it is appropriate to begin with simplified analytical models of the structure or component and the associated free-body diagram(s) to aid the student in understanding and applying the relevant theory in engineering analysis of the system. The text examples vary in length from one to four pages, depending upon the complexity of the material to be illustrated. When the emphasis is on concepts, the examples are worked out in symbolic terms so as to better illustrate the ideas, and when the emphasis is on problem-solving, the examples are numerical in character. In selected examples throughout the text, graphical display of results (e.g., stresses in beams) has been added to enhance the student's understanding of the problem results.

PROBLEMS

In all mechanics courses, solving problems is an important part of the learning process. This textbook offers more than 1230 problems for homework assignments and classroom discussions. The problems are placed at the end of each chapter so that they are easy to find and don't break up the presentation of the main subject matter. Also, problems are generally arranged in order of increasing difficulty thus alerting students to the time necessary for solution. Answers to all problems are listed near the back of the book. An Instructor Solution Manual (ISM) is available to registered instructors at the publisher's web site.

Considerable effort has been spent in checking and proofreading the text so as to eliminate errors. If you happen to find one, no matter how trivial, please notify me by e-mail (*bgoodno@ce.gatech.edu*). We will correct any errors in the next printing of the book.

UNITS

The International System of Units (SI) is used in all examples and problems. Tables containing properties of selected structural-steel shapes in SI units may be found in Appendix E; these tables will be useful in the solution of beam analysis and design examples and end-of-chapter problems in Chapter 5.

SUPPLEMENTS

INSTRUCTOR RESOURCES

An Instructor's Solutions Manual (ISM) is available in both print and digital versions. The digital version is accessible to registered instructors at the publisher's web site. This web site also includes both a full set of PowerPoint slides containing all graphical images in the text, and a set of LectureBuilder PowerPoint slides of all equations and Example Problems from the text, for use by instructors during lecture or review sessions.

To access additional course materials, please visit www.cengagebrain.com. At the cengagebrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where these resources can be found.

S.P. TIMOSHENKO (1878–1972) AND J.M. GERE (1925–2008)

Many readers of this book will recognize the name of Stephen P. Timoshenko—probably the most famous name in the field of applied mechanics. Timoshenko is generally recognized as the world's most outstanding pioneer in applied mechanics. He contributed many new ideas and concepts and became famous for both his scholarship and his teaching. Through his numerous textbooks he made a profound change in the teaching of mechanics not only in this country but wherever mechanics is taught. Timoshenko was both teacher and mentor to James Gere and provided the motivation for the first edition of this text, authored by James M. Gere and published in 1972; the second and each subsequent edition of this book were written by James Gere over the course of his long and distinguished tenure as author, educator and researcher at Stanford University. James Gere started as a doctoral student at Stanford in 1952 and retired from Stanford as a professor in 1988 having authored this and eight other well known and respected text books on mechanics, and structural and earthquake engineering. He remained active at Stanford as Professor Emeritus until his death in January of 2008.

A brief biography of Timoshenko appears in the first reference in the *References and Historical Notes* section, and also in an August 2007 *STRUCTURE* magazine article entitled "Stephen P. Timoshenko: Father of Engineering Mechanics in the U.S." by Richard G. Weingardt, P.E. This article provides an excellent historical perspective on this and the many other engineering mechanics textbooks written by each of these authors.

ACKNOWLEDGMENTS

To acknowledge everyone who contributed to this book in some manner is clearly impossible, but I owe a major debt to my former Stanford teachers, especially my mentor and friend, and lead author, James M. Gere.

I am grateful to my many colleagues teaching Mechanics of Materials at various institutions throughout the world who have provided feedback and constructive criticism about the text; for all those anonymous reviews, my thanks. With each new edition, their advice has resulted in significant improvements in both content and pedagogy.

My appreciation and thanks also go to the reviewers who provided specific comments for this eighth edition:

Jonathan Awerbuch, Drexel University

Henry N. Christiansen, Brigham Young University

Remi Dingreville, NYU—Poly

Apostolos Fafitis, Arizona State University

Paolo Gardoni, Texas A & M University

Eric Kasper, California Polytechnic State University, San Luis Obispo

Nadeem Khattak, University of Alberta

Kevin M. Lawton, University of North Carolina, Charlotte

Kenneth S. Manning, Adirondack Community College

Abulkhair Masoom, University of Wisconsin—Platteville

Craig C. Menzemer, University of Akron

Rungun Nathan, The Pennsylvania State University, Berks

Douglas P. Romilly, University of British Columbia

Edward Tezak, Alfred State College

George Tsiatis, University of Rhode Island

Xiangwu (David) Zeng, Case Western Reserve University

Mohammed Zikry, North Carolina State University

I wish to also acknowledge my Structural Engineering and Mechanics colleagues at the Georgia Institute of Technology, many of whom provided valuable advice on various aspects of the revisions and additions leading to the current edition. It is a privilege to work with all of these educators and to learn from them in almost daily interactions and discussions about structural engineering and mechanics in the context of research and higher education. I wish to extend my thanks to my many

current and former students who have helped to shape this text in its various editions. Finally, I would like to acknowledge the excellent work of German Rojas, PhD., PEng. who carefully checked the solutions of many of the new examples and end of chapter problems.

The editing and production aspects of the book were always in skillful and experienced hands, thanks to the talented and knowledgeable personnel of Cengage Learning. Their goal was the same as mine—to produce the best possible new edition of this text, never compromising on any aspect of the book.

The people with whom I have had personal contact at Cengage Learning are Christopher Carson, Executive Director, Global Publishing Program, Christopher Shortt, Publisher, Global Engineering Program, Randall Adams and Swati Meherishi, Acquisitions Editors, who provided guidance throughout the project; Hilda Gowans, Senior Developmental Editor, Engineering, who was always available to provide information and encouragement; Kristiina Paul who managed all aspects of new photo selection and permissions research; Andrew Adams who created the cover design for the book; and Lauren Betsos, Global Marketing Manager, who developed promotional material in support of the text. I would like to especially acknowledge the work of Rose Kernan of RPK Editorial Services, and her staff, who edited the manuscript and managed it throughout the production process. To each of these individuals I express my heartfelt thanks for a job well done. It has been a pleasure working with you on an almost daily basis to produce this eighth edition of the text.

I am deeply appreciative of the patience and encouragement provided by my family, especially my wife, Lana, throughout this project.

Finally, I am very pleased to continue this endeavor, at the invitation of my mentor and friend, Jim Gere. This eighth edition text has now reached its 40th year of publication. I am committed to its continued excellence and welcome all comments and suggestions. Please feel free to provide me with your critical input at *bgoodno@ce.gatech.edu*.

Barry J. Goodno
Atlanta, Georgia

CengageNOW—Just What You Need To Know and Do NOW!

CengageNOW Is An Online Teaching & Learning Resource
CengageNOW offers all of your teaching and learning resources in one intuitive program organized around the essential activities you perform for class—lecturing, creating assignments, grading, quizzing, and tracking student progress and performance. CengageNOW's intuitive "tabbed" design allows you to navigate to all key functions with a single click and a unique homepage tell you just what needs to be done and when. CengageNOW provides students access to an integrated eBook, interactive tutorials, and other multimedia tools that help them get the most out of your course.

CengageNOW Provides More Control In Less Time
CengageNOW's flexible assignment and gradebook options provide you more control while saving you valuable time in planning and managing your course assignments. With CengageNOW you can automatically grade all assignments, weigh grades, choose points or percentages and set the number of attempts and due dates per problem to best suit your overall course plan.

CengageNOW Delivers Better Student Outcomes
CengageNOW Personalized Study is a diagnostic tool consisting of a chapter-specific Pre-Test, Study Plan and Post-test that utilizes valuable text-specific assets to empower students to master concepts, prepare for exams and be more involved in class. It's easy to assign and if you want, results will automatically post to your gradebook. Results to Personalized Study provide immediate and ongoing feedback regarding what students are mastering and why they're not—to both you and the student. In most cases, Personalized Study, in most cases links to an integrated eBook so students can easily review topics. Best of all, CengageNOW Personalized Study is designed to help your students get a better grade in your class.

To purchase or access CengageNOW™ with eBook for Gere/Goodno, *Mechanics of Materials*, Eighth Edition go to www.cengagebrain.com.

SYMBOLS

A	area
A_f, A_w	area of flange; area of web
a, b, c	dimensions, distances
C	centroid, compressive force, constant of integration
c	distance from neutral axis to outer surface of a beam
D	diameter
d	diameter, dimension, distance
E	modulus of elasticity
E_r, E_t	reduced modulus of elasticity; tangent modulus of elasticity
e	eccentricity, dimension, distance, unit volume change (dilatation)
F	force
f	shear flow, shape factor for plastic bending, flexibility, frequency (Hz)
f_T	torsional flexibility of a bar
G	modulus of elasticity in shear
g	acceleration of gravity
H	height, distance, horizontal force or reaction, horsepower
h	height, dimensions
I	moment of inertia (or second moment) of a plane area
I_x, I_y, I_z	moments of inertia with respect to x, y, and z axes
I_{x1}, I_{y1}	moments of inertia with respect to x_1 and y_1 axes (rotated axes)
I_{xy}	product of inertia with respect to xy axes
I_{x1y1}	product of inertia with respect to x_1y_1 axes (rotated axes)
I_P	polar moment of inertia
I_1, I_2	principal moments of inertia
J	torsion constant
K	stress-concentration factor, bulk modulus of elasticity, effective length factor for a column
k	spring constant, stiffness, symbol for $\sqrt{P/EI}$
k_T	torsional stiffness of a bar
L	length, distance
L_E	effective length of a column
ln, log	natural logarithm (base e); common logarithm (base 10)
M	bending moment, couple, mass
M_P, M_Y	plastic moment for a beam; yield moment for a beam
m	moment per unit length, mass per unit length
N	axial force
n	factor of safety, integer, revolutions per minute (rpm)
O	origin of coordinates
O'	center of curvature
P	force, concentrated load, power
P_{allow}	allowable load (or working load)
P_{cr}	critical load for a column

P_P	plastic load for a structure
P_r, P_t	reduced-modulus load for a column; tangent-modulus load for a column
P_Y	yield load for a structure
p	pressure (force per unit area)
Q	force, concentrated load, first moment of a plane area
q	intensity of distributed load (force per unit distance)
R	reaction, radius
r	radius, radius of gyration ($r = \sqrt{I/A}$)
S	section modulus of the cross section of a beam, shear center
s	distance, distance along a curve
T	tensile force, twisting couple or torque, temperature
T_P, T_Y	plastic torque; yield torque
t	thickness, time, intensity of torque (torque per unit distance)
t_{f}, t_{w}	thickness of flange; thickness of web
U	strain energy
u	strain-energy density (strain energy per unit volume)
u_r, u_t	modulus of resistance; modulus of toughness
V	shear force, volume, vertical force or reaction
v	deflection of a beam, velocity
v', v'', etc.	dv/dx, d^2v/dx^2, etc.
W	force, weight, work
w	load per unit of area (force per unit area)
x, y, z	rectangular axes (origin at point O)
x_c, y_c, z_c	rectangular axes (origin at centroid C)
$\bar{x}$, $\bar{y}$, $\bar{z}$	coordinates of centroid
Z	plastic modulus of the cross section of a beam
α	angle, coefficient of thermal expansion, nondimensional ratio
β	angle, nondimensional ratio, spring constant, stiffness
β_R	rotational stiffness of a spring
γ	shear strain, weight density (weight per unit volume)
γ_{xy}, γ_{yz}, γ_{zx}	shear strains in xy, yz, and zx planes
γ_{x1y1}	shear strain with respect to x_1y_1 axes (rotated axes)
γ_θ	shear strain for inclined axes
δ	deflection of a beam, displacement, elongation of a bar or spring
ΔT	temperature differential
δ_P, δ_Y	plastic displacement; yield displacement
ε	normal strain
ε_x, ε_y, ε_z	normal strains in x, y, and z directions
ε_{x1}, ε_{y1}	normal strains in x_1 and y_1 directions (rotated axes)
ε_θ	normal strain for inclined axes
ε_1, ε_2, ε_3	principal normal strains
ε'	lateral strain in uniaxial stress
ε_T	thermal strain
ε_Y	yield strain
θ	angle, angle of rotation of beam axis, rate of twist of a bar in torsion (angle of twist per unit length)

θ_p	angle to a principal plane or to a principal axis
θ_s	angle to a plane of maximum shear stress
κ	curvature ($\kappa = 1/\rho$)
λ	distance, curvature shortening
ν	Poisson's ratio
ρ	radius, radius of curvature ($\rho = 1/\kappa$), radial distance in polar coordinates, mass density (mass per unit volume)
σ	normal stress
$\sigma_x, \sigma_y, \sigma_z$	normal stresses on planes perpendicular to x, y, and z axes
σ_{x1}, σ_{y1}	normal stresses on planes perpendicular to x_1y_1 axes (rotated axes)
σ_θ	normal stress on an inclined plane
$\sigma_1, \sigma_2, \sigma_3$	principal normal stresses
σ_{allow}	allowable stress (or working stress)
σ_{cr}	critical stress for a column ($\sigma_{\text{cr}} = P_{\text{cr}}/A$)
σ_{pl}	proportional-limit stress
σ_r	residual stress
σ_T	thermal stress
σ_U, σ_Y	ultimate stress; yield stress
τ	shear stress
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	shear stresses on planes perpendicular to the x, y, and z axes and acting parallel to the y, z, and x axes
τ_{x1y1}	shear stress on a plane perpendicular to the x_1 axis and acting parallel to the y_1 axis (rotated axes)
τ_θ	shear stress on an inclined plane
τ_{allow}	allowable stress (or working stress) in shear
τ_U, τ_Y	ultimate stress in shear; yield stress in shear
ϕ	angle, angle of twist of a bar in torsion
ψ	angle, angle of rotation
ω	angular velocity, angular frequency ($\omega = 2\pi f$)

GREEK ALPHABET

Α	α	Alpha	Ν	ν	Nu
Β	β	Beta	Ξ	ξ	Xi
Γ	γ	Gamma	Ο	o	Omicron
Δ	δ	Delta	Π	π	Pi
Ε	ε	Epsilon	Ρ	ρ	Rho
Ζ	ζ	Zeta	Σ	σ	Sigma
Η	η	Eta	Τ	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
Ι	ι	Iota	Φ	ϕ	Phi
Κ	κ	Kappa	Χ	χ	Chi
Λ	λ	Lambda	Ψ	ψ	Psi
Μ	μ	Mu	Ω	ω	Omega

*A star attached to a section number indicates a specialized.

Mechanics of Materials

Eighth Edition, SI

CHAPTER 1

Tension, Compression, and Shear

This telecommunications tower is an assemblage of many members that act primarily in tension or compression. (Péter budella/Shutterstock)

CHAPTER OVERVIEW

In Chapter 1, we are introduced to mechanics of materials, which examines the **stresses**, **strains**, **and displacements** in bars of various materials acted on by axial loads applied at the centroids of their cross sections. After a brief review of basic concepts presented in statics, we will learn about **normal stress (σ)** and **normal strain (ε)** in materials used for structural applications, then identify key properties of various materials, such as the modulus of elasticity (E) and yield (σ_y) and ultimate (σ_u) stresses, from plots of stress (σ) versus strain (ε). We will also plot shear stress (τ) versus shear strain (γ) and identify the shearing modulus of elasticity (G). If these materials perform only in the linear range, stress and strain are related by Hooke's Law for normal stress and strain ($\sigma = E \cdot \varepsilon$) and also for shear stress and strain ($\tau = G \cdot \gamma$). We will see that changes in lateral dimensions and volume depend upon Poisson's ratio (ν). Material properties E, G, and ν, in fact, are directly related to one another and are not independent properties of the material.

Assemblage of bars to form structures (such as trusses) leads to consideration of average shear (τ) and bearing (σ_b) stresses in their connections as well as normal stresses acting on the net area of the cross section (if in tension) or on the full cross-sectional area (if in compression). If we restrict maximum stresses at any point to **allowable** values by use of factors of safety, we can identify allowable levels of axial loads for simple systems, such as cables and bars. **Factors of safety** relate actual to required strength of structural members and account for a variety of uncertainties, such as variations in material properties and probability of accidental overload. Lastly, we will consider **design**: the iterative process by which the appropriate size of structural members is determined to meet a variety of both **strength** and **stiffness requirements** for a particular structure subjected to a variety of different loadings.

Chapter 1 is organized as follows:

1.1 INTRODUCTION TO MECHANICS OF MATERIALS

Mechanics of materials is a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading. Other names for this field of study are *strength of materials* and *mechanics of deformable bodies*. The solid bodies considered in this book include bars with axial loads, shafts in torsion, beams in bending, and columns in compression.

The principal objective of mechanics of materials is to determine the stresses, strains, and displacements in structures and their components due to the loads acting on them. If we can find these quantities for all values of the loads up to the loads that cause failure, we will have a complete picture of the mechanical behavior of these structures.

An understanding of mechanical behavior is essential for the safe design of all types of structures, whether airplanes and antennas, buildings and bridges, machines and motors, or ships and spacecraft. That is why mechanics of materials is a basic subject in so many engineering fields. Statics and dynamics are also essential, but those subjects deal primarily with the forces and motions associated with particles and rigid bodies. However, most problems in mechanics of materials begin with an examination of the external and internal forces acting on a stable deformable body. We first define the loads acting on the body, along with its support conditions, then determine reaction forces at supports and internal forces in its members or elements using the basic laws of static equilibrium (provided that it is statically determinate). A well-constructed free-body diagram is an essential part of the process of carrying out a proper static analysis of a structure.

In mechanics of materials we go beyond the concepts presented in statics to study the stresses and strains inside real bodies, that is, bodies of finite dimensions that deform under loads. To determine the stresses and strains, we use the physical properties of the materials as well as numerous theoretical laws and concepts. Later, we will see that mechanics of materials provides additional essential information, based on the deformations of the body, to allow us to solve so-called statically indeterminate problems (not possible if using the laws of statics alone).

Theoretical analyses and experimental results have equally important roles in mechanics of materials. We use theories to derive formulas and equations for predicting mechanical behavior, but these expressions cannot be used in practical design unless the physical properties of the materials are known. Such properties are available only after careful experiments have been carried out in the laboratory. Furthermore, not all practical problems are amenable to theoretical analysis alone, and in such cases physical testing is a necessity.

The historical development of mechanics of materials is a fascinating blend of both theory and experiment—theory has pointed the way to useful results in some instances, and experiment has done so in others. Such famous persons as Leonardo da Vinci (1452–1519) and Galileo Galilei (1564–1642) performed experiments to determine the strength of wires, bars, and beams, although they did not develop adequate theories (by today's standards) to explain their test results. By contrast, the famous mathematician Leonhard Euler (1707–1783) developed the mathematical theory of columns and calculated the critical load of a column in 1744, long before any experimental evidence existed to show the significance of his results. Without appropriate tests to back up his theories, Euler's results remained unused for over a hundred years, although today they are the basis for the design and analysis of most columns.*

Problems

When studying mechanics of materials, you will find that your efforts are divided naturally into two parts: first, understanding the logical development of the concepts, and second, applying those concepts to practical situations. The former is accomplished by studying the derivations, discussions, and examples that appear in each chapter, and the latter is accomplished by solving the problems at the ends of the chapters. Some of the problems are numerical in character, and others are symbolic (or algebraic).

An advantage of *numerical problems* is that the magnitudes of all quantities are evident at every stage of the calculations, thus providing an opportunity to judge whether the values are reasonable or not. The principal advantage of *symbolic problems* is that they lead to general-purpose formulas. A formula displays the variables that affect the final results; for instance, a quantity may actually cancel out of the solution, a fact that would not be evident from a numerical solution. Also, an algebraic solution shows the manner in which each variable affects the results, as when one variable appears in the numerator and another appears in the denominator. Furthermore, a symbolic solution provides the opportunity to check the dimensions at every stage of the work.

Finally, the most important reason for solving algebraically is to obtain a general formula that can be used for many different problems. In contrast, a numerical solution applies to only one set of circumstances. Because engineers must be adept at both kinds of solutions, you will find a mixture of numeric and symbolic problems throughout this book.

Numerical problems require that you work with specific units of measurement. This book utilizes the International System of Units (SI). A discussion of SI units appears in Appendix A, where you will also find many useful tables.

All problems appear at the ends of the chapters, with the problem numbers and subheadings identifying the sections to which they belong. The techniques for solving problems are discussed in detail in Appendix B. In addition to a list of sound engineering procedures, Appendix B includes sections on dimensional homogeneity and significant digits. These topics are especially important, because every equation must be dimensionally homogeneous and every numerical result must be expressed with the proper number of significant digits. In this book, final numerical results are usually presented with three significant digits when a number begins with the digits 2 through 9, and with four significant digits when a number begins with the digit 1. Intermediate values are often recorded with additional digits to avoid losing numerical accuracy due to rounding of numbers.

***The history of mechanics of materials, beginning with Leonardo and Galileo, is given in Refs. 1-1, 1-2, and 1-3.**

1.2 STATICS REVIEW

In your prerequisite course on statics, you studied the *equilibrium* of rigid bodies acted upon by a variety of different forces and supported or restrained in such a way that the body was stable and at rest. As a result, a properly restrained body could not undergo rigid-body motion due to the application of static forces. You drew *free-body diagrams* of the entire body, or of key parts of the body, and then applied the *equations of equilibrium* to find external reaction forces and moments or internal forces and moments at critical points. In this section, we will review the basic static equilibrium equations and apply them to the solution of example structures (both two and three dimensional) using both scalar and vector operations (both acceleration and velocity of the body will be assumed to be zero). Most problems in mechanics of materials require a static analysis as the first step, so all forces acting on the system and causing its deformation are known. Once all external and internal forces of interest have been found, we will be able to proceed with the evaluation of stresses, strains, and deformations of bars, shafts, beams, and columns in subsequent chapters.

Equilibrium Equations

The resultant force R and resultant moment M of *all* forces and moments acting on either a rigid or deformable body in equilibrium are both zero. The sum of the moments may be taken about any arbitrary point. The resulting equilibrium equations can be expressed in *vector form* as:

$$R = \Sigma F = 0 \tag{1-1}$$

$$M = \Sigma M = \Sigma(r \times F) = 0 \tag{1-2}$$

where F is one of a number of vectors of forces acting on the body and r is a position vector from the point at which moments are taken to a point along the line of application of any force F. It is often convenient to write the equilibrium equations in *scalar form* using a rectangular Cartesian coordinate system, either in two dimensions (x, y) or three dimensions (x, y, z) as

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_z = 0 \tag{1-3}$$

Eq. (1-3) can be used for two-dimensional or planar problems, but in three dimensions, three force and three moment equations are required:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{1-4}$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \tag{1-5}$$

If the number of unknown forces is equal to the number of independent equilibrium equations, these equations are sufficient to solve for all unknown reaction or internal forces in the body, and the problem is referred to as *statically determinate* (provided that the body is stable). If the body or structure is constrained by additional (or redundant) supports, it is *statically indeterminate*, and a solution is not possible using the laws of static equilibrium alone. For statically indeterminate structures, we must also examine the deformations of the structure, as will be discussed in the following chapters.

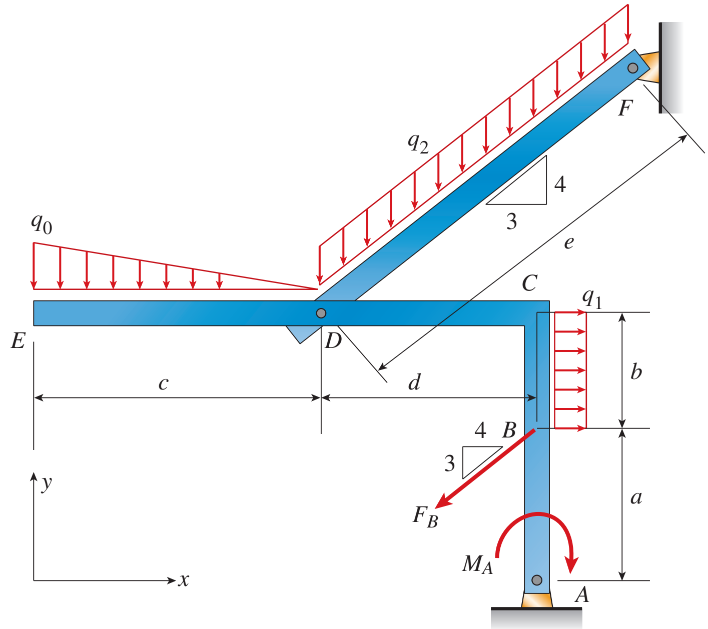


Fig. 1-1

Plane frame structure

Applied Forces

External loads applied to a body or structure may be either concentrated or distributed forces or moments. For example, force F_B (with units of pounds, lb, or newtons, N) in Fig. 1-1 is a point or concentrated load and is assumed to act at point B on the body, while moment M_A is a concentrated moment or couple (with units of lb-ft or N · m) acting at point A. Distributed forces may act alone or normal to a member and may have constant intensity, such as line load q_1 normal to member BC (Fig. 1-1) or line load q_2 acting in the $-y$ direction on inclined member DF; both q_1 and q_2 have units of force intensity (lb/ft or N/m). Distributed loads also may have a linear (or other) variation with some peak intensity q_0 (as on member ED in Fig. 1-1). Surface pressures p (with units of lb/ft^2 or Pa), such as wind acting on a sign (Fig. 1-2), act over a designated region of a body. Finally, a body force w (with units of force per unit volume, lb/ft^3 or N/m^3),

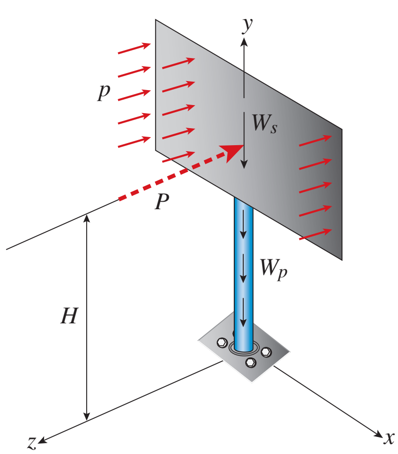


Fig. 1-2

Wind on sign

such as the distributed self-weight of the sign or post in Fig. 1-2, acts throughout the volume of the body and can be replaced by the component weight W acting at the center of gravity (c.g.) of the sign (W_s) or post (W_p). In fact, any distributed loading (line, surface, or body force) can be replaced by a statically equivalent force at the center of gravity of the distributed loading when overall static equilibrium of the structure is evaluated using Eqs. (1-1) to (1-5).

Free-Body Diagrams

A free-body diagram (FBD) is an essential part of a static analysis of a rigid or deformable body. All forces acting on the body, or component part of the body, must be displayed on the FBD if a correct equilibrium solution is to be obtained. This includes applied forces and moments, reaction forces and moments, and any connection forces between individual components. For example, an *overall* FBD of the plane frame in Fig. 1-1 is shown in Fig. 1-3a; all applied and reaction forces are shown on this FBD and statically equivalent concentrated loads are displayed for all distributed loads. Statically equivalent forces F_{q0}, F_{q1}, and F_{q2}, each acting at the c.g. of the corresponding distributed loading, are used in the equilibrium equation solution to represent distributed loads q_0, q_1, and q_2, respectively.

Next, the plane frame has been disassembled in Fig. 1-3b, so that *separate* FBD's can be drawn for each part of the frame, thereby exposing pin-connection forces at D (D_x, D_y). Both FBD's must show all applied forces as well as reaction forces A_x and A_y at pin-support joint A and F_x and F_y at pin-support joint F. The forces transmitted between frame elements EDC and DF at pin connection D must be determined if the proper interaction of these two elements is to be accounted for in the static analysis.

The plane frame structure in Fig. 1-1 will be analyzed in Example 1-2 to find reaction forces at joints A and F and also pin-connection forces at joint D using the equilibrium equations Eqs. (1-1) to (1-3). The FBD's presented in Figs. 1-3a and 1-3b are essential parts of this solution process. A *statics sign convention* is usually employed in the solution for support reactions; forces acting in the positive directions of the coordinate axes are assumed positive, and the right-hand rule is used for moment vectors.

Reactive Forces and Support Conditions

Proper restraint of the body or structure is essential if the equilibrium equations are to be satisfied. A sufficient number and arrangement of supports must be present to prevent rigid body motion under the action of static forces. A reaction force at a support is represented by a single arrow with a slash drawn through it (see Fig. 1-3) while a moment restraint at a support is shown as a double-headed or curved arrow with a slash. Reaction forces and moments usually result from the action of applied forces of the types described above (i.e., concentrated, distributed, surface, and body forces).

A variety of different support conditions may be assumed depending on whether the problem is 2D or 3D. Supports A and F in the 2D plane frame structure shown in Fig. 1-1 and Fig. 1-3 are pin supports,

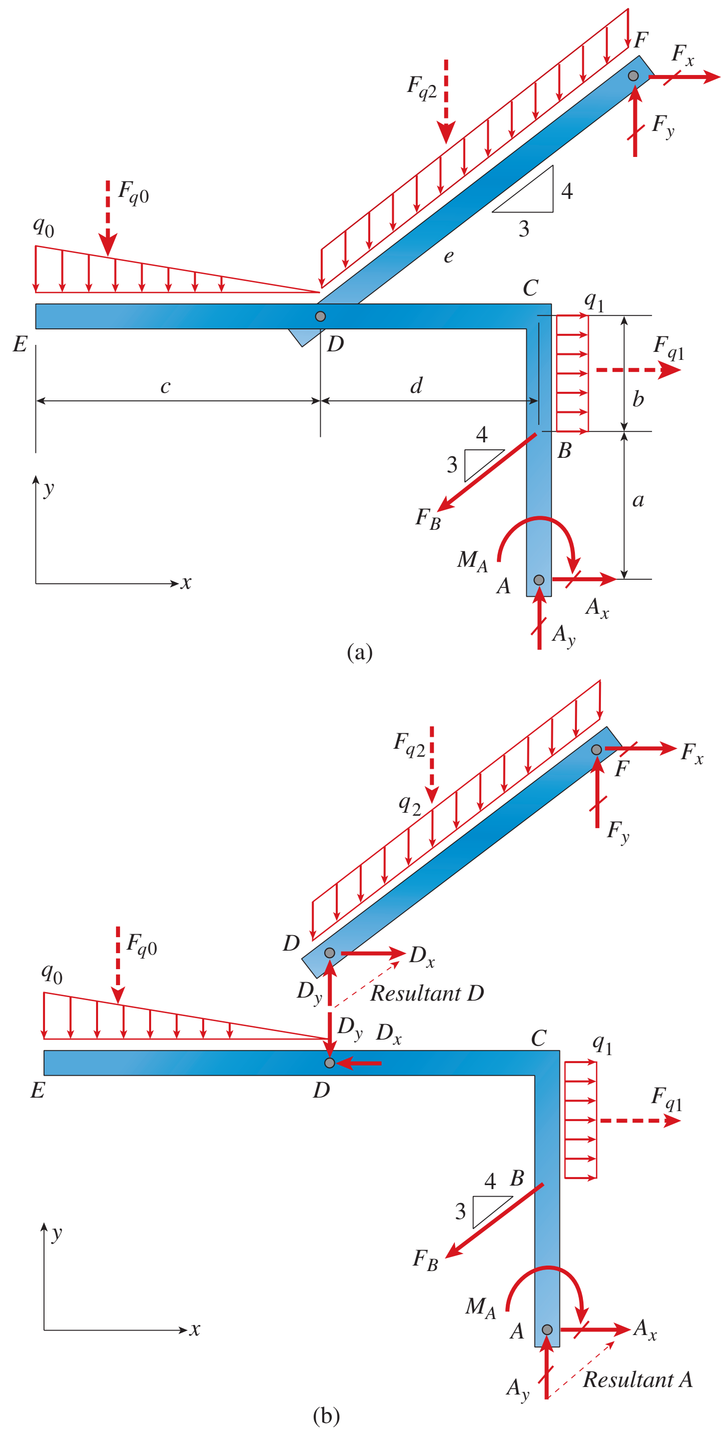


Fig. 1-3

(a) Overall FBD of plane frame structure from Fig. 1-1, and (b) Separate free-body diagrams of parts *A* through *E* and *DF* of the plane frame structure in Fig. 1-1

while the base of the 3D sign structure in Fig. 1-2 may be considered to be a fixed or clamped support. Some of the most commonly used idealizations for 2D and 3D supports, as well as interconnections between members or elements of a structure, are illustrated in Table 1-1. The

Table 1-1

Reaction and Connection Forces in 2D or 3D Static Analysis

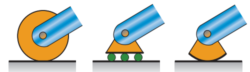
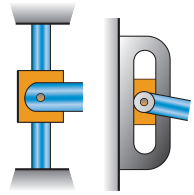
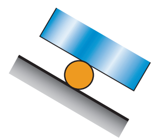
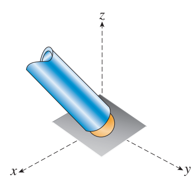
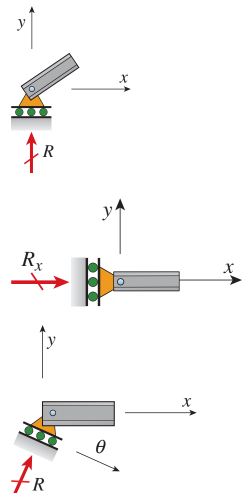
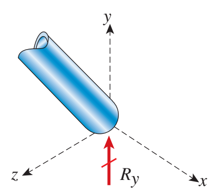
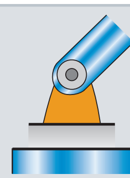
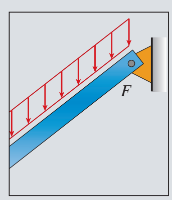
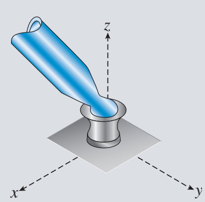
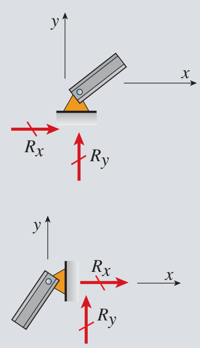
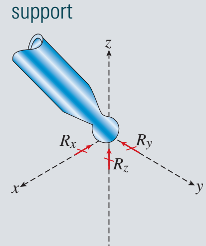
Type of support or connection	Simplified sketch of support or connection	Display of restraint forces and moments, or connection forces
(1) Roller support—horizontal, vertical, or inclined  Bridge with roller support (The Earthquake Engineering Online Archive)	 Horizontal roller support (constrains motion in both $+y$ and $-y$ directions)  Vertical roller restraints  Rotated or inclined roller support  	(a) Two-dimensional roller support  (b) Three-dimensional roller support 
(2) Pin support  Bridge with pin support (Courtesy of Joel Kerkhoff, P.Eng.)  Pin support on old truss bridge (© Barry Goodno)	  Pin support at F in Fig. 1-1  	(a) Two-dimensional pin support  (b) Three-dimensional pin support 

Table 1-1 (continued)

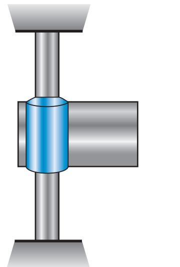
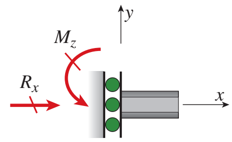
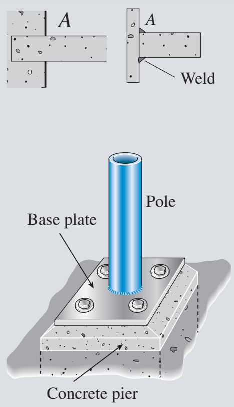
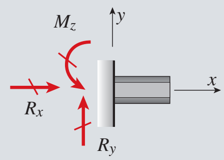
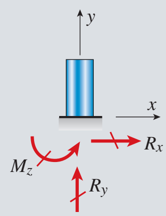
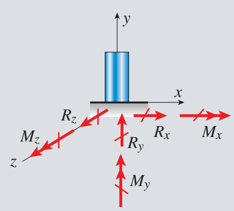
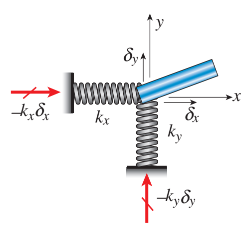
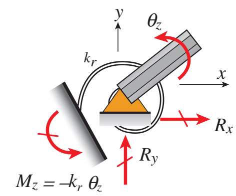
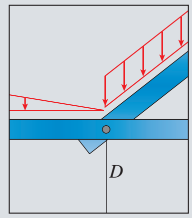
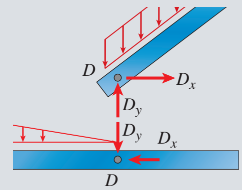
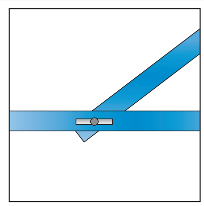
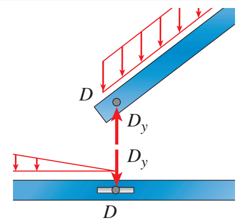
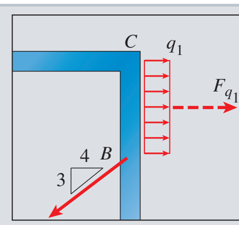
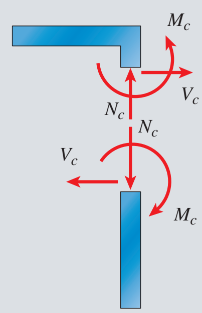
(3) Sliding support	 Frictionless sleeve on vertical shaft	
(4) Clamped or fixed support	 Fixed support at base of sign post (see Fig. 1-2)	(a) Two-dimensional fixed support   (b) Three-dimensional fixed support 
(5) Elastic or spring supports		(a) Translational spring (k) 

Table 1-1 (continued)

		(b) Rotational spring (k_r) 
(6) Pinned connection (from Figs. 1-1 and 1-3)  Pin connection on old bridge (© Barry Goodno)	 Pinned connection at *D* between members *EDC* and *DF* in plane frame (Fig. 1-1)	
(7) Slotted connection *(modified connection from that shown in Figs.* 1-1 *and* 1-3*)*	 Alternate slotted connection at *D* on plane frame (Note that the plane frame in Fig. 1-1 is *unstable* if this slotted connection is used instead of a pin at *D*.)	
(8) Rigid connection *(internal forces and moment in members joined at C of plane frame in Fig.* 1-1*)*	 Rigid connection at *C* on plane frame	

restraining or transmitted forces and moments associated with each type of support or connection are displayed in the third column of the table (these are not FBDs, however). The reactions forces and moments for the 3D sign structure in Fig. 1-2 are shown on the FBD in Fig. 1-4a;

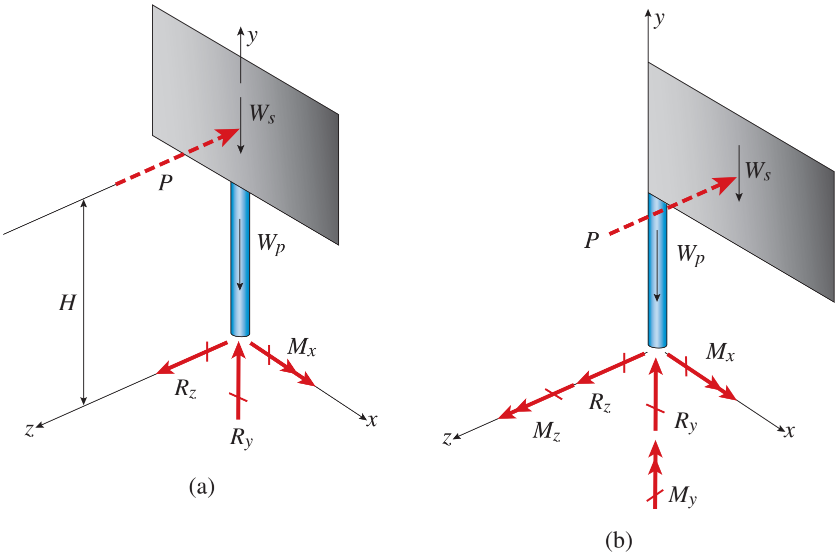


Fig. 1-4

(a) FBD of symmetric sign structure, and (b) FBD of eccentric sign structure

only reactions R_y, R_z, and M_x are non-zero because the sign structure and wind loading are symmetric with respect to the *yz* plane. If the sign is eccentric to the post (Fig. 1-4b), only reaction R_x is zero for the case of wind loading in the $-z$ direction. (See Prob. 1.7-16 at the end of Chapter 1 for a more detailed examination of the reaction forces due to wind pressure acting on the sign structure in Fig. 1-2; forces and stresses in the base plate bolts are also computed. Several eccentric sign structures are presented for analysis as end of chapter problems in Chapter 8.)

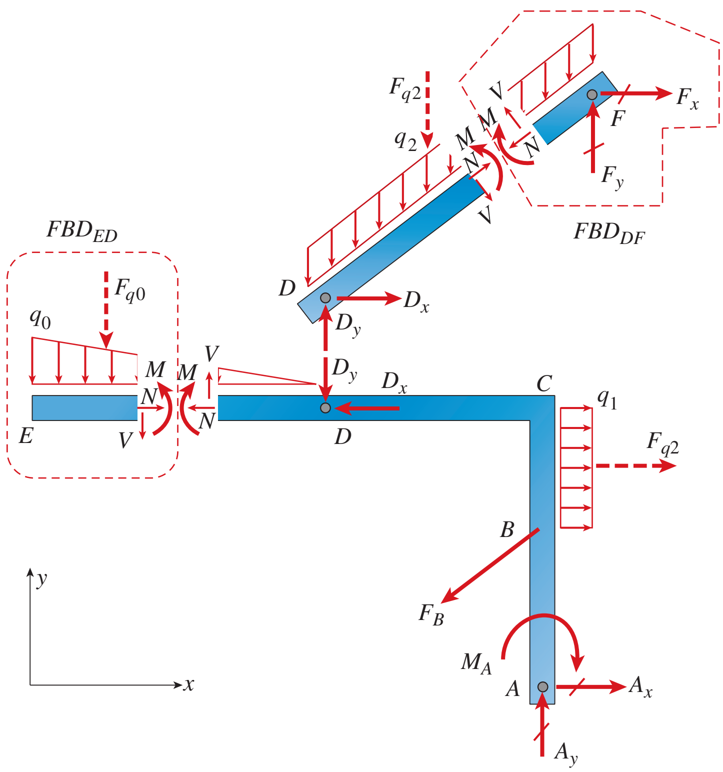
Internal Forces (Stress Resultants)

In our study of mechanics of materials, we will investigate the deformations of the members or elements which make up the overall deformable body. In order to compute the member deformations, we must first find the internal forces and moments (i.e., the internal stress resultants) at key points along the members of the overall structure. In fact, we will often create graphical displays of the internal axial force, torsional moment, transverse shear and bending moment along the axis of each member of the body so that we can readily identify critical points or regions within the structure. The first step is to make a section cut normal to the axis of each member so that a FBD can be drawn which displays the internal forces of interest. For example, if a cut is made at the top of member *BC* in the plane frame in Fig. 1-1, the internal *axial force* (N_c), *transverse shear force* (V_c) and *bending moment* (M_c) at joint *C* can be exposed as shown in the last row of Table 1-1. Fig. 1-5 shows two additional cuts made through members *ED* and *DF* in the plane frame; the resulting FBD's now can be used to find N, V, and M in members ED and DF of the plane frame. Stress resultants N, V, and M are usually taken along and normal to the member under consideration (i.e., local or member axes are used), and a *deformation sign convention* (e.g., tension is positive, compression is negative) is employed in their solution. In later chapters, we will see how these (and other) internal stress resultants are used to compute stresses in the member cross section.

The following examples are presented as a review of application of the equations of static equilibrium in the solution for external reactions and internal forces in truss, beam, circular shaft, and frame structures. First, a **truss structure** is considered and both scalar and vector solutions for reaction forces are reviewed. Then member forces are computed using the *method of joints*. Properly drawn FBD's are seen to be essential to the overall solution process. The second example involves static analysis of a **beam structure** to find reactions and internal forces at a particular section along the beam. In the third example, reactive and internal torsional moments in a **stepped shaft** are computed. And, finally, the fourth example presents the solution of the **plane frame structure** discussed here. Numerical values are assigned to applied forces and structure dimensions, and then reaction, pin connection, and selected internal forces in the frame are computed.

Fig. 1-5

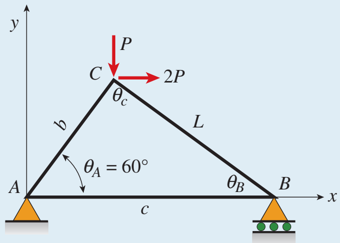
FBD's for internal stress resultants in *ED* and *DF*



Example 1-1

Fig. 1-6

Example 1-1: Plane truss static analysis for joint loads



The plane truss shown in Fig. 1-6 is pin supported at A and has a roller support at B. Joint loads $2P$ and $-P$ are applied at joint C. Find support reactions at joints A and B, then solve for forces in members AB, AC and BC. Use numerical properties given below.

Numerical data:

$$P = 160 \text{ kN} \quad L = 3 \text{ m} \quad \theta_A = 60° \quad b = 2.2 \text{ m}$$

Solution

(1) Use the law of sines to find angles θ_B and θ_C, then find the length (c) of member AB.
(2) Draw the FBD, then use equilibrium equations in scalar form [Eq. (1-3)] to find the support reactions.
(3) Find member forces using the method of joints.
(4) Repeat solution for support reactions using a vector solution.
(5) Solve for support reactions and member forces for a 3D version of this plane (2D) truss.

(1) Use the law of sines to find angles θ_B, θ_C then find the length (c) of member AB.

See **law of sines** in Appendix C:

$$\theta_B = \arcsin\left(\frac{b}{L}\sin(\theta_A)\right) = 39.426° \quad \text{so} \quad \theta_C = 180° - (\theta_A + \theta_B) = 80.574°$$

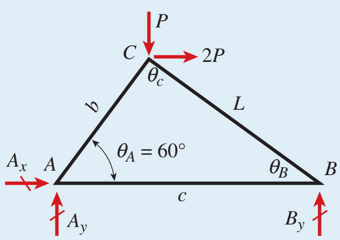
$$\text{and } c = L\left(\frac{\sin(\theta_C)}{\sin(\theta_A)}\right) = 3.417 \text{ m} \quad \text{or} \quad c = b\cos(\theta_A) + L\cos(\theta_B) = 3.417 \text{ m}$$

Note that the **Law of Cosines** also could be used:

$$c = \sqrt{b^2 + L^2 - 2bL\cos(\theta_C)} = 3.417 \text{ m}$$

Fig. 1-7

Example 1-1: FBD of plane truss



(2) Draw the FBD (Fig. 1-7), then use equilibrium equations in *scalar form* [Eq. (1-3)] to find the support reactions.

Note that **the plane truss is statically determinate** since there are ($m + r = 6$) unknowns (where m = number of member forces and r = number of reactions), but there are ($2j = 2 \times 3 = 6$) equations of statics from the method of joints (where j = number of joints).

Use equilibrium equations in scalar form to find support reactions.

Sum moments about A to get reaction B_y:

$$B_y = \frac{[Pb\cos(\theta_A) + (2P)b\sin(\theta_A)]}{c} = 230 \text{ kN}$$

Sum forces in y direction to get A_y:

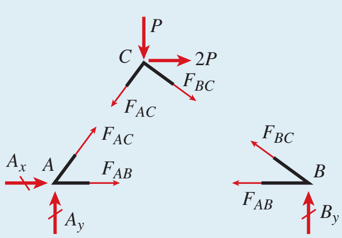
$$A_y = P - B_y = -70 \text{ kN}$$

Sum forces in x direction to get A_x:

$$A_x = -2P = -320 \text{ kN}$$

Fig. 1-8

Example 1-1: FBD of each joint of plane truss



(3) Find member forces using the method of joints.

Draw FBDs of each joint (Fig. 1-8) then sum forces in x and y directions to find member forces.

Continues

Example 1-1 - *Continued*

Sum forces in y direction at joint A:

$$F_{AC} = \frac{-A_y}{\sin(\theta_A)} = 80.7\text{ kN}$$

Sum forces in x direction at joint A:

$$F_{AB} = -A_x - F_{AC}\cos(\theta_A) = 280\text{ kN}$$

Sum forces in y direction at joint B:

$$F_{BC} = \frac{-B_y}{\sin(\theta_B)} \qquad F_{BC} = -362\text{ kN}$$

Check equilibrium at joint C. (First at x direction, then y direction.)

$$-F_{AC}\cos(\theta_A) + F_{BC}\cos(\theta_B) + 2P = 0 \qquad -F_{AC}\sin(\theta_A) - F_{BC}\sin(\theta_B) - P = 0$$

(4) Repeat solution for support reactions using a *vector solution* (show x, y, z components in vector format).

Position vectors to B and C from A:

$$r_{AB} = \begin{pmatrix} c \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.4173 \\ 0 \\ 0 \end{pmatrix}\text{ m} \qquad r_{AC} = \begin{pmatrix} b\cos(\theta_A) \\ b\sin(\theta_A) \\ 0 \end{pmatrix} = \begin{pmatrix} 1.1 \\ 1.9053 \\ 0 \end{pmatrix}\text{ m}$$

Force vectors at A, B, and C:

$$A = \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ B_y \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 2P \\ -P \\ 0 \end{pmatrix}$$

Sum moments about point A, then equate each expression to zero:

$$M_A = r_{AB} \times B + r_{AC} \times C = \begin{pmatrix} 0 \\ 0 \\ 3.417\text{ m } B_y - 785.7\text{ m kN} \end{pmatrix}$$

$$\text{so } B_y = \frac{785.7}{3.417} = 230\text{ kN}$$

$$\text{or}\ldots \left|\begin{pmatrix} i & j & k \\ c & 0 & 0 \\ 0 & B_y & 0 \end{pmatrix}\right| + \left|\begin{pmatrix} i & j & k \\ \frac{b}{2} & b\frac{\sqrt{3}}{2} & 0 \\ 2P & -P & 0 \end{pmatrix}\right| = -785.68\text{ k kN}\cdot\text{m} + 3.4173\text{ m } B_y\,k$$

Now sum forces and equate each expression to zero:

$$A + B + C = \begin{pmatrix} A_x + 320\text{ kN} \\ A_y + B_y - 160\text{ kN} \\ 0 \end{pmatrix} \quad so\ A_x = -320\text{ kN}$$

$$A_y = 160 - B_y = -70\text{ kN}$$

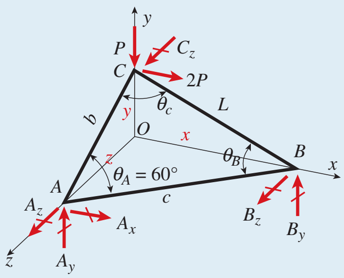
Reactions A_x, A_y, and B_y are the same as from the scalar solution approach.

(5) Solve for support reactions and member forces for a 3D version of plane (2D) truss.

To create a space truss from the plane truss, move joint A along the z axis a distance z while holding B on the x axis and constraining C to lie some

Fig. 1-9

Example 1-1: FBD of space truss (extended version of plane truss)



distance y along the y axis (see Fig. 1-9); hold member lengths (L, b, c) and angles (θ_A, θ_B, θ_C) to the values used for the plane truss. Apply joint loads $2P$ and $-P$ at joint C. Add 3D pin support at A, two restraints at B (B_y, B_z), and one restraint at C (C_z).

Note that **the space truss is statically determinate** since there are ($m + r = 9$) unknowns (where m = number of member forces and r = number of reactions), but there are ($3j = 3 \times 3 = 9$) equations of statics from the method of joints (where j = number of joints).

First, find x, y, and z projections of members along coordinate axes. Then find angles OBC, OBA, and OAC in each plane.

$$x = \sqrt{\frac{L^2 - b^2 + c^2}{2}} = 2.81408 \text{ m} \qquad y = \sqrt{\frac{L^2 + b^2 - c^2}{2}} = 1.03968 \text{ m}$$

$$z = \sqrt{\frac{-L^2 + b^2 + c^2}{2}} = 1.93883 \text{ m}$$

$$OBC = \arctan\left(\frac{y}{x}\right) = 20.277° \qquad OBA = \arctan\left(\frac{z}{x}\right) = 34.566°$$

$$OAC = \arctan\left(\frac{y}{z}\right) = 28.202°$$

Draw the overall FBD (see Fig. 1-9), then use a *scalar solution* to find reactions and member forces.

(1) Sum moments about a line through A, which is parallel to the y axis (this will isolate reaction B_z, giving us one equation with one unknown):

$$B_z x + (2P)z = 0 \qquad B_z = -2P\frac{z}{x} = -220 \text{ kN}$$

This is based on a *statics sign convention* so the negative sign means that force B_z acts in the $-z$ direction.

(2) Sum moments about the z-axis to find B_y, then sum forces in y direction to get A_y:

$$B_y = \frac{2P(y)}{x} = 118.2 \text{ kN} \quad \text{so} \quad A_y = P - B_y = 41.8 \text{ kN}$$

(3) Sum moments about the x axis to find C_z:

$$C_z = \frac{A_y z}{y} = 77.9 \text{ kN}$$

(4) Sum forces in the x and z directions to get A_x and A_z:

$$A_x = -2P = -320 \text{ kN} \qquad A_z = -C_z - B_z = 142.6 \text{ kN}$$

(5) Finally, use the method of joints to find member forces (a deformation sign convention is used here so positive (+) means tension and negative (−) means compression).

Sum forces in x direction at joint A:

$$\frac{x}{c}F_{AB} + A_x = 0 \qquad F_{AB} = \frac{-c}{x}A_x \qquad F_{AB} = 389 \text{ kN}$$

Sum forces in y direction at joint A:

$$\frac{y}{b}F_{AC} + A_y = 0 \qquad F_{AC} = \frac{b}{y}(-A_y) \qquad F_{AC} = -88.4 \text{ kN}$$

Continues

Example 1-1 - *Continued*

Sum forces in y direction at joint B:

$$\frac{y}{L}F_{BC} + B_y = 0 \qquad F_{BC} = \frac{-L}{y}B_y \qquad F_{BC} = -341 \text{ kN}$$

Re-compute reactions for the space truss using a vector solution.

Find position (**r**) and unit (**e**) vectors from joint A to joints B and C:

$$r_{AB} = \begin{pmatrix} x \\ 0 \\ -z \end{pmatrix} \qquad e_{AB} = \frac{r_{AB}}{|r_{AB}|} = \begin{pmatrix} 0.823 \\ 0 \\ -0.567 \end{pmatrix}$$

$$r_{AC} = \begin{pmatrix} 0 \\ y \\ -z \end{pmatrix} \qquad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} 0 \\ 0.473 \\ -0.881 \end{pmatrix}$$

Sum moments about point A, then equate each expression to zero:

$$M_A = r_{AB} \times B + r_{AC} \times C$$

$$M_A = r_{AB} \times \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} + r_{AC} \times \begin{pmatrix} 2P \\ -P \\ C_z \end{pmatrix}$$

$$= \begin{pmatrix} 1.9388 \text{ m } B_y + 1.0397 \text{ m } C_z - 310.21 \text{ kN}\cdot\text{m} \\ -2.8141 \text{ m } B_z - 620.43 \text{ kN}\cdot\text{m} \\ 2.8141 \text{ m } B_y - 332.7 \text{ kN}\cdot\text{m} \end{pmatrix}$$

$$\text{or} \quad \left|\begin{pmatrix} i & j & k \\ x & 0 & -z \\ 0 & B_y & B_z \end{pmatrix}\right| = 1.9388 \text{ m } B_y\, i - 2.8141 \text{ m } B_z\, j + 2.8141 \text{ m } B_y\, k$$

$$\text{and} \quad \left|\begin{pmatrix} i & j & k \\ 0 & y & -z \\ 2P & -P & C_z \end{pmatrix}\right| = 1.0397 \text{ m } C_z i - 310.21 \text{ kN}\cdot\text{m } i$$

$$-620.43 \text{ kN}\cdot\text{m } j - 332.7 \text{ kN}\cdot\text{m } k$$

Collecting coefficients of j and solving:

$$B_z = \frac{620.43}{-2.8141} = -220 \text{ kN}$$

Collecting coefficients of k and solving:

$$B_y = \frac{332.7}{2.8141} = 118.2 \text{ kN}$$

Collecting coefficients of i and solving:

$$C_z = \frac{310.21 - 1.9388\, B_y}{1.0397} = 77.9 \text{ kN}$$

Complete the solution by summing forces and equating to zero:

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} 2P \\ -P \\ C_z \end{pmatrix} = \begin{pmatrix} A_x + 320.0 \text{ kN} \\ A_y - 41.8 \text{ kN} \\ A_z - 142.6 \end{pmatrix}$$

$$A_x = -320 \text{ kN} \qquad A_y = 41.8 \text{ kN} \qquad A_z = 142.6 \text{ kN}$$

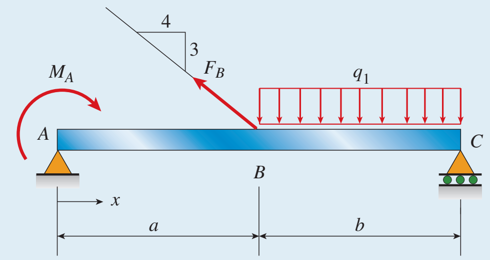
Reactions A_x, A_y, A_z and B_y, B_z are the same as from the scalar solution approach.

Example 1-2

The simply-supported beam structure shown in Fig. 1-10 is subjected to moment M_A at pin-supported joint A, inclined load F_B applied at joint B, and uniform load with intensity q_1 on member segment BC. Find support reactions at joints A and C, then solve for internal forces at the midpoint of BC. Use properly drawn free-body diagrams in your solution.

Fig. 1-10

Example 1-2: Beam static analysis for support reactions



Numerical data (Newtons and meters):

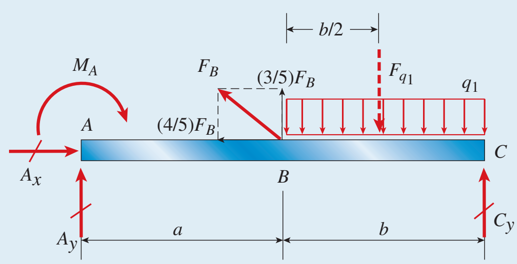
$$a = 3\,\text{m} \quad b = 2\,\text{m}$$
$$M_A = 380\,\text{N}\cdot\text{m} \quad F_B = 200\,\text{N} \quad q_1 = 160\,\text{N/m}$$

Solution

(1) Draw the FBD of the overall beam. The solution for reaction forces at A and C must begin with a proper drawing of the FBD of the overall beam (Fig. 1-11). The FBD shows all applied and reactive forces.

Fig. 1-11

Example 1-2: FBD of beam



(2) Determine statically equivalent concentrated forces. Distributed forces are replaced by their statical equivalents (F_{q1}) and the components of the inclined concentrated force at B may also be computed:

$$F_{q1} = q_1 b = 320\,\text{N} \qquad F_{Bx} = \frac{4}{5}F_B = 160\,\text{N} \qquad F_{By} = \frac{3}{5}F_B = 120\,\text{N}$$

(3) Sum the moments about A to find reaction force C_y. This structure is statically determinate because there are three available equations from statics ($\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$) and three reaction unknowns (A_x, A_y, C_y). It is convenient to start the static analysis using $\Sigma M_A = 0$, because we can isolate one equation with one unknown and then easily find reaction C_y. A statics sign convention is used (i.e., right-hand rule or CCW is positive).

$$C_y = \frac{1}{(a+b)}\left[M_A - F_{By}a + F_{q1}\left(a + \frac{b}{2}\right)\right] = 260\,\text{N}$$

Continues

Example 1-2 - *Continued*

(4) Sum the forces in *x* and *y* directions to find reaction forces at A. Now that C_y is known, we can complete the equilibrium analysis to find A_x and A_y using $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Then we can find the resultant reaction force at A using components A_x and A_y:

Sum forces in x direction: $A_x - F_{Bx} = 0 \qquad A_x = F_{Bx} \qquad A_x = 160 \text{ N}$

Sum forces in y direction: $A_y + F_{By} + C_y - F_{q1} = 0$

$$A_y = -F_{By} - C_y + F_{q1} \qquad A_y = -60 \text{ N}$$

Resultant force at A: $A = \sqrt{A_x^2 + A_y^2} \qquad A = 171 \text{ N}$

(5) Find the internal forces and moment at the midpoint of member segment *BC*. Now that reaction forces at A and C are known, we can cut a section through the beam midway between B and C, creating left and right FBDs (Fig. 1-12). Section forces N_c (axial) and V_c (shear) as well as section moment (M_c) are exposed and may be computed using statics. Either FBD may be used to find N_c, V_c, and M_c; the computed internal forces and moment N_c, V_c, and M_c will be the same.

Calculations based on ***left FBD***:

$$\Sigma F_x = 0 \quad N = F_{Bx} - A_x = 0 \text{ N}$$

$$\Sigma F_y = 0 \quad V = A_y + F_{By} - q_1\left(\frac{b}{2}\right) = -100 \text{ N}$$

$$\Sigma M = 0 \quad M = M_A + A_y\left(a + \frac{b}{2}\right) + F_{By}\left(\frac{b}{2}\right) - q_1\left(\frac{b}{2}\right)\left(\frac{b}{4}\right) = 180 \text{ N}\cdot\text{m}$$

Calculations based on ***right FBD***:

$$\Sigma F_x = 0 \quad N = 0$$

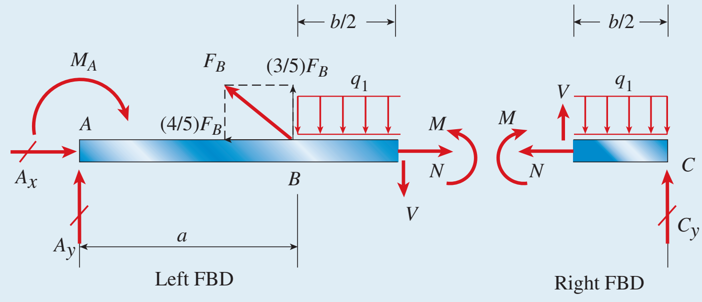
$$\Sigma F_y = 0 \quad V = q_1\left(\frac{b}{2}\right) - C_y = -100 \text{ N}$$

$$\Sigma M = 0 \quad M = C_y\left(\frac{b}{2}\right) - q_1\left(\frac{b}{2}\right)\left(\frac{b}{4}\right) = 180 \text{ N}\cdot\text{m}$$

The computed internal forces (N and V) and internal moment (M) are the same and can be determined using either the left or right FBD. This applies for any section taken through the beam at any point along its length. Later, we will create plots or diagrams which show the variation of N, V, and M over the length of the beam. These diagrams will be very useful in the design of beams, because they readily show the critical regions of the beam where N, V, and M have maximum values.

Fig. 1-12

Example 1-2: Left and right FBDs of beam

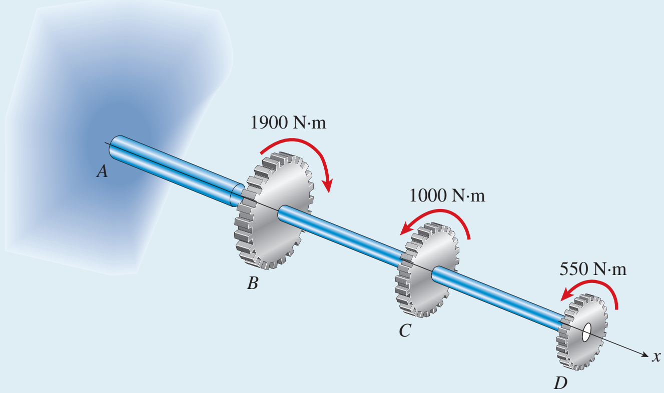


Example 1-3

A stepped circular shaft is fixed at *A* and has three gears that transmit the torques shown in Fig. 1-13. Find the reaction torque at *A*, then find the internal torsional moments in segments *AB*, *BC*, and *CD*. Use properly drawn free-body diagrams in your solution.

Fig. 1-13

Example 1-3: Stepped circular shaft in torsion

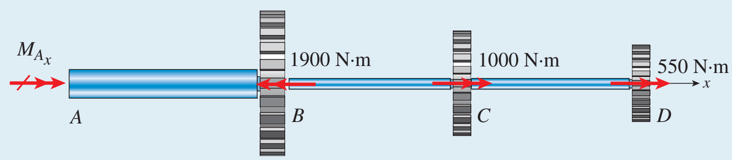


Solution

(1) Draw the FBD of the overall shaft structure. The cantilever shaft structure is statically determinate. The solution for the reaction moment at *A* must begin with a proper drawing of the FBD of the overall structure (Fig. 1-14). The FBD shows all applied and reactive torques.

Fig. 1-14

Example 1-3: FBD of overall shaft



(2) Sum the moments about the *x* axis to find the reaction moment M_{Ax}. This structure is statically determinate because there is one available equation from statics ($\Sigma M_x = 0$) and one reaction unknown (M_{Ax}). A statics sign convention is used (i.e., right-hand rule or CCW is positive).

$$M_{Ax} - 1900\ \text{N}\cdot\text{m} + 1000\ \text{N}\cdot\text{m} + 550\ \text{N}\cdot\text{m} = 0$$

$$M_{Ax} = -(-1900\ \text{N}\cdot\text{m} + 1000\ \text{N}\cdot\text{m} + 550\ \text{N}\cdot\text{m})$$

$$= 350\ \text{N}\cdot\text{m}$$

The computed result for M_{Ax} is positive, so the reaction moment vector is in the positive *x* direction as assumed.

(3) Find the internal torsional moments in each segment of the shaft. Now that reaction moment M_{Ax} is known, we can cut a section through the shaft in each segment creating left and right FBDs (Fig. 1-15). Internal

Continues

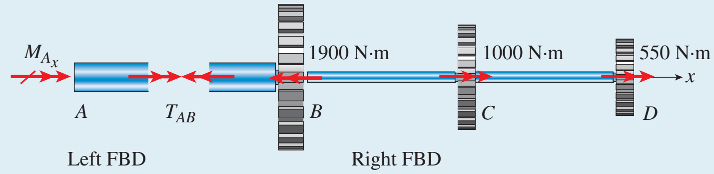
Example 1-3 - *Continued*

torsional moments then may be computed using statics. Either FBD may be used; the computed internal torsional moment will be the same.

Find the internal torque T_{AB} (Fig. 1-15a).

Fig. 1-15a

Example 1-3: Left and right FBDs of shaft for each segment



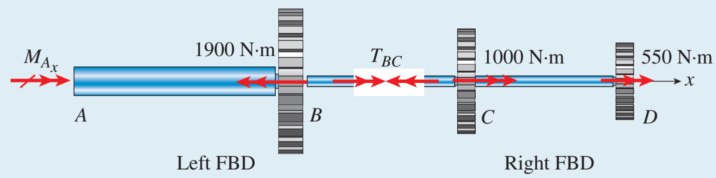
(a)

Left FBD:
$T_{AB} = -M_{Ax} = -350\ \text{N}\cdot\text{m}$

Right FBD:
$T_{AB} = -1900\ \text{N}\cdot\text{m} + 1000\ \text{N}\cdot\text{m} + 550\ \text{N}\cdot\text{m} = -350\ \text{N}\cdot\text{m}$

Find the internal torque T_{BC} (Fig. 1-15b).

Fig. 1-15b



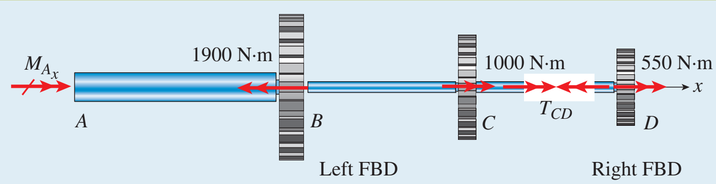
(b)

Left FBD:
$T_{BC} = -M_{Ax} + 1900\ \text{N}\cdot\text{m} = 1550\ \text{N}\cdot\text{m}$

Right FBD:
$T_{BC} = 1000\ \text{N}\cdot\text{m} + 550\ \text{N}\cdot\text{m} = 1550\ \text{N}\cdot\text{m}$

Find internal torque T_{CD} (Fig. 1-15c).

Fig. 1-15c



(c)

Left FBD:
$T_{CD} = -M_{Ax} + 1900\ \text{N}\cdot\text{m} - 1000\ \text{N}\cdot\text{m} = 550\ \text{N}\cdot\text{m}$

Right FBD:
$T_{CD} = 550\ \text{N}\cdot\text{m}$

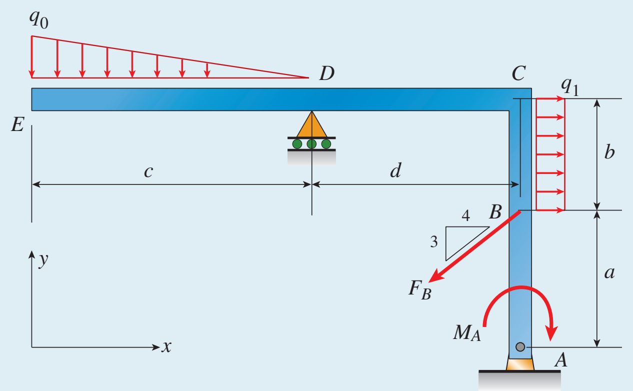
In each segment, the internal torsional moments computed using either the left or right FBDs are the same.

Example 1-4

The plane frame in Fig. 1-16 is a modified version of that shown in Fig. 1-1. Initially, member *DF* has been replaced with a roller support at *D*. Moment M_A is applied at pin-supported joint *A*, and load F_B is applied at joint *B*. A uniform load with intensity q_1 acts on member *BC*, and a linearly distributed load with peak intensity q_0 is applied downward on member *ED*. Find the support reactions at joints *A* and *D*, then solve for internal forces at the top of member *BC*. Use numerical properties given. As a final step, remove the roller at *D*, insert member *DF* (as shown in Fig. 1-1) and reanalyze the structure to find the reaction forces at *A* and *F*.

Fig. 1-16

Example 1-4: Plane frame static analysis for support reactions



Numerical data (Newtons and meters):

$$a = 3 \text{ m} \quad b = 2 \text{ m} \quad c = 6 \text{ m} \quad d = 2.5 \text{ m}$$

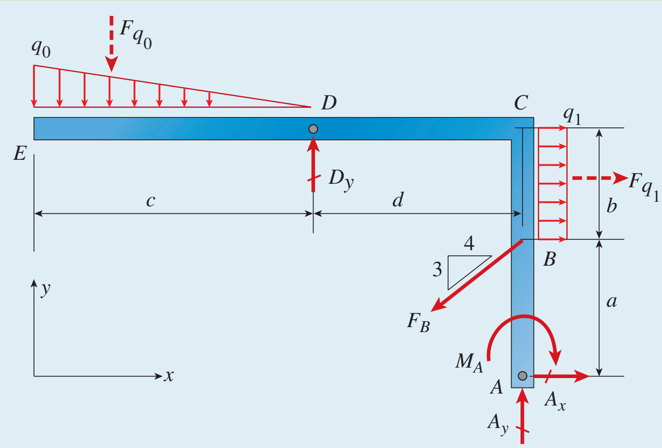
$$M_A = 380 \text{ N}\cdot\text{m} \quad F_B = 200 \text{ N} \quad q_0 = 80 \text{ N/m} \quad q_1 = 160 \text{ N/m}$$

Solution

(1) Draw the FBD of the overall frame. The solution for reaction forces at *A* and *D* must begin with a proper drawing of the FBD of the overall frame (Fig. 1-17). The FBD shows all applied and reactive forces.

Fig. 1-17

Example 1-4: FBD of plane frame



Continues

Example 1-4 - *Continued*

(2) **Determine the statically equivalent concentrated forces.** Distributed forces are replaced by their statical equivalents (F_{q0} and F_{q1}). The components of the inclined concentrated force at B also may be computed:

$$F_{q0} = \frac{1}{2} q_0 c = 240 \text{ N} \qquad F_{q1} = q_1 b = 320 \text{ N}$$

$$F_{Bx} = \frac{4}{5} F_B = 160 \text{ N} \qquad F_{By} = \frac{3}{5} F_B = 120 \text{ N}$$

(3) **Sum the moments about A to find reaction force D_y.** This structure is statically determinate because there are three available equations from statics ($\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$) and three reaction unknowns (A_x, A_y, D_y). It is convenient to start the static analysis using $\Sigma M_A = 0$, because we can isolate one equation with one unknown and then easily find reaction D_y.

$$D_y = \frac{1}{d}\left[-M_A + F_{Bx}a - F_{q1}\left(a + \frac{b}{2}\right) + F_{q0}\left(d + \frac{2}{3}c\right)\right] = 152 \text{ N}$$

(4) **Sum the forces in the x and y directions to find the reaction forces at A.** Now that D_y is known, we can find A_x and A_y using $\Sigma F_x = 0$ and $\Sigma F_y = 0$, and then find the resultant reaction force at A using components A_x and A_y.

Sum forces in x direction: $A_x - F_{Bx} + F_{q1} = 0 \quad A_x = F_{Bx} - F_{q1}$

$$A_x = -160 \text{ N}$$

Sum forces in y direction: $A_y - F_{By} + D_y - F_{q0} = 0 \quad A_y = F_{By} - D_y + F_{q0}$

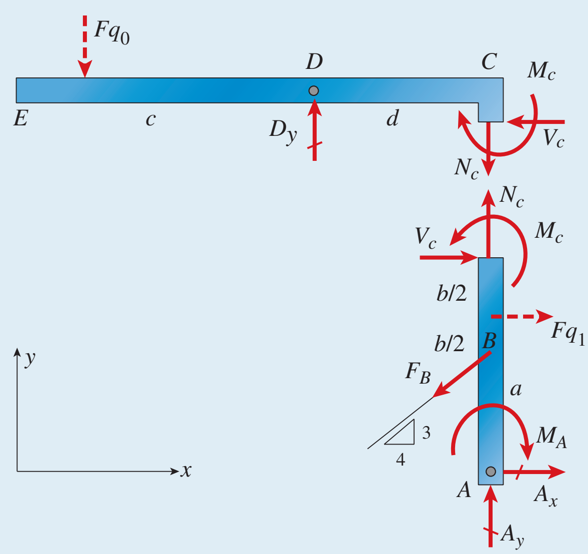
$$A_y = 208 \text{ N}$$

Resultant force at A: $A = \sqrt{A_x^2 + A_y^2} \qquad A = 262 \text{ N}$

(5) **Find the internal forces and moment at the top of member BC.** Now that reaction forces at A and D are known, we can cut a section through the frame just below joint C, creating upper and lower FBDs (Fig. 1-18).

Fig. 1-18

Example 1-4: Upper and lower FBDs of plane frame



Section forces N_c (axial) and V_c (shear) as well as section moment (M_c) are exposed and may be computed using statics. Either FBD may be used to find N_c, V_c, and M_c; the computed stress resultants N_c, V_c, and M_c will be the same.

Calculations based on ***upper FBD***:

$$\Sigma F_x = 0 \qquad V_c = 0$$

$$\Sigma F_y = 0 \qquad N_c = D_y - F_{q0} = -88 \text{ N}$$

$$\Sigma M_c = 0$$

$$M_c = -D_y d + F_{q0}\left(d + \frac{2}{3}c\right) = 1180 \text{ N}\cdot\text{m}$$

Calculations based on ***lower FBD***:

$$\Sigma F_x = 0 \qquad V_c = -F_{q1} + F_{Bx} - A_x = 0$$

$$\Sigma F_y = 0 \qquad N_c = F_{By} - A_y = -88 \text{ N}$$

$$\Sigma M_c = 0$$

$$M_c = -F_{q1}\frac{b}{2} + F_{Bx}b - A_x(a + b) + M_A = 1180 \text{ N}\cdot\text{m}$$

(6) Remove the roller at *D*, insert member *DF* (as shown in Fig. 1-1) and reanalyze the structure to find reaction forces at *A* and *F*. Member *DF* is pin-connected to *EDC* at *D*, has a pin support at *F*, and carries uniform load q_2 in the $-y$ direction. See Figs. 1-3a and 1-3b for the FBDs required in the solution. Note that there are now four unknown reaction forces (A_x, A_y, F_x and F_y) but only three equilibrium equations available ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$) for use with the overall FBD in Fig. 1-3a. To find another equation, we will have to separate the structure at pin connection *D* to take advantage of the fact that the moment at *D* is known to be zero (friction effects are assumed to be negligible); we can then use $\Sigma M_D = 0$ for either the upper FBD or the lower FBD in Fig. 1-3b to develop one more independent equation of statics. Recall that a ***statics sign convention*** is used here for all equilibrium equations.
Dimensions and loads for new member *DF*:

$$e = 5 \text{ m} \quad e_x = \frac{3}{5}e = 3 \text{ m} \quad e_y = \frac{4}{5}e = 4 \text{ m}$$

$$q_2 = 180 \text{ N/m} \quad F_{q2} = q_2 e = 900 \text{ N}$$

First, write equilibrium equations for the entire structure FBD (see Fig. 1-3a).

(a) Sum forces in *x* direction **for entire FBD**:

$$A_x + F_x - F_{Bx} + F_{q1} = 0 \qquad \textbf{(a)}$$

(b) Sum forces in *y* direction **for entire FBD**:

$$A_y + F_y - F_{q0} - F_{q2} - F_{By} = 0 \qquad \textbf{(b)}$$

Continues

Example 1-4 - *Continued*

(c) Sum moments about A **for entire FBD**:

$$-M_A - F_{q1}\left(a + \frac{b}{2}\right) - F_x(a + b + e_y) + F_y(e_x - d) + F_{Bx}a + F_{q0}\left(d + \frac{2}{3}c\right) + F_{q2}\left(d - \frac{e_x}{2}\right) = 0$$

$$\text{so} \quad -F_x(a + b + e_y) + F_y(e_x - d) = M_A + F_{q1}\left(a + \frac{b}{2}\right) - \left[F_{Bx}a + F_{q0}\left(d + \frac{2}{3}c\right) + F_{q2}\left(d - \frac{e_x}{2}\right)\right] \qquad \textbf{(c)}$$

Next, write another equilibrium equation for the upper FBD in Fig. 1-3b.

(d) Sum moments about D **on upper FBD**:

$$-F_xe_y + F_ye_x - F_{q2}\frac{e_x}{2} = 0 \quad \text{so} \quad -F_xe_y + F_ye_x = F_{q2}\frac{e_x}{2} \qquad \textbf{(d)}$$

Solving Eqs. (c) and (d) for F_x and F_y, we have

$$\begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{bmatrix} -(a + b + e_y) & e_x - d \\ -e_y & e_x \end{bmatrix}^{-1}$$

$$\begin{bmatrix} M_A + F_{q1}\left(a + \frac{b}{2}\right) - \left[F_{Bx}a + F_{q0}\left(d + \frac{2}{3}c\right) + F_{q2}\left(d - \frac{e_x}{2}\right)\right] \\ F_{q2}\frac{e_x}{2} \end{bmatrix} = \begin{pmatrix} 180.6 \\ 690.8 \end{pmatrix} \text{N}$$

Now, substitute solutions for F_x and F_y into Eqs. (a) and (b) to find reactions A_x and A_y:

$A_x = -(F_x - F_{Bx} + F_{q1})$ $\qquad A_x = -340.6$ N

$A_y = -F_y + F_{q0} + F_{q2} + F_{By}$ $\qquad A_y = 569.2$ N

The resultant force at A is $A = \sqrt{A_x^2 + A_y^2}$ $\qquad A = 663$ N

Sum the moments about D **on lower FBD** as a check; the lower FBD is in equilibrium as required:

$$F_{q0}\left(\frac{2}{3}c\right) + F_{q1}\frac{b}{2} - F_{Bx}b - F_{By}d - M_A + A_x(a + b) + A_yd = 0$$

(7) Finally, compute the resultant force in the pin connection at D. Use the summation of forces in the upper FBD to find component forces D_x and D_y, then find the resultant D (see Fig. 1-3b).

$\Sigma F_x = 0$ $\qquad D_x = -F_x = -180.6$ N

$\Sigma F_y = 0$ $\qquad D_y = -F_y + F_{q2} = 209.2$ N

The resultant force at D is $D = \sqrt{D_x^2 + D_y^2} = 276$ N.

1.3 NORMAL STRESS AND STRAIN

Now that statical equilibrium has been established and we have computed all required reaction forces and internal forces associated with the deformable body, we are ready to examine internal actions more closely. The most fundamental concepts in mechanics of materials are **stress** and **strain**. These concepts can be illustrated in their most elementary form by considering a prismatic bar subjected to axial forces. A **prismatic bar** is a straight structural member having the same cross section throughout its length, and an **axial force** is a load directed along the axis of the member, resulting in either tension or compression in the bar. Examples are shown in Fig. 1-19, where the tow bar is a prismatic member in tension and the landing gear strut is a member in compression. Other examples are the members of a bridge truss, connecting rods in automobile engines, spokes of bicycle wheels, columns in buildings, and wing struts in small airplanes.

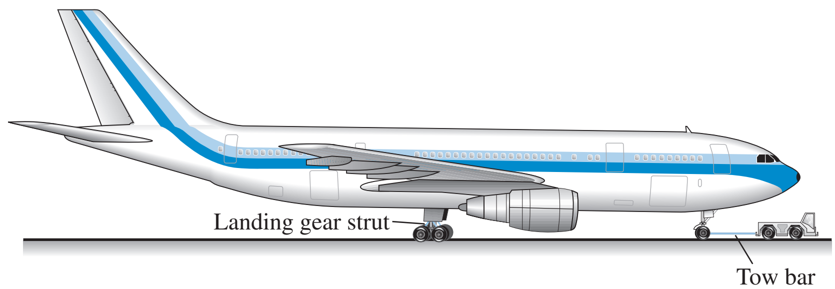


Fig. 1-19

Structural members subjected to axial loads (the tow bar is in tension and the landing gear strut is in compression)

For discussion purposes, we will consider the tow bar of Fig. 1-19 and isolate a segment of it as a free body (Fig. 1-20a). When drawing this free-body diagram, we disregard the weight of the bar itself and assume that the only active forces are the axial forces P at the ends. Next we consider two views of the bar, the first showing the same bar *before* the loads are applied (Fig. 1-20b) and the second showing it *after* the loads are applied (Fig. 1-20c). Note that the original length of the bar is denoted by the letter L, and the increase in length due to the loads is denoted by the Greek letter δ (delta).

The internal actions in the bar are exposed if we make an imaginary cut through the bar at section *mn* (Fig. 1-20c). Because this section is taken perpendicular to the longitudinal axis of the bar, it is called a **cross section**.

We now isolate the part of the bar to the left of cross section *mn* as a free body (Fig. 1-20d). At the right-hand end of this free body (section *mn*) we show the action of the removed part of the bar (that is, the part to the right of section *mn*) upon the part that remains. This action consists of continuously distributed *stresses* acting over the entire cross section, and the axial force P acting at the cross section is the *resultant* of those stresses. (The resultant force is shown with a dashed line in Fig. 1-20d.)